RAIRO-Oper. Res. 53 (2019) 157–178
https://doi.org/10.1051/ro/2019005

RAIRO Operations Research
www.rairo-ro.org

GOAL PROGRAMMING APPROACH FOR SOLVING MULTI-OBJECTIVE FRACTIONAL TRANSPORTATION PROBLEM WITH FUZZY PARAMETERS

Paraman Anukokila[1,*], Bheeman Radhakrishnan[2] and Antony Anju[1]

Abstract. In this paper, authors studied a goal programming approach for solving multi-objective fractional transportation problem by representing the parameters (γ, δ) in terms of interval valued fuzzy numbers. Fuzzy goal programming problem with multiple objectives is difficult for the decision makers to determine the goal valued of each objective precisely. The proposed model presents a special type of non-linear (hyperbolic) membership functions to solve multi-objective fractional transportation problem with fuzzy parameters. To illustrate the proposed method numerical examples are solved.

Mathematics Subject Classification. 90 C 08, 90 C 32, 90 C 70.

Received April 10, 2017. Accepted December 31, 2018.

1. Introduction

Transportation problem is one of the important application of the linear programming problem. The basic transportation problem was originally developed by Hitchcock [10]. It may be solved by using a simplified version of the simplex technique called transportation method. Transportation problem with fractional objective functions are widely used as performance measures in many real life situations. The problem of optimizing one or several ratios of functions are called fractional programming. Fractional programming has attracted the attention of many researchers in the past. The main reason for interest in fractional programming from the fact that linear fractional objective functions occur frequently as a measure of performance in a variety of circumstances such as when satisfying objectives under uncertainty. The fractional transportation problem (FTP) plays an important role in logistics and supply management for reducing cost and improving service. In real life, there are many diverse situations due to uncertainty in judgment, lack of evidence, etc., Fuzzy transportation problem is more appropriate to model and solve the real world problems.

Goal programming [GP] was introduced by Charnes and Cooper in 1961. It has widely applied to solve different problems which involve multiple objectives. GP requires decision maker to set an aspiration level for each good which can be a very difficult task as there are several of uncertainties in nature that must be considered. It is one of the powerful approach that has been proposed for the modeling, analysis and solution to multi-objective optimization problems.

Keywords. Transportation problem, fractional programming, goal programming.

[1] Department of Mathematics, PSG College of Arts and Science, 641014 Coimbatore, India.

[2] Department of Mathematics, PSG College of Technology, 641004 Coimbatore, India.

*Corresponding author: anuparaman@gmail.com

Article published by EDP Sciences

This paper is motivated by Gupta and Kumar [9] and Radhakrishnan and Anukokila [21] are pointed out and to overcome these shortcomings, a new method is proposed for finding the solution for linear multi-objective fractional transportation problem by representing the parameters (γ, δ) interval valued fuzzy numbers. The advantages of the proposed method over existing method are discussed in this article. Also the author extended to solve existing method using fuzzy goal programming (FGP) approach with interval cost. Here the objectives of the FGP model formulation problem are transformed into fuzzy goals by means of assigning an aspiration level to each of them. The achievement of the hyperbolic membership value to the extent possible of each of the fuzzy goal is considered. Lingo [16] software package is used to solve optimization problem. In the solution process, the under and over deviational variables of the membership goals associated with the fuzzy goals are introduced to transform the proposed method into an equivalent FGP model to solve the problem efficiently in the decision situation.

This paper is organized as follows: Section 2 discusses the review of literature of the proposed problem. Section 3 gives preliminary background of the paper. Section 4 analyzed problem formulation of linear multi-objective fractional transportation problems in crisp and fuzzy environment with goal programming and Min-max problem formulations are explained. Fractional goal programming approach applied in Section 5 also illustrative examples were solved and the optimal solution is compared with the proposed approach are given in Section 6.

2. Literature review

A lot of researchers have been studied the transportation problem, fractional transportation problem, goal programming which is one way or the other relates to this paper. Kocken and Sivri [14] presented a simple parametric method to generate all optimal solution of fuzzy solid transportation problem. Kumar and Kaur [2,9,13] proposed a new method for solving fuzzy transportation problem using ranking function and using generalized trapezoidal fuzzy numbers. Ebrahimnejad [7] developed a simplified new approach for solving fuzzy transportation problem with generalized trapezoidal fuzzy numbers. Ringuest and Rinks [23] proposed two interactive algorithms to obtain the solution of linear multi-objective transportation problems. Isermann [11] discussed an algorithm for identifying all the non-dominated solutions, for a linear multi-objective transportation problem. Kumar and Panda [3] proposed multi-objective optimization problem with bounded parameters. Anukokila *et al.* [19,21,22] discussed a fuzzy goal programming approach for solving multi-objective transportation problem with interval cost. Das *et al.* [6] presented a new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. Zangiabadi *et al.* [27] proposed an application of fuzzy goal programming to the linear multi-objective transportation problem. Ojha *et al.* [20] developed multi-objective geometric programming problem with Karush–Kuhn–Tucker condition using ($\in$) constraint method. Wahed [1] presented interactive fuzzy goal programming for a multi-objective transportation problem. Fegade *et al.* [8] discussed a method for solving fuzzy transportation using zero suffix method and robust ranking methodology. Liu *et al.* [15,17] proposed uncertain multi-objective programming and uncertain goal programming. Pal *et al.* [5,24] developed interval goal programming approach to multi-objective fuzzy goal programming with interval weights. Jahanshahloo *et al.* [12] discussed to find a solution for multi-objective linear fractional programming problem based on goal programming and data envelopment analysis. Upmanyu *et al.* [26] solved multi-objective set covering problem with imprecise linear fractional objectives. Chiang [4] found out the optimal solution of the transportation problem with fuzzy demand and fuzzy product.

