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Exponential behavior of neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process

Abstract: In this article, we are concerned with the neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process. By using resolvent operator and some analysis techniques, we ensure existence and uniqueness of solutions. Further, we investigate exponential stability of mild solutions. We have also given an example to illustrate our theoretical results.

Keywords: Existence results, Stability, Poisson jumps, Rosenblatt process, Resolvent operator

MSC: 60H15, 35R60

1 Introduction

In the past decades, the theory of nonlinear functional differential or integro-differential equations with resolvent operators has become an active research field due to their applications in many physical phenomena. The resolvent operator is comparable to the semigroup operator for abstract differential equations in Banach spaces. However, the resolvent operator does not satisfy semigroup properties. The study of deterministic neutral functional differential equations was initiated by Hale and Mayer [17]. For more details on the theory and their applications, we also refer the readers to Hale and Lunel [18], Kolmanovkii and Nosov [20] and so on. Deterministic and stochastic differential equations have gained great popularity in the last few years due to their use in many areas, such as physics, electronics, control theory, engineering and economics. Several authors have considered the existence, uniqueness and asymptotic behavior of mild solutions, and many important theory and applications findings have been obtained. For more details we refer to the papers by Ali et al. [4], El-Borai et al. [11], Gorec and Sathanantham [12], Gupta and Dabas [15], Gupta and al. [16], Laksmikantha [22], Ahmed [2], Ahmed et al. [3], Arthi and Balachandran [5],Gupta et al. [14], Levin et al. [24].

On the other side, fractional Brownian motion was intensively explored due to their applications in various domain. We point out that, a fractional Brownian motion (fBm) of Hurst index $H \in (0, 1)$ (see [21, 26] for

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more details) is a Gaussian process $B^H = \{B^H(t),\ t \geq 0\}$, centered, with the covariance function

$$R_H(t, s) = \mathbb{E}\left(B^H(t)B^H(s)\right) = \frac{1}{2}\left(t^{2H} + s^{2H} - |t - s|^{2H}\right).$$

We also mention that fBm is not a semimartingale and when $H = 1/2$, the fBm becomes standard Brownian motion. Further, if $H > 1/2$, the fBm B^H have a long-memory and this property makes it an ideal process to modeling in biology, mathematics finance [7, 8] etc.

Moreover, the fBm belongs to Hermite family processes, it's selfsimilar which being defined as limits that appear in the so-called Non-Central Theorem. For $d \geq 1$, they have the following representation

$$Y_t^{H,d} = c(H_0) \int_{\mathbb{R}} \dots \int_{\mathbb{R}} \left(\int_0^t \prod_{j=1}^d (s - x_j)_+^{H_0-1}\, ds\right) dBx_1 \dots dBx_d,\ \ \forall t > 0,$$

where $\{B_{x_i} : x_i \in \mathbb{R}, i = 1, \dots, d\}$ are some two-sided Brownian motions, $c(H_0)$ is a normalizing constant such that

$$\mathbb{E}|Y_1^{H,d}|^2 = 1,\ \ H_0 = \frac{1}{2} + \frac{H-1}{d},\ \ H \in (\frac{1}{2}, 1).$$

When $d = 1$ the process become a fractional Brownian motion, thus Taqqu [32] have named the Rosenblatt process when $d = 2$, which is not Gaussian process but they have stationary increments and long range-dependency. Recently, the Rosenblatt process have attracted attention of many authors due to their properties. For example, Meajima and Tudor [29], Veillette and Taqqu [36] have given many important properties of distributions, Bernet and Tudor [6], Viens and Tudor [35] established the construction of estimator for the self-similarity parameter H.

Based on the above works, we investigate the following neutral stochastic functional integro-differential equations with delay and impulses effects

$$\begin{cases} d\left[u(t) - q(t, u_t)\right] &= \left[A\left(u(t) - q(t, u_t)\right) + \int_0^t Y(t - s)\left[u(s) - q(s, u_s)\right] ds + f(t, u_t)\right. \\ &\quad \left. + \int_0^t g(t, s, u_s)ds\right]dt + \int_\theta h(t, u_t, v)\widetilde{N}(dt, dv) + \sigma(t)dZ_Q^H(t),\ t \in [0, T],\ t \neq t_k, \quad (1) \\ \Delta u(t) &= I_k(u(t_k^-)),\ \ t = t_k, k = 1, 2, \cdots, \\ u_0(t) &= \phi(t) \in \mathcal{PC}\left([-\tau, 0], \mathbb{H}\right),\ \ -\tau \leq t \leq 0, \end{cases}$$

where $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ is the infinitesimal generator of a C_0-semigroup $(T(t))_{t\geq0}$ of bounded linear operators in a Hilbert space $\mathbb{H}$; for $t \geqslant 0$, $Y(t)$ a closed linear operator on $\mathbb{H}$, with $D(A) \subset D(Y)$. The impulsive moments t_k satisfy the condition $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, $I_k : \mathbb{H} \to \mathbb{H}$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, $u(t_k^+)$ and $u(t_k^-)$ are the right and left limits of $u(t)$ at t_k, respectively which is the jump size of the state u at t_k. For $\phi \in \mathcal{PC}$, $||\phi|| = \sup_{s \in [-r,0]} ||\phi(s)|| < \infty$, where $\mathcal{PC} = \{\phi : [-r, 0] \to \mathbb{H}, \phi(t)$ is continuous everywhere except a finite number of points $\tilde{t}$ at which $\phi(\tilde{t}_k^-)$, $\phi(\tilde{t}_k^+)$ exist and $\phi(\tilde{t}_k^-) = \phi(\tilde{t})\}$.

For any $t \in [0, T]$ and any continuous function u, the element of $\mathcal{PC}$ is defined by $u_t(\theta) = u(t + \theta), -\tau \leq \theta \leq 0$. The functions $q, f : [0, +\infty) \times \mathcal{PC} \to \mathbb{H}$, $g : [0, +\infty) \times [0, +\infty) \times \mathcal{PC} \to \mathbb{H}$, $h : [0, +\infty) \times \mathbb{H} \times \theta \to \mathbb{H}$ and $\sigma : [0, +\infty) \to L_Q^0(Y, \mathbb{H})$ are appropriate functions, and Z_Q^H is assumed to be a Rosenblatt process.

The particular case $Y(t) = 0$ and $h = 0$ of Eq. (1) has been considered by Ma et al [25], where the authors used an impulsive integral inequality to prove their result. It should be mentioned that there is no work yet reported on the exponential stability of neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process. Motivated by this facts, our main objective is to study the exponential stability for a class of neutral impulsive stochastic integro-differential equations(1). In this paper, we derive existence and exponential results for the system (1) with the help of resolvent operator and fixed point techniques.

In the first result, we obtain the sufficient conditions proving existence and uniqueness of the mild solution

of (1) by utilizing Banach fixed point theorem under Lipschitz conditions on nonlinear terms. While in the second result, we have proved the exponential stability of mild solution via an integral inequality. Our article, expands the usefulness of stochastic integro-differential equations, since the literature shows results for existence and exponential stability for such equations in the case of semigroup only (see [2, 3, 11, 14, 16] and the references therein). The results obtained improve, extend and complete many other important ones in the literature.

The following is the organization of this paper. We recall some preliminary definitions and outcomes in Section 2. Section 3 is devoted to investigate existence and uniqueness of mild solution. The exponential stability for the mild solution is also discussed. We give an example in the fourth section to illustrate the results. The last section is dedicated to conclude this paper.

2 Preliminaries

In this section, we provided some basic results about Poisson process, Resolvent operator and Rosenblatt process.

2.1 poisson jumps process

Let $\mathcal{B}_\sigma(\mathcal{H})$ the Borel σ-algebra of $\mathcal{H}$. Let $(p(t))$, $(t \geq 0)$ be an $\mathcal{H}$-valued, σ-finite stationary $\Im_t$-adapted Poisson point process on $(\Omega, \Im, \mathbb{P})$. The counting random measure N defined by

$$N((t_1, t_2] \times \theta)(w) \quad = \quad \sum_{t_1 < s \leq t_2} 1_{\mathcal{U}}(p(s)(w)),$$

for any $\mathcal{U} \in \mathcal{B}_\sigma(\mathcal{H} - \{0\})$. where $0 \notin \bar{\theta}$ is called the Poisson random measure associated to Poisson point process p. The following notation is used

$$N(t, \theta) \quad = \quad N((0, t] \times \theta).$$

Then it is known that there exists a σ-finite measure θ such that

$$\mathbb{E}(N(t, \theta)) \quad = \quad \nu(\theta)t,$$
$$\mathbf{P}(N(t, \theta) = k) \quad = \quad \frac{\exp(-t\nu(\theta))(t\nu(\theta))^k}{k!}.$$

This measure ν is said Levy measure. Then the measure $\widetilde{N}$ is defined by

$$\widetilde{N}((0, t] \times \theta) \quad = \quad N((0, t] \times \theta) - t\nu(\theta).$$

This measure $\widetilde{N}(dt, du)$ is called the compensated Poisson random measure, and $dt\nu(\theta)$ is called the compensator (see [30]).

Definition 2.1. *Let $\theta \in \mathcal{B}_\sigma(\mathcal{H} - \{0\})$. $P^2([0, T] \times \theta; \mathcal{H})$ is the space of all predictable mappings $h : [0, T] \times \theta \times \Omega \to \mathcal{H}$ for which*

$$\int_0^T \int_\theta \mathbb{E} \left\| h(t, v) \right\|^2 dt\lambda(dv) < \infty.$$

We may then define the $\mathcal{H}$-valued stochastic integral $\int_0^T \int_\theta L(t, v)\widetilde{N}(dt, dv)$, which is a centered square-integral martingale [30].

2.2 Rosenblatt process

In this subsection, we recall some basic concepts on the Rosenblatt process as well as the Wiener integral with respect to it. Consider $(\xi_n)_{n\in Z}$ a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that $R(n) := \mathbb{E}(\xi_0\xi_n) = n^{\frac{2H-2}{k}}L(n)$, with $H \in (\frac{1}{2}, 1)$ and L is a slowly varying function at infinity. Let g be a function of Hermite rank k, that is, if g admits the following expansion in Hermite polynomials

$$g(x) = \sum_{j\geq 0} c_j H_j(x), \quad c_j = \frac{1}{j!}\mathbb{E}(g(\xi_0 H_j(\xi_0))),$$

then $k = \min\{j|c_j \neq 0\} \geq 1$, where $H_j(x)$ is the Hermite polynomial of degree j given by $H_j(x) = (-1)^j e^{\frac{x^2}{2}}\frac{d^j}{dx^j}e^{-\frac{x^2}{2}}$. Then, the Non-Central Limit Theorem (see, for example, Dobrushin & Major [10]) says $\frac{1}{n^H}\sum_{j=1}^{[nt]}g(\xi_j)$ converges as $n \to \infty$, in the sense of finite dimensional distributions, to the process

$$Z_H^k(t) = c(H, k)\int_{\mathbb{R}^k}\int_0^t \left(\prod_{j=1}^k (s - y_j)_+^{\left(-\frac{1}{2}+\frac{1-H}{k}\right)}\right) ds\,dB(y_1)\cdots dB(y_k), \tag{2}$$

where the above integral is a Wiener-Itô multiple integral of order k with respect to the standard Brownian motion $(B(y))_{y\in\mathbb{R}}$ and $c(H, k)$ is a positive normalization constant depending only on H and k. The process $(Z_H^k(t))_{t\geq 0}$ is called as the Hermite process and it is H self-similar in the sense that for any $c > 0$, $(Z_H^k(ct)) \stackrel{d}{=} (c^H Z_H^k(t))$ and it has stationary increments.

The fractional Brownian motion (which is obtained from (2) when $k = 1$) is the most used Hermite process to study evolution equations due to its large range of applications. When $k = 2$ in (2), Taqqu [32] named the process as the Rosenblatt process. The stationarity of increments, self-similarity and long range dependence (see Tindel, Tudor and Viens [33]) were made that the Rosenblatt process is very important in practical applications. However, it is noted that Rosenblatt process is not Gaussian. In fact, due to their properties (long range dependence, self-similarity), the fractional Brownian motion process has large utilization in practical models, for instance in telecommunications and hydrology. So, many researchers prefer to use fractional Brownian motion than other processes because it is Gaussian and it facilitate calculations. However in concrete situations when the Gaussianity is not plausible for the model, one can use the Rosenblatt process. In recent years, there exists many works that investigated on diverse theoretical aspects of the Rosenblatt process. For example, Leonenko and Ahn [23] gave the rate of convergence to the Rosenblatt process in the Non-Central Limit Theorem and the wavelet-type expansion has been presented by Abry and Pipiras [1]. Tudor [34] established, the representation as a Wiener-Itô multiple integral with respect to the Brownian motion on a finite interval and developed the stochastic calculus with respect to it by using both pathwise type calculus and Malliavin calculus (see also Maejima and Tudor [27]). For more details on Rosenblatt process, we refer the reader to Maejima and Tudor [28, 29]), Pipiras and Taqqu [31] and the references therein.

Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{Z_H(t), t \in [0, T]\}$ be a one-dimensional Rosenblatt process with parameter $H \in (\frac{1}{2}, 1)$. According to the work of Tudor [34], the Rosenblatt process with parameter $H > \frac{1}{2}$ can be written as

$$Z_H(t) = d(H)\int_0^t\int_0^t\left[\int_{y_1 \vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u, y_1)\frac{\partial K^{H'}}{\partial u}(u, y_2)du\right]dB(y_1)dB(y_2), \tag{3}$$

where $K^H(t, s)$ is given by

$$K^H(t, s) = c_H s^{\frac{1}{2}-H}\int_s^t (u - s)^{H-3/2}u^{H-1/2}du \text{ for } t > s,$$

with

$$c_H = \sqrt{\frac{H(2H - 1)}{\beta(2 - 2H, H - \frac{1}{2})}},$$

$\beta(.,.)$ denotes the Beta function, $K^H(t, s) = 0$ when $t \leq s$, $(B(t), t \in [0, T])$ is a Brownian motion, $H' = \frac{H+1}{2}$ and $d(H) = \frac{1}{H+1} \sqrt{\frac{H}{2(2H-1)}}$ is a normalizing constant. The covariance of the Rosenblatt process $\{Z_H(t), t \in [0, T]\}$ satisfy

$$\mathbb{E}(Z_H(t)Z_H(s)) = \frac{1}{2}\left(s^{2H} + t^{2H} - |s - t|^{2H}\right).$$

The covariance structure of the Rosenblatt process allows to construct Wiener integral with respect to it. We refer to Maejima and Tudor [27] for the definition of Wiener integral with respect to general Hermite processes and to Kruk, Russo, and Tudor [19] for a more general context (see also Tudor [34]).

Note that

$$Z_H(t) = \int_0^T \int_0^T I(1_{[0,t]})(y_1, y_2) dB(y_1) dB(y_2),$$

where the operator I is defined on the set of functions $f : [0, T] \to \mathbb{R}$, which takes its values in the set of functions $g : [0, T]^2 \to \mathbb{R}^2$ and is given by

$$I(f)(y_1, y_2) = d(H) \int_{y_1 \vee y_2}^T f(u) \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2) du.$$

Let f be an element of the set $\mathcal{E}$ of step functions on $[0, T]$ of the form

$$f = \sum_{i=0}^{n-1} a_i 1_{(t_i, t_{i+1}]}, \; t_i \in [0, T].$$

Then, it is natural to define its Wiener integral with respect to Z_H as

$$\int_0^T f(u) dZ_H(u) := \sum_{i=0}^{n-1} a_i(Z_H(t_{i+1}) - Z_H(t_i)) = \int_0^T \int_0^T I(f)(y_1, y_2) dB(y_1) dB(y_2).$$

Let $\mathcal{H}$ be the set of functions f such that

$$\|f\|_{\mathcal{H}}^2 := 2 \int_0^T \int_0^T (I(f)(y_1, y_2))^2 dy_1 dy_2 < \infty.$$

It follows that (see Tudor[34])

$$\|f\|_{\mathcal{H}}^2 = H(2H - 1) \int_0^T \int_0^T f(u)f(v)|u - v|^{2H-2} du dv.$$

It has been proved in Maejima and Tudor [27] that the mapping

$$f \to \int_0^T f(u) dZ_H(u)$$

defines an isometry from $\mathcal{E}$ to $L^2(\Omega)$ and it can be extended continuously to an isometry from $\mathcal{H}$ to $L^2(\Omega)$ because $\mathcal{E}$ is dense in $\mathcal{H}$. We call this extension as the Wiener integral of $f \in \mathcal{H}$ with respect to Z_H. It is noted that the space $\mathcal{H}$ contains not only functions but its elements could be also distributions. Therefore it is suitable to know subspaces $|\mathcal{H}|$ of $\mathcal{H} : |\mathcal{H}| = \left\{f : [0, T] \to \mathbb{R} | \int_0^T \int_0^T |f(u)||f(v)||u - v|^{2H-2} du dv < \infty\right\}$. The space $|\mathcal{H}|$ is not complete with respect to the norm $\|.\|_{\mathcal{H}}$ but it is a Banach space with respect to the norm

$$\|f\|_{|\mathcal{H}|}^2 = H(2H - 1) \int_0^T \int_0^T |f(u)||f(v)||u - v|^{2H-2} du dv.$$

As a consequence, we have

$$L^2([0, T]) \subset L^{1/H}([0, T]) \subset |\mathcal{H}| \subset \mathcal{H}.$$

For any $f \in L^2([0, T])$, we have

$$\|f\|_{|\mathcal{H}|}^2 \le 2HT^{2H-1} \int_0^T |f(s)|^2 ds$$

and

$$\|f\|_{|\mathcal{H}|}^2 \le C(H)\|f\|_{L^{1/H}([0,T])}^2, \tag{4}$$

for some constant $C(H) > 0$. Let $C(H) > 0$ stands for a positive constant depending only on H and its value may be different in different appearances.