3. Preliminaries

In this section, some necessary background and notions of interval valued fuzzy numbers are reviewed.

Definition 3.1. If the membership function of the fuzzy set $\widetilde{A}$ on R is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{\gamma(x-p)}{(q-p)}, & p < x \le q, \\ \dfrac{\gamma(r-x)}{(r-q)}, & q \le x \le r, \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < \gamma \le 1$, then $\widetilde{A}$ is called a level γ fuzzy numbers and it is denoted as $\widetilde{A} = (p, q, r; \gamma)$.

Definition 3.2. An interval valued fuzzy set $\widetilde{A}$ on R is given by

$$\widetilde{A} \triangleq \left\{ \left(x, \left[\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^U}(x)\right]\right) : x \in R \right\},$$

where $\mu_{\widetilde{A}^L}(x)$, $\mu_{\widetilde{A}^U}(x) \in [0,1]$ and $\mu_{\widetilde{A}^L}(x) \le \mu_{\widetilde{A}^U}(x)\ \forall\ \ x \in R$ and is denoted as $\widetilde{A} = [\widetilde{A}^L, \widetilde{A}^U]$. This means that the grade of membership of x belongs to the interval $\left[\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^U}(x)\right]$, the lower grade of membership at x is $\mu_{\widetilde{A}^L}(x)$ and the upper grade of membership at x is $\mu_{\widetilde{A}^U}(x)$. Let

$$\mu_{\widetilde{A}^L}(x) = \begin{cases} \dfrac{\gamma(x-p)}{(q-p)}, & p < x \le q, \\ \dfrac{\gamma(r-x)}{(r-q)}, & q \le x < r, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\widetilde{A}^L = (p, q, r; \gamma)$. Let

$$\mu_{\widetilde{A}^U}(x) = \begin{cases} \dfrac{\delta(x-l)}{(m-l)}, & l < x \le m \\ \dfrac{\delta(n-x)}{(n-m)}, & m \le x < n \\ 0, & \text{otherwise.} \end{cases}$$

Then $\widetilde{A}^U = (l, q, n; \delta)$. Here $0 < \gamma \le \delta \le 1$ and $l < p < q < r < n$. Then interval valued fuzzy set is

$$\widetilde{A} \triangleq \left\{ \left(x, \left[\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^U}(x)\right]\right) : x \in R \right\},$$

is also denoted as,

$$\widetilde{A} = [(p, q, r; \gamma), (l, q, n; \delta)] = \left[\widetilde{A}^L, \widetilde{A}^U\right]$$

$\widetilde{A}$ is called a level (γ, δ) interval valued fuzzy number (Fig. 1).

In Figure 1, interval values are marked in the x axis and level (γ, δ) are marked on the y axis. Lower grade membership function (γ) and upper grade membership function (δ) are marked in the figure. General form is represented as $[(p, q, r; \gamma), (l, q, n; \delta)] = \left[\widetilde{A}^L, \widetilde{A}^U\right]$.

Property 3.3. Let $F_{IV}(\gamma, \delta) = \{[(p, q, r; \gamma), (l, q, n; \delta)] : \forall\ \ l < p < q < r < n\}$, $0 < \gamma \le \delta \le 1$ be the family of (γ, δ) interval valued fuzzy numbers. Let $\widetilde{A} = [(p, q, r; \gamma), (l, q, n; \delta)]$ and $\widetilde{B} = [(p_1, q_1, r_1; \gamma), (l_1, q_1, n_1; \delta)] \in F_{IV}(\gamma, \delta)$ be two interval valued fuzzy numbers. The arithmetic operations between interval valued fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ are defined as follows:

$$\begin{aligned} \widetilde{A} \oplus \widetilde{B} &= [(p + p_1, q + q_1, r + r_1; \gamma), (l + l_1, m + q_1, n + n_1; \delta)], \\ K\widetilde{A} &= [(Kp, Kq, Kr; \gamma), (Kl, Kq, Kn; \delta)],\ \ K > 0, \\ K\widetilde{A} &= [(Kr, Kq, Kp; \gamma), (Kn, Kq, Kl; \delta)],\ \ K < 0, \\ K\widetilde{A} &= [(0, 0, 0; \gamma), (0, 0, 0; \delta)], K = 0. \end{aligned}$$

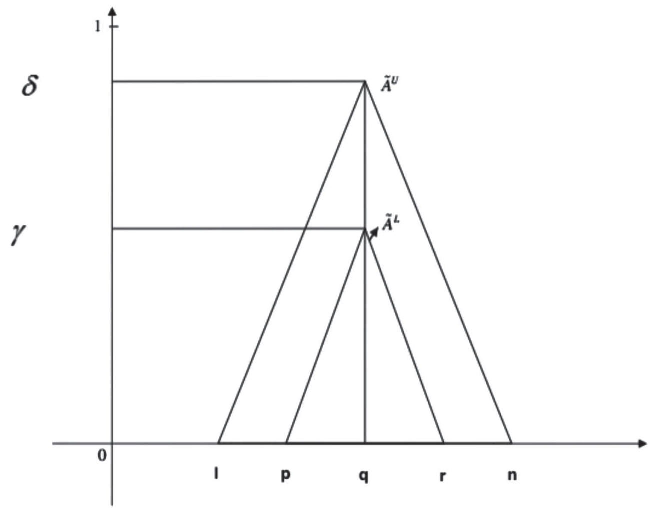

FIGURE 1. Level (γ, δ) interval valued fuzzy number.