Define the linear operator $K_H^\star$ from $\mathcal{E}$ to $L^2([0, T])$ by

$$(K_H^\star f)(y_1, y_2) = \int_{y_1 \vee y_2}^T f(t) \frac{\partial \mathcal{K}}{\partial t}(t, y_1, y_2) dt,$$

where $\mathcal{K}$ is the kernel of Rosenblatt process in representation (3)

$$\mathcal{K}(t, y_1, y_2) = 1_{[0,t]}(y_1)1_{[0,t]}(y_2) \int_{y_1 \vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u, y_1)\frac{\partial K^{H'}}{\partial u}(u, y_2) du.$$

Note that $(K_H^\star 1_{[0,t]})(y_1, y_2) = \mathcal{K}(t, y_1, y_2)1_{[0,t]}(y_1)1_{[0,t]}(y_2)$. The operator $K_H^\star$ is an isometry between $\mathcal{E}$ to $L^2([0, T])$, which can be extended to the Hilbert space $\mathcal{H}$. In fact, for any $s, t \in [0, T]$ we have

$$
\begin{aligned}
\left\langle K_H^\star 1_{[0,t]}, K_H^\star 1_{[0,s]} \right\rangle_{L^2([0,T])} &= \left\langle \mathcal{K}(t, ., .)1_{[0,t]}, \mathcal{K}(s, ., .)1_{[0,s]} \right\rangle_{L^2([0,T])} \\
&= \int_0^{t \wedge s} \int_0^{t \wedge s} \mathcal{K}(t, y_1, y_2)\mathcal{K}(s, y_1, y_2) dy_1 dy_2 \\
&= H(2H-1) \int_0^t \int_0^s |u - v|^{2H-2} du dv \\
&= \left\langle 1_{[0,t]}, 1_{[0,s]} \right\rangle_{\mathcal{H}}.
\end{aligned}
$$

Moreover, for $f \in \mathcal{H}$, we have

$$Z_H(f) = \int_0^T \int_0^T (K_H^\star f)(y_1, y_2) dB(y_1) dB(y_2).$$

Let $\{Z_n(t)\}_{n \in \mathbb{N}}$ be a sequence of two-sided one dimensional Rosenblatt process mutually independent on $(\Omega, \mathcal{F}, \mathbb{P})$. We consider a K-valued stochastic process $Z_Q(t)$ given by the following series:

$$Z_Q(t) = \sum_{n=1}^\infty z_n(t)Q^{1/2}e_n, \quad t \ge 0.$$

Moreover, if Q is a non-negative self-adjoint trace class operator, then this series converges in the space K, that is, it holds that $Z_Q(t) \in L^2(\Omega, K)$. Then, we say that the above $Z_Q(t)$ is a K-valued Q- Rosenblatt process with covariance operator Q. For instance, if $\{\sigma_n\}_{n \in \mathbb{N}}$ is a bounded sequence of non-negative real numbers such that $Qe_n = \sigma_n e_n$, by assuming that Q is a nuclear operator in K, then the stochastic process

$$Z_Q(t) = \sum_{n=1}^\infty z_n(t)Q^{1/2}e_n = \sum_{n=1}^\infty \sqrt{\sigma_n}z_n(t)e_n, \quad t \ge 0, \tag{5}$$

is well-defined as a K-valued Q- Rosenblatt process.

Definition 2.2. *(Tudor[34]). Let* $\varphi : [0, T] \to L_Q^0(K, \mathbb{H})$ *such that* $\sum_{n=1}^{\infty} \|K_H^{\star}(\varphi Q^{1/2} e_n)\|_{L^2([0,T];\mathbb{H})} < \infty$. *Then, its stochastic integral with respect to the Rosenblatt process* $Z_Q(t)$ *is defined, for* $t \geq 0$, *as follows :*

$$\int_0^t \varphi(s) dZ_Q(s) := \sum_{n=1}^{\infty} \int_0^t \varphi(s) Q^{1/2} e_n dz_n(s) = \sum_{n=1}^{\infty} \int_0^t \int_0^t (K_H^{\star}(\varphi Q^{1/2} e_n))(y_1, y_2) dB(y_1) dB(y_2). \quad (6)$$

Lemma 2.1. *For* $\psi : [0, T] \to L_Q^0(K, \mathbb{H})$ *such that* $\sum_{n=1}^{\infty} \|\psi Q^{1/2} e_n\|_{L^{1/H}([0,T];\mathbb{H})} < \infty$ *holds, and for any* $a, b \in [0, T]$ *with* $b > a$, *we have*

$$\mathbb{E} \left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 \leq c(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \|\psi(s) Q^{1/2} e_n\|^2 ds.$$

If, in addition,

$$\sum_{n=1}^{\infty} \|\psi(t) Q^{1/2} e_n\| \text{ is uniformly convergent for } t \in [0, T],$$

then, it holds that

$$\mathbb{E} \left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 \leq C(H)(b-a)^{2H-1} \int_a^b \|\psi(s)\|^2_{L_Q^0(K,\mathbb{H})} ds.$$

Proof. Let $\{e_n\}_{n \in \mathbb{N}}$ be the complete orthonormal basis of K introduced above. Applying (6) and Hölder inequality, we have

$$
\begin{aligned}
\mathbb{E} \left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 &= \mathbb{E} \left\| \sum_{n=1}^{\infty} \int_a^b \psi(s) Q^{1/2} e_n dz_n(s) \right\|^2 \\
&= \sum_{n=1}^{\infty} \mathbb{E} \left\| \int_a^b \psi(s) Q^{1/2} e_n dz_n(s) \right\|^2 \\
&= \sum_{n=1}^{\infty} H(2H-1) \int_a^b \int_a^b \|\psi(s) Q^{1/2} e_n\| \|\psi(t) Q^{1/2} e_n\| |t-s|^{2H-2} ds dt \\
&\leq C(H) \sum_{n=1}^{\infty} \left(\int_a^b \|\psi(s) Q^{1/2} e_n\|^{1/H} ds \right)^{2H} \\
&\leq C(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \|\psi(s) Q^{1/2} e_n\|^2 ds.
\end{aligned}
$$

$\square$

Lemma 2.2. *[37] Let a function* $\Gamma : [-r, +\infty) \to [0, +\infty)$ *be such that there exist positive constants* $\omega > 0$, $\alpha_j (j = 1, 2, 3)$, *and* $\beta_i (i = 1, 2, \cdots)$ *such that*

$$
\Gamma(t) \leq
\begin{cases}
\alpha_1 e^{-\omega t} \text{ for } t \in [-r, 0], \\
\\
\alpha_1 e^{-\omega t} + \alpha_2 \sup_{\theta \in [-r,0]} \Gamma(t + \theta) + \alpha_3 \int_0^t e^{-\omega(t-s)} \sup_{\theta \in [-r,0]} \Gamma(t + \theta) ds \\
\\
+ \sum_{t_i < t} \beta_i e^{-\omega(t-t_i)} \Gamma(t_i^-) \text{ for } t \geq 0.
\end{cases}
$$

If $\alpha_2 + \frac{\alpha_3}{\omega} + \sum_{i=1}^{+\infty} \beta_i < 1$, the $\Gamma(t) \leq Ne^{-\gamma t}$ for $t \geq -r$, where $\Gamma > 0$ is the unique solution to the equation

$$\alpha_2 + \frac{\alpha_3}{(\omega-\gamma)}e^{\gamma r} + \sum_{i=1}^{+\infty} \beta_i = 1 \text{ and } N = \max\{\alpha_1, \frac{\alpha_1(\omega-\gamma)}{\alpha_3 e^{\gamma r}}\}) > 0.$$

2.3 Partial integro-differential equations in Banach spaces

In this section, we recall some fundamental results needed to establish our main results. For the theory of resolvent operators we refer the reader to [13]. Throughout this paper, $\mathbb{H}$ is a Banach space, A and $Y(t)$ are closed linear operators on $\mathbb{H}$. Y represents the Banach space $D(A)$ equipped with the graph norm defined by

$$|y|_Y := |Ay| + |y| \quad \text{for } y \in Y.$$

The notations $C([0, +\infty); Y)$, $\mathcal{B}(Y, \mathbb{H})$ stand for the space of all continuous functions from $[0, +\infty)$ into Y, the set of all bounded linear operators from Y into $\mathbb{H}$, respectively. We consider the following Cauchy problem

$$\begin{cases} \rho'(t) &= A\rho(t) + \int_0^t Y(t-s)\rho(s)ds, \quad \text{for } t \geq 0, \\ \rho(0) &= v_0 \in \mathbb{H}. \end{cases} \tag{7}$$

Definition 2.3. *([13]). A resolvent operator for Eq. (7) is a bounded linear operator valued function $\Pi(t) \in \mathcal{L}(H)$ for $t \geq 0$, satisfying the following properties:*

(i) $\Pi(0) = I$ and $|\Pi(t)| \leq Me^{\beta t}$ for some constants M and β.
(ii) For each $x \in \mathbb{H}$, $\Pi(t)x$ is strongly continuous for $t \geq 0$.
(iii) $\Pi(t) \in \mathcal{L}(Y)$ for $t \geq 0$. For $x \in Y$, $\Pi(\cdot)x \in C^1([0, +\infty); \mathbb{H}) \cap C([0, +\infty); Y)$ and

$$\begin{aligned} \Pi'(t)x &= A\Pi(t)x + \int_0^t Y(s)(t-s)\Pi(s)xds \\ &= \Pi(t)Ax + \int_0^t \Pi(t-s)Y(s)xds \quad \text{for } t \geq 0. \end{aligned}$$

Remark 2.1. *There exist a constant $\hat{M} > 0$ such that $||\Pi(t)|| \leq \hat{M}$, for $t \in [0, T]$.*

The following assumptions are imposed on the system under consideration:

(H1) A is the infinitesimal generator of a strongly C_0-semigroup $\{S(t)\}_{t \geq 0}$ on $\mathbb{H}$.

(H2) For all $t \geq 0$, $Y(t)$ is a closed linear operator from $D(A)$ to $\mathbb{H}$, and $Y(t) \in \mathcal{B}(Y, \mathbb{H})$. For any $y \in Y$, the map $t \to Y(t)y$ is bounded, differentiable and the derivative $t \to Y'(t)y$ is bounded and uniformly continuous on $\mathbb{R}^+$.

Theorem 2.3. *([13, Theorem 3.7]) Assume that **(H1)-(H2)** hold. Then there exists a unique resolvent operator for the Cauchy problem (7).*

3 Main Results

3.1 Existence of mild solution

In this section, we present and prove the existence and uniqueness of mild solutions of Eq.(1) by means of the theory of resolvent operator and contraction mapping principle. First of all, we begin with the definition of mild solution for Eq.(1)

Definition 3.1. *An $\mathbb{H}$-valued stochastic process $u(t)$, $t \in [-\tau, T]$, is called a mild solution of Eq.(1) if*

1. $u(\cdot) \in \mathcal{P}C([-\tau, T], L^2(\Omega, \mathbb{H}))$,
2. *for $t \in [-\tau, 0]$, $u(t) = \phi(t)$,*
3. *for $t \in [0, T]$, $u(t)$ satisfies the following integral equation:*

$$
\begin{aligned}
u(t) = \ & \Pi(t) \left[\phi(0) - q(0, \phi) \right] + q(t, u_t) \\
& + \int_0^t \Pi(t-s) f(s, u_s) ds + \int_0^t \Pi(t-s) \int_0^s g(s, \eta, u_\eta) d\eta ds \\
& + \sum_{0 < t_k < t} \Pi(t - t_k) I_k(u(t_k^-)) + \int_0^t \int_\theta \Pi(t-s) h(s, u_s, v) \widetilde{N}(ds, dv) \\
& + \int_0^t \Pi(t-s) \sigma(s) dZ_Q^H(s) \quad P - a.s.
\end{aligned}
\tag{8}
$$

In order to attain the result, we impose the following assumptions:

(H3)(i) For all $t \in [0, T]$, there exist constants $0 < K_1 < 1$ such that, for $\psi_j \in \mathcal{P}C$, $j = 1, 2$, the $\mathbb{H}$-valued function $q : [0, +\infty) \times \mathcal{P}C \to \mathbb{H}$ satisfies the condition

$$
||q(t, \psi_1) - q(t, \psi_2)|| \leq K_1 ||\psi_1 - \psi_2||.
$$

Also, $\tilde{K}_1 = \sup_{t \in [0,T]} ||q(t, 0)||$.

(ii) The function q is continuous in the quadratic mean sense. $\forall \varphi \in \mathcal{P}C$,

$$
\lim_{t \to s} \mathbb{E}||q(t, \varphi) - q(s, \varphi)||^2 = 0.
$$

(H4) There exists a constant $K_2 > 0$ such that, for $\psi_j \in \mathcal{P}C$, $j = 1, 2$, the mapping $f : [0, +\infty) \times \mathcal{P}C \to \mathbb{H}$ satisfies the following Lipschitz condition for all $t \in [0, T]$:

$$
||f(t, \psi_1) - f(t, \psi_2)|| \leq K_2 ||\psi_1 - \psi_2||.
$$

Here $\tilde{K}_2 = \sup_{t \in [0,T]} ||f(t, 0)||$.

(H5) The mapping $g : [0, +\infty) \times [0, +\infty) \times \mathcal{P}C \to \mathbb{H}$ satisfies the following Lipschitz condition. For $t \in [0, T]$, there exists a constant $K_3 > 0$ such that, for $\psi_j \in \mathcal{P}C$, $j = 1, 2$,

$$
\left|\left| \int_0^t \left[g(t, s, \psi_1) - g(t, s, \psi_2) \right] ds \right|\right| \leq K_3 ||\psi_1 - \psi_2||.
$$

Here $\tilde{K}_3 = T \sup_{0 \leq s \leq t \leq T} ||g(t, s, 0)||$.

(H6) The mapping $h : [0, +\infty) \times \theta \times \mathcal{PC} \to \mathbb{H}$ satisfies the following Lipschitz condition. For $t \in [0, T]$, there exists a constant $K_4 > 0$ such that, for $\psi_j \in \mathcal{PC}, j = 1, 2,$

$$\int_\theta ||h(t, \psi_1, v) - f(t, \psi_2, v)||^2 \lambda(dv) \vee \left(\int_\theta ||h(t, \psi_1, v) - f(t, \psi_2, v)||^2 \lambda(dv) \right)^{1/2} \leq K_4 ||\psi_1 - \psi_2||.$$

$$\left(\int_\theta ||h(t, \psi_1, v) - f(t, \psi_2, v)||^2 \lambda(dv) \right)^{1/2} \leq K_4 ||\psi_1 - \psi_2||.$$

(H7) The impulsive function $I_k : \mathbb{H} \to \mathbb{H}$ is continuous. There exists constants $d_k > 0$ ($k = 1, 2, \ldots$) satisfying $\sum_{k=1}^{+\infty} d_k < \infty$, such that

$$||I_k(\psi_1) - I_k(\psi_2)|| \leq d_k ||\psi_1 - \psi_2||, \ \ ||I_k(0)|| = 0 \text{ for all } \psi_1, \psi_2 \in \mathcal{PC}.$$

(H8) The function $\sigma : [0, +\infty) \to L_Q^0(Y, \mathbb{H})$ satisfies

$$\int_0^t ||\sigma(s)||_{L_Q^0}^2 ds < \infty, \ \forall \ t \in [0, T].$$

We have the following two conditions for the complete orthonormal basis $\{a_n\}_{n \in \mathbb{N}}$ in Y.

(C.1) $\sum_{n=1}^{\infty} ||\sigma Q^{\frac{1}{2}} a_n||_{L^2([0,T];\mathbb{H})} < \infty,$

(C.2) $\sum_{n=1}^{\infty} ||\sigma(t) Q^{\frac{1}{2}} a_n||_{\mathbb{H}}$ is uniformly convergent for all $t \in [0, T]$.

We now establish the existence and uniqueness results for Eq.(1).

Theorem 3.1. *Assume that hypotheses **(H1)**−**(H7)** are satisfied for all $\phi \in \mathcal{PC}, \ T > 0$, and*

$$K_1 + \frac{4M^2}{(1 - K_1)} \left(\sum_{i=1}^{+\infty} d_i \right)^2 < 1. \tag{9}$$

Then, the Eq.(1) has a unique mild solution on $[-\tau, T]$.

Proof. To begin, we introduce $\Lambda_T := \mathcal{PC}([-\tau, T], L^2(\Omega, \mathbb{H}))$ the Banach space of all continuous functions from $[-\tau, T]$ into $L^2(\Omega, \mathbb{H})$ equipped with the supremum norm $||\xi||_{\Lambda_T}^2 = \sup_{s \in [-\tau, T]} (\mathbb{E}||\xi(s)||^2)$. Now consider the closed subset $\hat{\Lambda}_T = \left\{ u \in \Lambda_T : u(\tau) = \phi(\tau) \text{ for } \tau \in [-\tau, 0] \right\}$ of Λ_T endowed with the norm $|| \cdot ||_{\Lambda_T}$. The problem (1) is transformed into a fixed point problem. We define the operator $\Psi : \hat{\Lambda}_T \to \hat{\Lambda}_T$ by $(\Psi u)(t) = \phi(t), \ t \in [-\tau, 0]$ and for $t \in [0, T]$

$$(\Psi u)(t) = \Pi(t) \left[\phi(0) - g(0, \phi) \right] + g(t, u_t)$$

$$+ \int_0^t \Pi(t - s) f(s, u_s) ds + \int_0^t \Pi(t - s) \int_0^s h(s, \eta, u_\eta) d\eta ds$$

$$+ \sum_{0 < t_k < t} \Pi(t - t_k) I_k(u(t_k^-)) + \int_0^t \Pi(t - s) \sigma(s) dZ_Q^H(s) \ \ P - a.s.$$

Now, to prove the existence result of mild solution of Eq.(1), it is sufficient to show that Ψ has a fixed point. To this end we subdivide the proof into two steps.

Step 1: First, we show that the map $t \to (\Psi x)(t)$ is continuous on the interval $[0, T]$. Let $\left|\tilde{l}\right|$ be sufficently small, for $u \in \hat{\Lambda}_T$ and $0 < t < T$. We get

$$
\mathbb{E}||(\Psi u)(t + \tilde{l}) - (\Psi u)(t)||^2 \leq 12\mathbb{E}\left|\left|\left(\Pi(t + \tilde{l}) - \Pi(t)\right)\left[\phi(0) - q(0, \phi)\right]\right|\right|^2 + 12\mathbb{E}||q(t + \tilde{l}, u_{t+\tilde{l}}) - q(t, u_t)||^2
$$

$$
+ 12\mathbb{E}\left|\left|\int_0^t \left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)f(s, u_s)ds\right|\right|^2 + 12\mathbb{E}\left|\left|\int_t^{t+\tilde{l}} \Pi(t + \tilde{l} - s)f(s, u_s)ds\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\int_0^t \left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)\int_0^s g(s, \eta, u_\eta)d\eta\, ds\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\int_t^{t+\tilde{l}} \Pi(t + \tilde{l} - s)\int_0^s g(s, \eta, u_\eta)d\eta\, ds\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\int_0^t \int_\theta \left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)h(s, u_s, v)\widetilde{N}(ds, du)\right|\right|^2
$$

$$
+ 10\mathbb{E}\left|\left|\int_t^{t+\tilde{l}} \int_\theta \Pi(t + \tilde{l} - s)h(s, u_s, v)\widetilde{N}(ds, dv)\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\sum_{0<t_k<t} \left(\Pi(t + \tilde{l} - t_k) - \Pi(t - t_k)\right)I_k(u(t_k^-))\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\sum_{t<t_k<t+\tilde{l}} \Pi(t + \tilde{l} - t_k)I_k(u(t_k^-))\right|\right|^2
$$

$$
+ 12\mathbb{E}\left|\left|\int_0^t \Pi(t + \tilde{l} - s) - \Pi(t - s)\sigma(s)dZ_Q^H(s)\right|\right| + 12\mathbb{E}\left|\left|\int_t^{t+\tilde{l}} \Pi(t + \tilde{l} - s)\sigma(s)dZ_Q^H(s)\right|\right|^2
$$

$$
\leq 12\sum_{j=1}^{12} P_j.
$$

By Definition 2.3-(ii), we have

$$
\lim_{\tilde{l}\to 0} P_1 = 0.
$$

By using (ii) of **(H3)** we obtain that

$$
\lim_{\tilde{l}\to 0} P_2 = 0.
$$

By using Hölder inequality we have:

$$
P_3 \leq t\mathbb{E}\int_0^t \left|\left|\left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)f(s, u_s)\right|\right|^2 ds.
$$