Definition 3.4. Let $\widetilde{A} = [(p, q, r; \gamma), (l, q, n; \delta)] \in F_{IV}(\gamma, \delta), 0 < \gamma \leq \delta \leq 1$, so the signed distance of $\widetilde{A}$ from $\widetilde{0}$ (y-axis) is given as:

$$d_0(\widetilde{A}, \widetilde{0}) = \frac{1}{8}\left[6q + p + r + 4l + 4n + 3(2m - l - n)\frac{\gamma}{\delta}\right]. \tag{3.1}$$

Remark 3.5. If $\widetilde{A} = [(p - \alpha_3, p, p + \alpha_4; \gamma), (p - \alpha_1, p, p + \alpha_2; \delta)]$, where $0 < \alpha_3 < \alpha_1 < p$, $0 < \alpha_4 < \alpha_2$, then equation (3.1) reduces to $d_0(\widetilde{A}, \widetilde{0}) = 2p + \frac{1}{8}\left[(\alpha_4 - \alpha_3) + (4 - \frac{3\lambda}{p})(\alpha_2 - \alpha_1)\right]$.

Property 3.6. Let $\widetilde{A} = [(p, q, r; \gamma), (l, q, n; \delta)]$ and $\widetilde{B} = [(p, q, r; \gamma), (l, q, n; \delta)] \in F_{IV}(\gamma, \delta)$. Then,

$$d_0(\widetilde{A} \oplus \widetilde{B}, \widetilde{0}) = d_0(\widetilde{A}, \widetilde{0}) + d_0(\widetilde{B}, \widetilde{0})$$
$$d_0(K\widetilde{A}, \widetilde{0}) = K d_0(\widetilde{A}, \widetilde{0}), \ K > 0.$$

It is clear from property, that the signed distance ranking is a linear ranking on $F_{IV}(\gamma, \delta)$.

Definition 3.7. Let $(+, -, ., \div)$ be a binary operation on the set of real numbers. Consider X and Y are the closed intervals, then $X * Y = (x * y : x \in X, y \in y)$ defines a binary operation on the set of closed intervals. For division it is assumed that $0 \notin Y$. The interval operations are mentioned as follows:

$$X + Y = \langle x_c, x_w \rangle + \langle y_c, y_w \rangle = \langle x_c + y_c, x_w + y_w \rangle,$$
$$kX = k[x_L, x_R] = ([kx_L, kx_R] \text{ for } k \geq 0, [kx_R, kx_L] \text{ for } k < 0),$$
$$kX = k\langle x_c, x_w \rangle = \langle kx_c, |k|x_w \rangle,$$

where k is real number.

Definition 3.8. $x^0 \in S$ (S is the feasible region) yields optimal solution iff there is no other $x \in S$ such that,

$$\begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x^0_{ij}, \ \forall \ q, \\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x^0_{ij}, \ \text{for some} \ \ q, \ q = 1, 2, \ldots, \epsilon. \end{cases}$$

Definition 3.9. A point $x^0 \in S$ is efficient iff there does not exist another $x \in S$ and

$$\begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x^0_{ij},\ \forall\ q, \\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x_{ij} \neq \sum_{i=1}^{m}\sum_{j=1}^{n}\left[P^q_{L_{ij}}, P^q_{R_{ij}}\right] x^0_{ij},\ \forall\ q, q = 1, 2, \ldots, \epsilon. \end{cases}$$

Otherwise x^0 is an inefficient solution.

Definition 3.10. $x^0 \in S$ is an optimal solution iff there is no other solution $x \in S$ which satisfies $Z(x) <_{RC} Z(x^0)$. The right limit $Z^q_R(x)$ of the interval objective function $Z^q_R(x)$ may be elicited from Definition 3.7.

$$Z^q_R = \sum_{i=1}^{m}\sum_{j=1}^{n} p^q_{C_{ij}} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} p^q_{W_{ij}} \left|x_{ij}\right|,$$

where $p^q_{C_{ij}}$ is the center and $p^q_{W_{ij}}$ is the half width of the coefficient p^q_{ij} of $Z^q(x)$. In the case when $x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$. $Z^q_R(x)$ can be modified as

$$Z^q_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} p^q_{C_{ij}} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} p^q_{W_{ij}} x_{ij}.$$

The center of the objective function Z^p_C can be elicited as $Z^p_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} p^q_{C_{ij}} x_{ij}$.