By Definition 2.3-(ii), we have

$$
\lim_{\tilde{l}\to 0}\left(\left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)f(s, u_s)\right) = 0
$$

and by combining the assumption **(H4)** and Definition 2.3-(ii), we obtain

$$
\left|\left|\left(\Pi(t + \tilde{l} - s) - \Pi(t - s)\right)f(s, u_s)\right|\right|^2 \leq 2M^2(e^{2(t+\tilde{l})\beta} + e^{2t\beta})(K_2||u_t||^2 + \tilde{K}_2),
$$

The Lebesgue dominated theorem, implies that

$$\lim_{\tilde{l}\to 0} P_3 = 0.$$

Using Hölder inequality, we have

$$
\begin{aligned}
P_4 &\leq \mathbb{E}\left(\int_t^{t+\tilde{l}} \left\|\Pi(t+\tilde{l}-s)f(s,u_s)\right\| ds\right)^2 \\
&\leq \int_t^{t+\tilde{l}} ||\Pi(t+\tilde{l}-s)||^2 ds\, \mathbb{E}\int_t^{t+\tilde{l}} ||f(s,u_s)||^2 ds \\
&\leq \tilde{l}\hat{M}^2 \int_t^{t+\tilde{l}} \mathbb{E}||f(s,u_s)||^2 ds.
\end{aligned}
$$

Thus, we have

$$\lim_{\tilde{l}\to 0} P_4 = 0.$$

By using similar arguments to P_2 and combining assumption **(H5)** we obtain

$$\lim_{\tilde{l}\to 0} P_5 = 0.$$

Also, with the same argument to P_3 and using assumption **(H5)** we obtain that

$$\lim_{\tilde{l}\to 0} P_6 = 0.$$

Similarly, with the same argument to P_3 and using assumption **(H5)** we obtain that

$$\lim_{\tilde{l}\to 0} P_{11} = 0, \quad \lim_{\tilde{l}\to 0} P_{12} = 0.$$

For P_7, application of Lemma 2.1 gives

$$P_7 \leq 2cH(2H-1)t^{2H-1}\int_0^t ||\left(\Pi(t+\tilde{l}-s)-\Pi(t-s)\right)\sigma(s)||_{L_Q^0}^2 ds.$$

By Definition 2.3-(ii) we have

$$\left(\Pi(t+\tilde{l}-s)-\Pi(t-s)\right)\sigma(s) \to 0, \text{ as } \tilde{l} \to 0$$

and by Definition 2.3-(i) we have the inequality :

$$\left\|\Pi(t+\tilde{l}-s)-\Pi(t-s)\sigma(s)\right\|_{L_Q^0}^2 \leq 2M^2(e^{2(t+\tilde{l})\beta}+e^{2t\beta})||\sigma(s)||_{L_Q^0}^2,$$

and by the Lebesgue dominated convergence theorem, we get

$$\lim_{\tilde{l}\to 0} P_7 = 0.$$

By Lemma 2.1 we have

$$P_8 \leq 2cH(2H-1)\tilde{l}^{2H-1}\int_t^{t+\tilde{l}} ||\Pi(t-s)\sigma(s)||_{L_Q^0}^2 ds,$$

since $s \in [0, T]$, we have

$$P_8 \quad \leq \quad 2cH(2H-1)\tilde{l}^{2H-1}M^2 \int_t^{t+\tilde{l}} ||\sigma(s)||^2_{L^0_Q} \, ds,$$

and regarding to **(H7)**, we get

$$\lim_{\tilde{l}\to 0} P_8 = 0.$$

By using Definition 2.3-(i) we have

$$\left|\left|\left(\Pi(t+\tilde{l}-t_k) - \Pi(t-t_k)\right) I_k(u(t_k^-))\right|\right|^2 \leq 2M^2(e^{2(t+\tilde{l})\beta} + e^{2t\beta})d_k||u(t_k^-)||^2.$$

By combining assumption **(H6)** and Definition 2.3-(i), we have

$$\lim_{\tilde{l}\to 0} P_9 = 0.$$

By using assumption **(H6)** and Definition 2.3-(i), we get

$$\lim_{\tilde{l}\to 0} P_{10} = 0.$$

Therefore,we can conclude

$$\lim_{\tilde{l}\to 0} \mathbb{E}||(\Psi x)(t+\tau) - (\Psi x)(t)||^2 = 0.$$

Hence, the above arguments imply that function $t \to (\Psi x)(t)$ is continuous on the interval $[0, T]$.
Step 2: In this part of the proof, we will verify that Ψ is contraction mapping in $\hat{\Lambda}_{T_1}$ with some $T_1 \leq T$ to be specified later. Let $u, v \in \hat{\Lambda}_{T_1}$ and $t \in [0, T]$. By virtue of elementary inequality we obtain

$$
\begin{aligned}
||(\Psi u)(t) - (\Psi v)(t)||^2 \quad \leq \quad & \frac{1}{K_1}||q(t, u_t) - g(t, v_t)||^2 + \frac{4}{1-K_1}\left\{ \left|\left| \int_0^t \Pi(t-s) \left[f(s, u_s) - f(s, v_s)\right] ds \right|\right|^2 \right. \\
& + \left|\left| \int_0^t \Pi(t-s) \left[\int_0^s g(s, \eta, u_\eta)d\eta - \int_0^s g(s, \eta, v_\eta)d\eta \right] ds \right|\right|^2 \\
& + \left|\left| \int_0^t \int_\theta \Pi(t-s) \left[h(s, u_s, v) - h(s, v_s, v)\widetilde{N}(ds, dv)\right] ds \right|\right|^2 \\
& \left. + \left|\left| \sum_{0<t_k<t} \Pi(t-t_k) \left[I_k(x(t_k^-)) - I_k(y(t_k^-))\right] \right|\right|^2 \right\}.
\end{aligned}
$$

By using assumptions **(H3)**-**(H6)**, Defintion 2.3 together with Hölder's inequality, we get

$$
\begin{aligned}
\mathbb{E}||(\Psi u)(t) - (\Psi v)(t)||^2 \quad \leq \quad & K_1\mathbb{E}||u_t - v_t||^2 + \frac{4M^2K_2^2t}{1-k_1}\int_0^t \mathbb{E}||u_s - v_s||^2 ds + \frac{4M^2K_3^2t}{1-K_1}\int_0^t \mathbb{E}||u_s - v_s||^2 ds \\
& + \frac{4M^2K_4^2}{1-k}\int_0^t \mathbb{E}||u_s - v_s||^2 ds + \frac{4M^2}{1-k_1}\left(\sum_{i=1}^\infty d_j\right)^2 \mathbb{E}||u(t_k^-) - v(t_k^-)||^2.
\end{aligned}
$$

Hence, we have

$$\sup_{s\in[-\tau,t]} \mathbb{E}||(\Psi u)(s) - (\Psi v)(s)||^2 \leq \theta(t) \sup_{s\in[-\tau,t]} \mathbb{E}||u(s) - v(s)||^2,$$

where

$$\theta(t) = K_1 + 4M^2\frac{\left[K_2^2t + K_3^2t + K_4^2\right]t}{1-K_1} + \frac{4M^2}{(1-K_1)}\left(\sum_{i=1}^{+\infty} d_i\right)^2.$$

By inequality (9), we have

$$\theta(0) = K_1 + \frac{4M^2}{(1 - K_1)} \left(\sum_{k=1}^{+\infty} d_i \right)^2 < 1.$$

Then there exists $0 < T_1 \le T$ such that $0 < \theta(T_1) < 1$ and the operator Ψ is a contraction on $\hat{\Lambda}_{T_1}$ and hence it has a unique fixed point on $[-\tau, T_1]$, which is a mild solution of Eq.(1) on the interval $[-\tau, T_1]$. By repeating a similar process the solution can be extended to the entire interval $[-\tau, T]$. $\square$

Remark 3.1. *Notice that we can extend the solution for $t \ge T$. Indeed, if we assume that the constants K_1, K_2, K_3, K_4 which appear in assumptions **(H3)-(H6)** are independent of $T > 0$, then the mild solution is defined for all $t \in [-r, T]$, for each $T > 0$. This will play a crucial role in our analysis of stability. Therefore, in the next section we will assume that the solutions are defined globally in time (for instance, under the previous assumptions).*

3.2 Exponential stability

In this subsection, it is established the exponential stability in the mean square moment of the mild solution for Eq.(1), we need to state the following additional assumptions.

(H9) The corresponding resolvent operator $(\Pi(t))_{t\ge}$ of Eq. (7) verifies the following : There exist $\gamma > 0$ and $M > 0$ such that $||\Pi(t)|| \le Me^{-\gamma t}$, for all $t \ge 0$.

(H10) There exist nonnegative real numbers $R_i \ge 0$ and continuous functions
$\xi_i : [0, +\infty) \to \mathbb{R}_+$, $\xi_i(t) \le \alpha_i e^{-\gamma t}$, $i = 1, 2, 3, 4$, $\alpha_i > 0$ such that for all $t \ge 0$ and $\psi_i \in \mathcal{PC}$

$$||q(t, \psi_1)||^2 \le R_1 ||\psi_1||^2 + \xi_1(t),$$
$$||f(t, \psi_2)||^2 \le R_2 ||\psi_2||^2 + \xi_2(t),$$
$$\left|\left| \int_0^t h(t, s, \psi_3)ds \right|\right|^2 \le R_3 ||\psi_3||^2 + \xi_3(t),$$
$$\int_\theta \left|\left| h(t, \psi_4, v) \right|\right|^2 \lambda(dv) \vee \left(\int_\theta \left|\left| h(t, \psi_4, v) \right|\right|^2 \lambda(dv) \right)^{1/2} \le R_4 ||\psi_4||^2 + \xi_4(t).$$

(H11) The function $\sigma : [0, +\infty) \to L_Q^0(Y, \mathbb{H})$ satisfies the following condition in addition to assumptions (C.1) and (C.2):

$$\int_0^{+\infty} e^{\gamma s} ||\sigma(s)||_{L_Q^0}^2 ds < \infty.$$

Theorem 3.2. *Assume that the conditions of Theorem 3.1 are satisfied and **(H9)**−**(H11)** are fulfilled. Then the mild solution of Eq.(1) is exponentially stable in mean square moment provided*

$$\frac{5\{[M^2(R_2 + R_3 + R_4)/\gamma^2] + [M^2(\sum_{k=1}^{+\infty} d_k)^2]\}}{(1 - k)^2} < 1, \tag{10}$$

where $k := \sqrt{R_1}$.

Proof. From (10), it is possible to find a suitable number $\tilde{l} > 0$ small enough such that

$$k + \frac{5M^2(R_2 + R_3 + R_4)}{\gamma(\gamma - \tilde{l})(1 - k)} + \frac{5M^2(\sum_{k=1}^{+\infty} d_k)^2}{(1 - k)} < 1.$$

Let assume that $\mu = \gamma - \tilde{l}$ and $u(t)$ be a mild solution of Eq.(1). Then, from (8) we have

$$
\begin{aligned}
\mathbb{E}||u(t)||^2 \quad \leq \quad & \frac{1}{k}\mathbb{E}||q(t, u_t)||^2 + \frac{6}{1-k}\mathbb{E}\left\{||\Pi(t)[\phi(0) - q(0, \phi)]||^2 \right.\\
& + \left\|\int_0^t \Pi(t-s)f(s, u_s)ds\right\|^2 + \left\|\int_0^t \Pi(t-s)\sigma(s)dZ_Q^H(s)\right\|^2 \\
& + \left\|\int_0^t \Pi(t-s)\int_0^s g(s, \eta, u_\eta)d\eta ds\right\|^2 + \left\|\int_0^t\int_\theta \Pi(t-s)h(s, u_s, v)\widetilde{N}(ds, dv)\right\|^2 \\
& \left. + \left\|\sum_{0<t_k<t} \Pi(t-t_k)I_k(u(t_k^-))\right\|^2\right\} \\
\leq \quad & \sum_{j=1}^7 J_j(t).
\end{aligned}
$$

By assumption **(H10)** we get

$$
\begin{aligned}
J_1(t) \quad = \quad & \frac{1}{k}\mathbb{E}||q(t, u_t)||^2 \\[2ex]
\leq \quad & \frac{1}{k}\{R_1\mathbb{E}||u_t||^2 + \xi_1(t)\} \\[2ex]
\leq \quad & k\mathbb{E}||u_t||^2 + \lambda_1 e^{-\gamma t},
\end{aligned}
\tag{11}
$$

where $\lambda_1 = \frac{\alpha_1}{k}$.

According to **(H9)** and **(H10)** we obtain

$$
\begin{aligned}
J_2 \quad \leq \quad & \frac{10}{1-k}\mathbb{E}||\Pi(t)\phi(0)||^2 + \frac{10}{1-k}\mathbb{E}||\Pi(t)q(0, \phi)||^2 \\[2ex]
\leq \quad & \frac{10M^2}{1-k}e^{-2\gamma t}\mathbb{E}||\phi(0)||^2 + \frac{10M^2}{1-k}e^{-2\gamma t}\{R_1\mathbb{E}||\phi||^2 + \xi_1(t)\} \\[2ex]
\leq \quad & \lambda_2 e^{-\mu t},
\end{aligned}
\tag{12}
$$

where $\lambda_2 = \frac{10M^2}{1-k}[\mathbb{E}||\phi(0)||^2 + \{R_1\mathbb{E}||\phi||^2 + \alpha_1\}]$.

Employing **(H9)** , **(H10)** and Hölder's inequality, we get

$$
\begin{aligned}
J_3(t) \quad = \quad & \frac{5}{1-k}\mathbb{E}||\int_0^t \Pi(t-s)f(s, u_s)ds||^2 \\[2ex]
\leq \quad & \frac{5}{1-k}\mathbb{E}\left(\int_0^t Me^{-\gamma(t-s)}||f(s, u_s)||ds\right)^2 \\[2ex]
\leq \quad & \frac{5M^2R_2}{\gamma(1-k)}\int_0^t e^{-\gamma(t-s)}\mathbb{E}||u_s||^2 ds + \lambda_3 e^{-\mu t}
\end{aligned}
\tag{13}
$$

where $\lambda_3 = \frac{5M^2R_3}{\gamma(1-k)}\frac{\alpha_3}{\gamma-\mu}$.

Also with the same arguments, we have

$$
\begin{aligned}
J_4(t) \;\le\;& \frac{5}{1-k}\mathbb{E}\left(\int_0^t Ne^{-\gamma(t-s)}\left\|\int_0^t g(s,\eta,x_\eta)d\eta\right\|ds\right)^2 \\[2mm]
\le\;& \frac{5M^2 R_3}{\gamma(1-k)}\int_0^t e^{-\gamma(t-s)}\mathbb{E}\|u_s\|^2 ds + \lambda_4 e^{-\mu t},
\end{aligned}
\tag{14}
$$

where $\lambda_4 = \dfrac{5M^2 R_4}{\gamma(1-k)}\dfrac{\alpha_4}{\gamma-\mu}$.

By Lemma 2.1 and **(H11)** we have

$$
\begin{aligned}
J_5 \;\le\;& \frac{5}{1-k}M^2 c(H)t^{2H-1}\int_0^t e^{-2\gamma(t-s)}\|\sigma(s)\|_{L_Q^0}^2 ds \\[2mm]
\le\;& e^{-\mu t}\frac{5M^2}{1-k}c(H)t^{2H-1}e^{-\tilde{l}t}\int_0^t e^{\gamma s}\|\sigma(s)\|_{L_Q^0}^2 ds.
\end{aligned}
$$

According to assumption **(H11)**, there exist a constant $\gamma_5 > 0$ such that, for all $t \ge 0$,

$$
\frac{5M^2}{1-k}c(H)t^{2H-1}e^{-\tilde{l}t}\int_0^t e^{\gamma s}\|\sigma(s)\|_{L_Q^0}^2 ds \le \lambda_5.
\tag{15}
$$

Thus

$$
J_5(t) \le \lambda_5 e^{-\mu t}.
$$

and

$$
\begin{aligned}
J_6(t) \;\le\;& \frac{6}{1-k}\mathbb{E}\left(\left\|\int_0^t\int_\theta h(s,u_s,v)\widetilde{N}(ds,dv)\right\|\right)^2 \\[2mm]
\le\;& \frac{6M^2}{\gamma(1-k)}R^4\int_0^t e^{-\gamma(t-s)}\mathbb{E}\|u\|_s^2 ds + \lambda_6 e^{-\mu t},
\end{aligned}
\tag{16}
$$

where $\lambda_6 = \dfrac{6M^2}{\gamma(1-k)}\dfrac{\alpha_5}{\gamma-\mu}$.

By using **(H7)** and **(H9)** we have

$$
\begin{aligned}
J_7(t) \;\le\;& \frac{5M^2}{1-k}\left(\sum_{i=1}^{+\infty}d_i\right)^2 e^{-2\gamma(t-t_k)}\mathbb{E}\|u(t_k^-)\|^2 \\[2mm]
\le\;& \frac{5M^2}{1-k}\left(\sum_{i=1}^{+\infty}d_i\right)\left(\sum_{i=1}^{+\infty}d_i\right)e^{-\gamma(t-t_k)}\mathbb{E}\|u(t_k^-)\|^2.
\end{aligned}
\tag{17}
$$

The above inequalities (11)-(17) together with Lemma 2.2, imply that

$$
\mathbb{E}\|u(t)\|^2 \le v e^{-\mu t} \quad\text{for}\qquad t\in[-\tau,0]
$$

and

$$
\begin{aligned}
\mathbb{E}\|u(t)\|^2 \;\le\;& v e^{-\mu t} + k\sup_{-\tau\le\theta\le 0}\mathbb{E}\|u(t+\theta)\|^2 + \hat{\kappa}\int_0^t e^{-\mu(t-s)}\sup_{-\tau\le\theta\le 0}\mathbb{E}\|u(t+\theta)\|^2 ds \\[2mm]
& + \sum_{i=1}^{+\infty}\omega_k e^{-\mu(t-t_k)}\mathbb{E}\|u(t_k^-)\|^2, \qquad t\ge 0
\end{aligned}
$$

where

$$\hat{\kappa} = \frac{5M^2(R_2 + R_3 + R_4)}{\gamma(1 - k)} \quad \text{and} \quad \nu = \max\left(\sum_{i=1}^{6} \lambda_j, \sup_{-\tau \le \theta \le 0} \mathbb{E}\|\phi(\theta)\|^2\right).$$

And we observe that $k + \frac{\hat{k}}{M} + \sum_{i=1}^{+\infty} \omega_k < 1$.