4. Problem formulation

4.1. Linear multi-objective FTP

Linear programming formulation of linear multi-objective transportation problems in crisp environment are presented in this section. It can be stated mathematically as:

$$P_1: \begin{cases} \text{Minimize } Z^q(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} C^q_{ij} x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} D^q_{ij} x_{ij} + \beta}, \text{ where } q = 1, 2, \ldots, Q \\ \text{subject to} \\ \qquad \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \\ \qquad \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \\ \qquad x_{ij} \geq 0, \quad \forall\ i, j, \end{cases}$$

where $Z(x) = \left\{Z^1(x), Z^2(x), \ldots, Z^q(x)\right\}$ is a vector of q objective functions. x_{ij} represents the amount of the product to be shipped from ith source to the jth destination. C^q_{ij} is the penalty associated with transporting a unit of the product from ith source to the jth destination according to penalty criterion q. a_i represents the availability at ith source and b_j represents the demand at jth destination. Without loss of generality, it may be assumed that $a_i,\ b_j > 0,\ \ \forall\ \ i, j$ and $C^q_{ij},\ D^q_{ij} \geq 0,\ \forall\ \ i, j$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

4.2. Fuzzy environment

The linear multi-objective fractional transportation problems in fuzzy environment is presented in this section. Let all the parameters (cost, availability and demand) by level (γ, δ) interval valued fuzzy numbers. Then P_1 in fuzzy environment is:

$$P_2 : \begin{cases} \text{Minimize } \widetilde{Z}^q(x) \approx \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{C}_{ij}^q x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{D}_{ij}^q x_{ij} + \beta}, \text{ where } q = 1, 2, \dots, Q \\ \text{subject to} \\ \qquad \sum_{j=1}^{n} x_{ij}\widetilde{1} \approx \widetilde{a}_i, \quad i = 1, 2, \dots, m \\ \qquad \sum_{i=1}^{m} x_{ij}\widetilde{1} \approx \widetilde{b}_j, \quad j = 1, 2, \dots, n \\ \qquad\quad x_{ij} \geq 0, \quad \forall \ i, j \end{cases}$$

with $\sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} \widetilde{b}_j$, where $\widetilde{a}_i = [(a_i - \alpha_{3i}, a_i, a_i + \alpha_{4i}; \gamma), (a_i - \alpha_{1i}, a_i, a_i + \alpha_{2i}; \delta)]$ and $0 < \alpha_{3i} < \alpha_{1i} < a_i$, $0 < \alpha_{4i} < \alpha_{2i}, i = 1, 2, \dots, m$. Similarly, interval valued fuzzy numbers $\widetilde{b}_j$ and $\widetilde{C}_{ij}^q$ are given as:

$$\widetilde{b}_j = [(b_j - \beta_{3j}, b_j, b_j + \beta_{4j}; \gamma), (b_j - \beta_{1j}, b_j, b_j + \beta_{2j}; \delta)]$$

and

$$\begin{aligned} 0 < \beta_{3j} < \beta_{1j} < b_j, \quad 0 < \beta_{4j} < \beta_{2j}, \quad j = 1, 2, \dots, n \\ \widetilde{C}_{ij}^q = \left[(C_{ij}^q - \Delta_{3ij}^q, C_{ij}^q, C_{ij}^q + \Delta_{4ij}^q; \gamma), (C_{ij}^q - \Delta_{1ij}^q, C_{ij}^q, C_{ij}^q + \Delta_{2ij}^q; \delta)\right] \\ \widetilde{D}_{ij}^q = \left[(D_{ij}^q - \mu_{3ij}^q, D_{ij}^q, D_{ij}^q + \mu_{4ij}^q; \gamma), (D_{ij}^q - \mu_{1ij}^q, D_{ij}^q, D_{ij}^q + \mu_{2ij}^q; \delta)\right] \end{aligned}$$

and

$$\begin{gathered} 0 < \Delta_{3ij}^q < \Delta_{1ij}^q < C_{ij}^q, 0 < \Delta_{4ij}^q < \Delta_{2ij}^q, \\ 0 < \mu_{3ij}^q < \mu_{1ij}^q < D_{ij}^q, 0 < \mu_{4ij}^q < \mu_{2ij}^q \text{ for } i = 1, 2, \dots, m, \ j = 1, 2, \dots, n \ \text{ and } \ q = 1, 2, \dots, Q; \\ \widetilde{1} = [(1, 1, 1; \gamma), (1, 1, 1; \delta)]. \end{gathered}$$

4.3. Existing method

The shortcomings of the existing methods [9] are pointed out. It can be applied for solving single objective fractional transportation problem of the type:

$$P_3 : \begin{cases} \text{Minimize } Z(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} D_{ij} x_{ij} + \beta}, \\ \text{subject to} \\ \qquad \sum_{j=1}^{n} x_{ij}\widetilde{1} \approx \widetilde{a}_i, \ i = 1, 2, \dots, m \\ \qquad \sum_{i=1}^{m} x_{ij}\widetilde{1} \approx \widetilde{b}_j, \ j = 1, 2, \dots, n \\ \qquad\quad x_{ij} \geq 0, \quad \forall \ i, j \end{cases}$$

along with the conditions: $\sum\limits_{i=1}^{m}\widetilde{a_i} \approx \sum\limits_{j=1}^{n}\widetilde{b_j}$ and $\sum\limits_{i=1}^{m}a_i = \sum\limits_{j=1}^{n}b_j$,
where $\widetilde{a_i} = [(a_i-\alpha_{3i}, a_i, a_i+\alpha_{4i};\gamma),(a_i-\alpha_{1i}, a_i, a_i+\alpha_{2i};\delta)]$ and $0<\alpha_{3i}<\alpha_{1i}<a_i$, $0<\alpha_{4i}<\alpha_{2i}$, $\widetilde{b_j} = [(b_j-\beta_{3j}, b_j, b_j+\beta_{4j};\gamma),(b_j-\beta_{1j}, b_j, b_j+\beta_{2j};\delta)]$ and $0<\beta_{3j}<\beta_{1j}<b_j$, $0<\beta_{4j}<\beta_{2j}$, $\widetilde{1} = [(1,1,1;\gamma),(1,1,1;\delta)]$.