Thus, the mild solution of Eq.(1) is exponentially stable in mean square moment, since $k + \frac{\hat{k}}{\mu} + \sum_{k=1}^{+\infty} \omega_k < 1$ and by Lemma 2.2 we have the existence of two positive constants C and r such that $\mathbb{E}\|u(t)\|^2 \le Ce^{-rt}$ for any $t \ge -\tau$, where $\theta > 0$ is the unique solution to the equation $k + \frac{k}{\mu-\theta}e^{r\tau} + \sum_{k=1}^{+\infty} \omega_k = 1$ and $C = \max\{\nu, \frac{\nu(\mu-r)}{\hat{k}e^{r\tau}}\} > 0$.

This completes the proof of the theorem.   □

4 Illustration

This part consist to make an application of the theory studied above. Consider the impulsive neutral stochastic partial integrodifferential equations of the form

$$
\begin{cases}
d[w(t, \xi) - \beta_1(t, w(t - r, \xi))] = \left[\frac{\partial^2}{\partial\xi^2}[w(t, \xi) - \beta_1(t, w(t - r, \xi))]\right. \\
\quad + \int\limits_0^t b(t - s)\frac{\partial^2}{\partial\xi^2}[w(s, \xi) - \beta_1(s, w(s - r, \xi))]ds \\
\quad + \beta_2(t, w(t - r, \xi)) + \left.\int\limits_0^t \beta_3(t, s, w(s - r, \xi))ds\right]dt \\
\quad + \int\limits_\theta \beta_4(t, x(t - s), z)\tilde{N}(ds, dz), +\sigma(t)dZ_Q^H(t), \quad 0 \le \xi \le \pi, \quad t \ne t_k, \quad t \in [0, T], \\
w(t, 0) = w(t, \pi) = 0, \quad 0 \le t \le T, \\
\Delta w(t_k, \cdot)(\xi) = \frac{\beta_5}{k^2}w(t_k^-, \xi), \qquad t = t_k, \, k = 1, 2, \cdots, \\
w(t, \xi) = \varphi(t, \xi) \in \mathcal{PC}([-\tau, 0], L^2[0, \pi]), \quad -\tau \le t \le 0,
\end{cases}
\tag{18}
$$

where Z_Q^H is Rosenblatt process define on probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $\beta_1, \beta_2 : \mathbb{R}^+ \times \mathbb{R} \mapsto \mathbb{R}$, $\beta_3, \beta_4 : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R} \mapsto \mathbb{R}$, $b : \mathbb{R}^+ \mapsto \mathbb{R}$ are continuous functions, $\beta_5 \ge 0$ constant and $\sigma : [0, \infty) \mapsto L_Q^0\left(L^2([0, \pi]), L^2([0, \pi])\right)$.

Let $\mathbb{H} = Y = L^2[0, \pi]$ with the norm $\|\cdot\|$ and $e_n(x) = \sqrt{\frac{2}{\pi}}\sin nx$, $n = 1, 2, \cdots$. Then $(e_n)_{n \in \mathbb{N}}$ is a complete orthogonal basis in Y. In order to define the operator $Q : Y \to Y$, we chose a sequence $(\sigma_n)_{n \ge 1} \subset \mathbb{R}^+$ and set $Qe_n = \sigma_n e_n$, and assume that $\sum_{n=1}^\infty \sqrt{\sigma_n} < \infty$. Define the process Z^H by

$$Z^H(t) = \sum_{n=1}^\infty \sqrt{\sigma_n}z_n^H e_n,$$

where $H \in (\frac{1}{2}, 1)$ and $\left\{z_n^H\right\}_{n \in \mathbb{N}}$ is a sequence of two-side one-dimensional Rosenblatt process mutually independent.

Define the operator $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ by $A = \frac{\partial^2}{\partial\tau^2}$ with domain $D(A) = H_0^1(0, \pi) \cap H^2(0, \pi)$.

It is well-known that A is the infinitesimal generator of a strong continuous semigroup $\{T(t)\}_{t \ge 0}$ on $\mathbb{H}$, which is given by

$$T(t)x = \sum_{n=1}^\infty e^{-n^2 t}\langle x, e_n\rangle e_n \quad \text{and} \quad Ax = -\sum_{n=1}^{+\infty} n^2\langle x, e_n\rangle e_n, \quad x \in D(A).$$

We suppose that:

(i) There exist a positive constant l_1, $0 < \pi l_1^2 < 1$ such that

$$\left|\beta_1(t, y_1) - \beta_1(t, y_2)\right| \le l_1 |y_1 - y_2|, \ t \ge 0, \ y_1, y_2 \in \mathbb{R}.$$

(ii) There exist a positive constant, $l_2 > 0$ such that

$$\left|\beta_2(t, y_1) - \beta_2(t, y_2)\right| \le l_2 |y_1 - y_2|, \ t \ge 0, \ y_1, y_2 \in \mathbb{R}.$$

(iii) There exist a positive constant, $l_3 > 0$ such that

$$\left| \int_0^t \left[\beta_3(t, y_1) - \beta_3(t, y_2)\right] ds \right| \le l_3 |y_1 - y_2|, \ t \ge 0, \ y_1, y_2 \in \mathbb{R}.$$

(iv) There exist a positive constant, $l_4 > 0$ such that

$$\left| \int_\theta \left[\beta_4(t, y_1, v) - \beta_4(t, y_2, v)\right] \widetilde{N}(ds, dv) \right| \le l_4 |y_1 - y_2|, \ t \ge 0, \ y_1, y_2 \in \mathbb{R}.$$

(v) There exist nonnegative real numbers $Q_1, Q_2, Q_3, Q_4 > 0$ and functions $\xi_1, \xi_2, \xi_3, \xi_4 : [0, \infty) \mapsto \mathbb{R}^+$ with $\xi_i(t) \le P_i e^{-\lambda t}$ ($i = 1, 2, 3, 4$), $P_i > 0$, such that

$$\left|\beta_1(t, y)\right|^2 \le Q_1 |y|^2 + \xi_1(t),$$
$$\left|\beta_2(t, y)\right|^2 \le Q_2 |y|^2 + \xi_2(t),$$
$$\left| \int_0^t \beta_3(t, s, y)ds \right|^2 \le Q_3 |y|^2 + \xi_3(t),$$
$$\left| \int_\theta \beta_4(t, v, y)\widetilde{N}(ds, dv) \right|^2 \le Q_4 |y|^2 + \xi_4(t).$$

(v) The function $\sigma : [0, \infty) \mapsto L_Q^0 \left(L^2([0, \pi]), L^2([0, \pi]) \right)$ satisfies

$$\int_0^t \|\sigma(s)\|_{L_Q^0}^2 ds < \infty, \ t \in [0, T] \ \text{ and } \ \int_0^\infty e^{\lambda s} \|\sigma(s)\|_{L_Q^0}^2 ds < \infty.$$

Let $Y : D(A) \subset \mathbb{H} \to \mathbb{H}$ the operator define by $Y(t)(z) = b(t)Az$ for $t \ge 0$, $z \in D(A)$. For $\xi \in [0, \pi]$, we define the operators $q, f : [0, \infty) \times \mathbb{H} \mapsto \mathbb{H}$, $g : [0, \infty) \times [0, \infty) \times \mathbb{H} \mapsto \mathbb{H}$, $h : [0, \infty) \times \theta \times \mathbb{H} \mapsto \mathbb{H}$ and $I_k : \mathbb{H} \mapsto \mathbb{H}$ by

$$G(t, \phi)(\xi) = \beta_1(t, \phi(\xi)),$$
$$F(t, \phi)(\xi) = \beta_2(t, \phi(\xi)),$$
$$\int_0^t \widetilde{H}(t, s, \phi)(\xi)ds = \int_0^t \beta_3(t, s, \phi(\xi))ds,$$
$$\int_\theta h(t, \phi)(\xi) = \beta_4(t, \phi(\xi)),$$
$$I_k(\phi)(\xi) = \frac{\beta_5}{k^2} \phi(\xi) \qquad (k = 1, 2, \cdots).$$

If we put

$$\begin{cases} u(t)(\xi) = w(t, \xi), \ \text{ for } t \ge 0, \ \text{ and } \ \xi \in [0, \pi], \\ \varphi(\theta)(\xi) = u_0(\theta, \xi), \ \text{ for } -\tau \le \theta \le 0, \ \text{ and } \ \xi \in [0, \pi], \end{cases}$$

then, Eq.(18) takes the following abstract form

$$
\begin{cases}
d\left[u(t) - q(t, u_t)\right] &= \left[A\left(u(t) - q(t, u_t)\right) + \int\limits_0^t Y(t - s)\left[u(s) - q(s, u_s)\right] ds + f(t, u_t)\right. \\
& \left. + \int\limits_0^t g(t, s, u_s)ds\right]dt + \int\limits_\theta h(t, u_t, v)\widetilde{N}(dt, dv) + \sigma(t)dZ_Q^H(t), \ t \in [0, T], \ t \neq t_k, \\
\Delta u(t) &= I_k(u(t_k^-)), \quad t = t_k, k = 1, 2, \cdots, \\
u_0(t) &= \phi(t) \in \mathcal{P}C\left([-\tau, 0], \mathbb{H}\right), \quad -\tau \leq t \leq 0,
\end{cases}
$$