If the conditions $\sum_{i=1}^{m}a_i = \sum_{j=1}^{n}b_j$ is not satisfied in problem P_3, then the existing method [9] cannot be used for solving problems of the type P_3, because the conditions $\sum_{i=1}^{m}\widetilde{a_i} \approx \sum_{j=1}^{n}\widetilde{b_j}$ and $\sum_{i=1}^{m}a_i \neq \sum_{j=1}^{n}b_j$ lead to violation of consistency condition given by the equation number (26) in [4].

4.4. Proposed method

A new method is proposed for the solution of linear multi-objective FTP in P_2 which all the parameters (cost, availability and demand) are represented by (γ,δ) interval valued fuzzy numbers. Following are the steps to find the solution of problem P_2 are:

Step 1:

$$\begin{cases} \text{Minimize } d_0(\widetilde{Z}^q(x),\widetilde{0}) = d_0\left(\dfrac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\widetilde{C}_{ij}^q x_{ij}+\alpha}{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\widetilde{D}_{ij}^q x_{ij}+\beta}, 0\right), \quad q=1,2,\ldots,Q. \\ \text{subject to} \\ \qquad d_0\left(\sum\limits_{j=1}^{n}x_{ij}\widetilde{1},\widetilde{0}\right) = d_0(\widetilde{a_i},\widetilde{0}), \quad i=1,2,\ldots,m \\ \qquad d_0\left(\sum\limits_{i=1}^{m}x_{ij}\widetilde{1},\widetilde{0}\right) = d_0(\widetilde{b_j},\widetilde{0}), \quad j=1,2,\ldots,n \\ \qquad x_{ij}\geq 0 \ \ \forall\ i,j, \end{cases}$$

for consistency $d_0\left(\sum\limits_{i=1}^{m}\widetilde{a_i},\widetilde{0}\right) = d_0\left(\sum\limits_{j=1}^{n}\widetilde{b_j},\widetilde{0}\right)$.

Step 2: Using Property 3.6 and Remark 3.5, the crisp model of problem P_2 is

$$P_4: \begin{cases} \text{Minimize } Z^q(x) = \dfrac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(C_{ij}^q)^1 x_{ij}}{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(D_{ij}^q)^1 x_{ij}}, \quad q=1,2,\ldots,Q. \\ \text{subject to} \\ \qquad \sum\limits_{j=1}^{n}x_{ij} = (a_i)^1, \quad i=1,2,\ldots,m \\ \qquad \sum\limits_{i=1}^{m}x_{ij} = (b_j)^1, \quad j=1,2,\ldots,n \\ \text{for consistency } \sum\limits_{i=1}^{m}(a_i)^1 = \sum\limits_{j=1}^{n}(b_j)^1 \\ \qquad x_{ij}\geq 0, \quad \forall \quad i,j, \end{cases}$$

where $Z^q(x) = d_0(\widetilde{Z}^q(x), \widetilde{0})$.

$$(C_{ij}^q)^1 = 2C_{ij}^q + \frac{1}{8}\left[(\Delta_{4ij}^q - \Delta_{3ij}^q) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\Delta_{2ij}^q - \Delta_{1ij}^q)\right];$$
$$(D_{ij}^q)^1 = 2D_{ij}^q + \frac{1}{8}\left[(\mu_{4ij}^q - \mu_{3ij}^q) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\mu_{2ij}^q - \mu_{1ij}^q)\right];$$
$$(a_i)^1 = a_i + \frac{1}{16}\left[(\alpha_{4i} - \alpha_{3i}) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\alpha_{2i} - \alpha_{1i})\right];$$
$$(b_j)^1 = b_j + \frac{1}{16}\left[(\beta_{4j} - \beta_{3j}) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\beta_{2j} - \beta_{1j})\right].$$

Step 3: Now P_4 is classical multi-objective fractional transportation problem and can be solved by any classical multi-objective linear fractional programming approach.

4.5. Fractional transportation problem (FTP)

The FTP is the problem of minimizing q interval valued objective functions with interval cost. When the objective functions coefficients $\frac{C_{ij}^q}{D_{ij}^q}$, A_i is the source parameters, B_j is the destination parameter and C_q, D_q are the conveyance parameters, which are in the form of interval, where $A_i = [s_{L_i}, s_{R_i}], i = 1, 2, \ldots, m$ and $B_j = [t_{L_j}, t_{R_j}], j = 1, 2, \ldots, n$, are interval values of source and destination. The formulation for interval fuzzy problem is

$$\begin{cases} \text{Minimize } Z^q(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{L_{ij}}^q, C_{R_{ij}}^q\right]x_{ij}+\alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left[D_{L_{ij}}^q, D_{R_{ij}}^q\right]x_{ij}+\beta}, q = 1, 2, \ldots, Q \\ \text{subject to} \\ \quad \sum_{j=1}^{n} x_{ij} = A_i = [s_{L_i}, s_{R_i}], i = 1, 2, \ldots, m \\ \quad \sum_{i=1}^{m} x_{ij} = B_j = [t_{L_j}, t_{R_j}], j = 1, 2, \ldots, n \\ \quad x_{ij} \geq 0, \quad \forall \ \ i, j. \end{cases}$$