(19)

Moreover , if b is bounded and C^1 function such that b' is bounded and uniformly continuous, then **(H1)** and **(H2)** are satisfied, and hence, by Theorem 2.3, Eq.(7) has a resolvent operator $\left(\Pi(t)\right)_{t \geq 0}$ on $\mathbb{H}$. Using Lemma 5.2 [9], let $\mu > \delta > 1$ and $b(t) < \exp(-\beta t)$, for all $t \geq 0$. Then the above resolvent operator decays exponentially to zero. Specifically $\|\Pi(t)\| \leq \exp(-at)$ where $a = 1 - 1/\delta$.
For $(t, s, \phi_i) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{H}$, $j = 1, 2$, respectively by assumptions (i), (ii), (iii) and (iv), we get

$$\|q(t, \phi_1) - q(t, \phi_2)\| \leq l_1 \|\phi_1 - \phi_2\|,$$

$$\|f(t, \phi_1) - f(t, \phi_2)\| \leq l_2 \|\phi_1 - \phi_2\|,$$

$$\left\|\int\limits_0^t \left[g(t, s, \phi_1) - g(t, s, \phi_2)\right] ds\right\| \leq l_3 \|\phi_1 - \phi_2\|,$$

$$\left\|\int\limits_Z \left[h(t, \phi_1, v) - h(t, \phi_2, v)\widetilde{N}(dt, dv)\right]\right\| \leq l_4 \|\phi_1 - \phi_2\|,$$

$$\|q(t, \phi_1)\|^2 \leq Q_1 \|\phi_1\|^2 + \xi_1(t),$$

$$\|f(t, \phi_1)\|^2 \leq Q_2 \|\phi_1\|^2 + \xi_2(t),$$

$$\left\|\int\limits_0^t g(t, s, \phi_1)ds\right\|^2 \leq Q_3 \|\phi_1\|^2 + \xi_3(t),$$

$$\left\|\int\limits_\theta h(t, \phi_1, v)ds\right\|^2 \leq Q_4 \|\phi_1\|^2 + \xi_4(t).$$

We have also that

$$\left\|I_k(\phi_1) - I_k(\phi_2)\right\| \leq d_k \|\phi_1 - \phi_2\|,$$

where $d_k = \frac{\beta_5}{k^2}$, k=1,2,…and $\sum\limits_{k=1}^\infty d_k = \sum\limits_{k=1}^\infty \frac{\beta_5}{k^2} < \infty$.

Thus, all assumptions of Theorem 3.1 are fulfilled. Therefore, the existence of a mild solution of Eq.(18) follows. In addition, by Theorem 3.2, we easily see that the mild solution of Eq.(18) is exponentially stable in the 2nd moment.

5 Conclusion

In this article, we showed existence and unicity of mild solution to Eq.(1) by using Banach fixed point theorem. Further, we investigated exponential stability of mild solutions. The last part of this paper is devoted to an example to illustrate our results.

References

[1]   P. Abry, V. Pipiras, Wavelet-based synthesis of the Rosenblatt process. Signal Processing. 86(2006), 2326-2339.

[2]   H. M. Ahmed, Fractional neutral evolution equations with nonlocal conditions. Advances in Difference Equations. (2013), 2013-117.

[3]   H. M. Ahmed, M. M. El-Borai, Hilfer fractional stochastic integro-differential equations. Applied Mathematics and Computation. 331(2018), 182-189.

[4]   P. Ali, K. Shah, F. Jarad, V. Gupta, T. Abdeljawad, Existence and stability analysis to a coupled system of implicit type impulsive boundary value problems of fractional-order differential equations. Advances in Difference Equations. (2019), 1-21.

[5]   G. Arthi, K. Balachandran, Controllability of damped second-order neutral functional differential systems withimpulses. Taiwanese Journal of Mathematics. 16(2012), 89-106.

[6]   J. M. Bardet, C. A Tudor, A wavelet analysis of the Rosenblatt process: chaos expansion and estimation of the self-similarity parameter. Stochastic Processes and their Applications. 120(2010), 2331-2362.

[7]   S. Boudrahem, P. R. Rougier, Relation between postural control assessment with eyes open and centre of pressure visual feed back effects in healthy individuals. Experimental Brain Research. 195(2009), 145-152.

[8]   F. Comte, E. Renault, Long memory continuous time models. Journal of Econometrics. 73(1996), 101-149.

[9]   M. Dieye, M. A. Diop, K. Ezzinbi, On exponential stability of mild solutions for some stochastics partial integrodifferential equations. Statistics and Probability Letters. 123(2017), 61-76.

[10]   R. L. Dobrushin, P. Major, Non-central limit theorems for non-linear functionals of Gaussian fields. Zeitschrift fur Wahrscheinlichkeits theorie and Verwandte Gebiete. 50(1979), 27-52.

[11]   M. M. El-Borai, O. L. Moustafa, H. M. Ahmed, Asymptotic stability of some stochastic evolution equations. Applied Mathematics and Computation. 144(2003), 273-286.

[12]   J. Golec, S. Sathananthan, Strong approximations of stochastic integrodifferential equations. Dynamics of Continuous, Discrete and Impulsive Systems, Series B. 8(2001), 139-151.

[13]   R. C. Grimmer, Resolvent opeators for integral equations in a Banach space. Transactions of the American Mathematical Society. 273(1982), 333-349.

[14]   V. Gupta, S. N. Bora, J. J. Nieto, Dhage iterative principle for quadratic perturbation of fractional boundary value problems with finite delay. Mathematical Methods in the Applied Sciences. (2019), 1-12.

[15]   V. Gupta, J. Dabas, Existence results for fractional order boundary value problem with integrable impulse, Dynamics of Continuous, Discrete and Impulsive Systems Series A:Mathematical Analysis. 25(2018), 267-285.

[16]   V. Gupta, J. Dabas, Michal Feckan, Existence results of solutions for impulsive fractional differential equations. Nonautonomous Dynamical Systems. 5(2018), 35-51.

[17]   J. K. Hale, K. R. Meyer, A class of functional equations of neutral type. Memoirs of the American Mathematical Society. 76(1967), 1-65.

[18]   J. K. Hale, S. M. Verduyn Lunel, Introduction to Functional Differential Equations. Springer-Verlag, New York, 1993.

[19]   I. Kruk, F. Russo, C. A. Tudor, Wiener integrals, Malliavin calculus and covariance measure structure. Journal of Functional Analysis. 249(2007), 92-142.

[20]   V. B. Kolmanovskii, V. R. Nosov, Stability of neutral type functional differential equations. Nonlinear Analysis. 6(1982), 873-910.

[21]   A. N. Kolmogorov, The Wiener spiral and some other interesting curves in Hilbert space. Doklady Akademii Nauk SSSR. 26(1940), 115-118.

[22]   V. Laksmikantham, M. R.M Rao, Theory of Integro-Differential Equations. Gordon and Breach Publishers, London, 1995.

[23]   N. Leonenko, V. Ahn, Rate of convergence to the Rosenblatt distribution for additive functionals of stochastic processes with long-range dependence. Journal of Applied Mathematics and Stochastic Analysis. 14(2001), 27-46.

[24]   J. J. Levin, J. A. Nohel, The integro-differential equations of a class of nuclear reactors with delayed neutrons. Archive for Rational Mechanics and Analysis. 31(1968), 151-172.

[25]   Y. K. Ma, G. Arthi, S. M. Anthoni, Exponential stability behavior of neutral stochastic integrodifferential equations with fractional Brownian motion and impulsive effects. Advances in Difference Equations. (2018), 2018-110.

[26]   B. B. Mandelbrot, J. Van Ness, Fractional Brownian motions, fractional noises and applications. SIAM Review. 10(1968), 422-437.

[27]   M. Maejima, C. A. Tudor, Wiener integrals with respect to the Hermite process and a non central limit theorem. Stochastic Analysis and Applications. 25(2007), 1043-1056.

[28]   M. Maejima, C. A. Tudor, Selfsimilar processes with stationary increments in the second Wiener chaos. Probability and Mathematical Statistics. 32(2012), 167-186.

[29]   M. Maejima, C. A. Tudor, On the distribution of the Rosenblatt process. Statistics and Probability Letters. 83(2013), 1490-1495.

[30]   P. E Protter, Stochastic integration and differential equations. 2nd edition, Springer, New York, 2004.

[31] V. Pipiras, M. S. Taqqu, Regularization and integral representations of Hermite processes. Statistics and Probability Letters. 80(2010), 2014-2023.

[32] M. Taqqu, Weak convergence to the fractional Brownian motion and to the Rosenblatt process. Zeitschrift fur Wahrscheinlichkeits theorie and Verwandte Gebiete. 31(1975), 287-302.

[33] S. Tindel, C. A. Tudor, F. Viens, Stochastic evolution equations with fractional Brownian motion. Probability Theory and Related Fields. 127(2003), 186-204.

[34] C. A. Tudor, Analysis of the Rosenblatt process. ESAIM:Probability and Statistics. 12(2008), 230-257.

[35] C. A. Tudor, F. G. Viens, Variations and estimators for self-similarity parameters via Malliavin calculus. The Annals of Probability. 37(2009), 2093-2134.

[36] M. Veillette, M. S. Taqqu, Properties and numerical evaluation of the Rosenblatt distribution. Bernoulli. 19(2013), 982-1005.

[37] H. Yang, F. Jiang, Exponential stability of mild solutions to impulsive stochastic neutral partial differential equations with memory. Advances in Difference Equations. 148(2013), 2013.