The balanced condition is a necessary and sufficient condition for the existence of a feasible solution

$$\left[C_{L_{ij}}^q, C_{R_{ij}}^q\right] \text{ and } \left[D_{L_{ij}}^q, D_{R_{ij}}^q\right], \ (q = 1, 2, \ldots, Q),$$
$$P_{ij}^q = \left[P_{L_{ij}}^q, \ P_{R_{ij}}^q\right] = \frac{C_{ij}^q}{D_{ij}^q} = \frac{C_{L_{ij}}^q, C_{R_{ij}}^q}{D_{L_{ij}}^q, D_{R_{ij}}^q}$$

is an interval representing the uncertain cost for the transportation problem. When the feasible solutions are uncertain cost for the transportation problem. Uncertainty specifically concerns right hand side constraints and objective functions. When the set of feasible solutions is uncertain, we identify a class of linear programs for which these classical approaches are no longer relevant. However it is possible to compute the worst optimum solution. By the above definition the equivalent multi-objective deterministic transportation problem as

$$\begin{cases} \text{Minimize} \quad Z_R^q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} P_{C_{ij}}^q x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} P_{W_{ij}}^q x_{ij} \\ \text{Minimize} \quad Z_C^q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} P_{C_{ij}}^q x_{ij} \\ \text{subject to} \\ \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \le s_{R_i}, \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \le s_{L_i} \\ \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \le t_{R_j}, \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \le t_{L_j} \\ \qquad\qquad x_{ij} \ge 0, \quad \forall \;\; i, \; j. \end{cases}$$

4.6. Goal programming

Fuzzy goal programming involves applying the fuzzy set theory to goal programming. The fuzzy goals are then characterized by the membership functions which are transformed into fuzzy flexible membership goals by means of introducing negative and positive deviational variables and assigning highest membership value to each of them. The main purpose is to minimize the deviations between the achievement of goals $Z^q(x)$ and aspiration levels G_1 and G_2. A mathematical formulation of goal programming is given below:

$$\begin{cases} \text{Minimize } Z^q(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} \left| \left[C_{L_{ij}}^q, C_{R_{ij}}^q\right] x_{ij} + \alpha - G_1 \right|}{\sum_{i=1}^{m}\sum_{j=1}^{n} \left| \left[D_{L_{ij}}^q, D_{R_{ij}}^q\right] x_{ij} + \beta - G_2 \right|} \\ \text{subject to} \\ \qquad \sum_{j=1}^{n} x_{ij} = A_i = [s_{L_i}, s_{R_i}], i = 1, 2, \ldots, m, \\ \qquad \sum_{i=1}^{m} x_{ij} = B_j = [t_{L_j}, t_{R_j}], j = 1, 2, \ldots, n, \\ \qquad\quad x_{ij} \ge 0, \quad \forall \;\; i, j, \quad x \in F \quad (F \text{ is a feasible set}), \end{cases}$$

where $Z^q(x)$ is the linear function of the qth goal, G_q is the aspiration level of the qth goal. To solve the goal programming, let the function $Z^q(x) = D_q^+ - D_q^- + G_q$. Then the achievement function can be formulated as,

$$\begin{cases} \text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} \left(D_q^+ - D_q^-\right) \\ \text{subject to} \\ \qquad \dfrac{\left[C_{L_{ij}}^q, C_{R_{ij}}^q\right] x_{ij} - G_1}{\left[D_{L_{ij}}^q, D_{R_{ij}}^q\right] x_{ij} - G_2} = D_q^+ - D_q^-, q = 1, 2, \ldots, Q \\ \qquad X \in F \;\; (F \text{ is a feasible set}) \\ \qquad D_q^+ - D_q^- \ge 0, \quad q = 1, 2, \ldots, Q. \end{cases}$$

4.6.1. Min–Max approach

There are number of different methods have been developed goal programming, pre-emptive goal programming and min–max goal programming are some of them. Among these the min–max approach of the fuzzy goal programming by Zimmermann [28], convert to the following model.

$$\begin{cases} \text{Minimize } \psi \\ \text{subject to} \\ \quad \sum_{j=1}^{n} x_{ij} = [s_{L_i}, s_{R_i}] = A_i, i = 1, 2, \ldots, m \\ \quad \sum_{i=1}^{m} x_{ij} = [t_{L_j}, t_{R_j}] = B_j, j = 1, 2, \ldots, n \\ \quad Z^q(x) + D_q^- - D_q^+ = G_q^-, \quad q = 1, 2, \ldots, Q \\ \qquad \psi \geq D_q^+, \quad q = 1, 2, \ldots, Q \\ \quad D_q^+, D_q^- \geq 0, D_q^+ D_q^- = 0, \quad q = 1, 2, \ldots, Q \\ \qquad x_{ij} \geq 0, \quad \forall\, i, j \end{cases}$$

where the equilibrium condition $\sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j$ is satisfied.

4.6.2. Hyperbolic membership function

Membership function is one of the most important element of the fuzzy approach and it allows a fuzzy approach to evaluate uncertain and ambiguous matters. The role of the membership function is to represent an individual and subjective human perception as a member of a fuzzy set. Besides an exponential membership function hyperbolic function are non-linear function, a fuzzy mathematical programming with a non-linear membership function is non-linear programming. Hyperbolic functions arise from simple combinations of the exponential function. The hyperbolic function is convex over a part of the objective function value and is concave over the remaining part.

Hyperbolic membership function can be defined as,

$$\mu_q^H(Z^q(x)) = \frac{1}{2}\tanh\left[\frac{U_q + L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n} Z^q(x) x_{ij}\right] q + \frac{1}{2}$$

where $\alpha_q = \frac{6}{U_q - L_q}$. The fuzzy programming with hyperbolic membership functions for obtaining efficient solutions as well as the best compromise solution of a multi-objective fractional transportation problem. It is used to represent objective functions into fuzzy environment. This membership function has the following properties.

(1) $\mu_q^H(X^q(x))$ is strictly monotonically decreasing function with respect to $Z^q(x)$.
(2) $\mu_q^H(Z^q(x)) = \frac{1}{2}$ iff $Z^q(x) = \frac{1}{2}(U^q + L^q)$.
(3) $\mu_q^H(Z^q(x))$ is strictly convex for $Z^q(x) \leq \frac{1}{2}(U_q + L_q)$.
(4) $\mu_q^H(Z^q(x))$ satisfies $0 < \mu_q^H(Z^q(x)) < 1$ for $L_q < F^q(x) < 1$ for $L_q < Z^q(x) < U_q$.

An hyperbolic function of the qth objective function can be defined as

$$\mu_q^H Z^q(x) = \begin{cases} 1, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{C_R^q, C_C^q}{D_R^q, D_C^q} x_{ij} \leq L_q \\ \frac{1}{2}\tanh\left[\left(\frac{U_q + L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{C_R^q, C_C^q}{D_R^q, D_C^q} x_{ij}\right)\alpha_q\right] + \frac{1}{2}, & \text{if } L_q < \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{C_R^q, C_C^q}{D_R^q, D_C^q} x_{ij} < U_q \\ 0, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{C_R^q, C_C^q}{D_R^q, D_C^q} x_{ij} \geq U_q \end{cases}$$

where $\alpha_q = \frac{6}{U_q - L_q}$. Then an equivalent crisp model for the fuzzy model can be formulated as

$$\begin{cases} \text{Maximize } \phi \\ \text{subject to} \\ \qquad \phi \leq \frac{1}{2}\tanh\left[\left(\frac{U_q+L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{C_R^q, C_C^q}{D_R^q, D_C^q}x_{ij}\right)\alpha_q\right] + \frac{1}{2} \\ \sum_{j=1}^{n} x_{ij} = [s_{L_i}, s_{R_i}] = A_i, i = 1, 2, \ldots, m \\ \sum_{i=1}^{m} x_{ij} = [t_{L_j}, t_{R_j}] = B_j, j = 1, 2, \ldots, n \\ x_{ij} \geq 0, \quad \forall \;\; i, j \geq 0. \end{cases}$$

5. Fractional goal programming approach

In this section, Mohamed in [18] used linear membership functions, in which he introduced fuzzy goal programming approach to a multi-objective linear programming problem. In this paper we introduce the deviational variables $D_q^-, D_q^+ \geq 0$ corresponding to the qth hyperbolic membership function, the flexible membership goal with aspired level can be presented as,

$$\frac{1}{2}\tanh\left[\frac{U_q + L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{C_R^q, C_C^q}{D_R^q, D_C^q}x_{ij}\right]\alpha_q + \frac{1}{2} + D_q^- - D_q^+ = 1$$

where $D_q^- D_q^+ = 0$. Now, we apply the min–max form of goal programming to the fuzzy model of multi-objective transportation problem, with the hyperbolic membership function leads the following model:

$$\begin{cases} \text{Minimize } \phi \\ \text{subject to} \\ \frac{1}{2}\tanh\left[\left(\frac{U_q+L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{C_R^q, C_C^q}{D_R^q, D_C^q}x_{ij}\right)\alpha_q\right] + \frac{1}{2} + D_q^- - D_q^+ = 1 \\ \qquad \phi \geq D_q^-, \quad q = 1, 2, \ldots, Q \end{cases}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = [s_{L_i}, s_{R_i}] = A_i, \;\; i = 1, 2, \ldots, m \\ \sum_{i=1}^{m} x_{ij} = [t_{L_j}, t_{R_j}] = B_j, \;\; j = 1, 2, \ldots, n \\ \qquad \phi \leq 1, \phi \geq 0, x_{ij} \geq 0 \quad \forall \;\; i, j. \end{cases}$$

The equilibrium condition $\sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j$ is satisfied. When the objective function coefficients C_{ij}^q are in the form of interval, that is $C_{ij}^q = \left[C_{L_{ij}}^q, C_{R_{ij}}^q\right]$ and the constraints are deterministic, interval valued transportation problem will be defined as,

$$\begin{cases} \text{Minimize } Z^q(x) = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\left[C^q_{L_{ij}}, C^q_{R_{ij}}\right] x_{ij}}{\left[D^q_{L_{ij}}, D^q_{R_{ij}}\right] x_{ij}}, \; q = 1, 2, \ldots, Q \\ \text{subject to} \\ \qquad \displaystyle\sum_{j=1}^{n} x_{ij} = A_i, i = 1, 2, \ldots, m, \\ \qquad \displaystyle\sum_{i=1}^{m} x_{ij} = B_j, j = 1, 2, \ldots, n, \\ \qquad x_{ij} \geq 0, \quad \forall \; i, j. \end{cases}$$

6. Numerical example

The existing methods [9] cannot be used for solving single objective transportation problem by using existing method is illustrated here:

$$\begin{cases} \text{Minimize } Z(x) = \dfrac{\begin{array}{c} 11x_{11} + 8x_{12} + 14x_{13} + 19x_{14} + 6x_{21} + 18x_{22} + 9x_{23} + 4x_{24} \\ + 16x_{31} + 18x_{32} + 8x_{33} + 12x_{34} + 12x_{41} + 12x_{42} + 12x_{43} + 16x_{44} + \alpha \end{array}}{\begin{array}{c} 16x_{11} + 38x_{12} + 34x_{13} + 22x_{14} + 6x_{21} + 28x_{22} + 32x_{23} + 22x_{24} \\ + 2x_{31} + 9x_{32} + 6x_{33} + 6x_{34} + 12x_{41} + 14x_{32} + 22x_{43} + 11x_{44} + \beta \end{array}} \\ \text{subject to} \quad \displaystyle\sum_{j=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{a_i}, \quad i = 1, 2, 3, 4, \\ \qquad\qquad \displaystyle\sum_{i=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{b_j}, \quad j = 1, 2, 3, 4, \\ \qquad\qquad x_{ij} \geq 0 \quad \forall \; i, j, \end{cases}$$

with $\sum_{i=1}^{4} \widetilde{a_i} \approx \sum_{j=1}^{4} \widetilde{b_j}$ where the values of $\widetilde{a_i}$, and $\widetilde{b_j}$, for $i, j = 1, 2, 3, 4$ are given as

$$\begin{array}{ll} \widetilde{a_1} = [(3, 10, 12; 0.6), (1, 10, 14; 0.9)] & \widetilde{a_2} = [(15, 18, 20; 0.6), (13, 18, 22; 0.9)] \\ \widetilde{a_3} = [(17, 22, 24; 0.6), (15, 22, 26; 0.9)] & \widetilde{a_4} = [(10, 12, 15; 0.6), (7, 12, 15; 0.9)] \\ \widetilde{b_1} = [(13, 18, 20; 0.6), (12, 18, 21; 0.9)] & \widetilde{b_2} = [(9, 13, 18; 0.6), (6, 13, 19; 0.9)] \\ \widetilde{b_3} = [(10, 12, 14; 0.6), (9, 12, 16; 0.9)] & \widetilde{b_4} = [(13, 15, 20; 0.6), (10, 15, 23; 0.9)]\,. \end{array} \tag{6.1}$$

The existing method cannot be used for solving single objective transportation problem as all the parameters $c_{ij}, \; a_i, \; b_j$ are represented by (γ, δ) interval valued fuzzy numbers

$$\begin{cases} \text{Minimize } \widetilde{Z}(x) \approx \dfrac{\sum_{i=1}^{4}\sum_{j=1}^{4} \widetilde{C}_{ij} x_{ij} + \alpha}{\sum_{i=1}^{4}\sum_{j=1}^{4} \widetilde{D}_{ij} x_{ij} + \beta} \\ \text{subject to} \\ \qquad \displaystyle\sum_{j=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{a_i}, \quad i = 1, 2, 3, 4. \\ \qquad \displaystyle\sum_{i=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{b_j}, \quad j = 1, 2, 3, 4. \\ \qquad x_{ij} \geq 0 \quad \forall \; i, j, \end{cases}$$

with $\sum_{i=1}^{4} \widetilde{a}_i \approx \sum_{j=1}^{4} \widetilde{b}_j$ where the values of $\widetilde{C}_{ij}, \widetilde{D}_{ij}$ for $i = 1,2,3,4, j = 1,2,3,4$, are:

$$\begin{aligned}
\widetilde{C}_{11} &= [(3,6,8;0.6),(2,6,9;0.9)] & \widetilde{C}_{12} &= [(1,3,8;0.6),(1,3,9;0.9)]\\
\widetilde{C}_{13} &= [(4,6,12;0.6),(2,6,16;0.9)] & \widetilde{C}_{14} &= [(6,7,8;0.6),(5,7,29;0.9)]\\
\widetilde{C}_{21} &= [(2,3,4;0.6),(1,3,5;0.9)] & \widetilde{C}_{22} &= [(6,8,12;0.6),(5,8,18;0.9)]\\
\widetilde{C}_{23} &= [(2,3,4;0.6),(1,3,18;0.9)] & \widetilde{C}_{24} &= [(0.5,1,5.5;0.6),(0.25,1,7.75;0.9)]\\
\widetilde{C}_{31} &= [(5,7,13;0.6),(3,7,17;0.9)] & \widetilde{C}_{32} &= [(8.5,9,9.5;0.6),(7,9,11;0.9)]\\
\widetilde{C}_{33} &= [(2,3,4;0.6),(1,3,13;0.9)] & \widetilde{C}_{34} &= [(5,6,7;0.6),(3,6,9;0.9)]\\
\widetilde{C}_{41} &= [(3,6,9;0.6),(2,6,12;0.9)] & \widetilde{C}_{42} &= [(5,6,9;0.6),(3,6,12;0.9)]\\
\widetilde{C}_{43} &= [(2,5,9;0.6),(1,5,17;0.9)] & \widetilde{C}_{44} &= [(2,7,12;0.6),(1,7,19;0.9)]
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}_{11} &= [(6,7,12;0.6),(5,7,15;0.9)] & \widetilde{D}_{12} &= [(17,20,21;0.6),(11,20,22;0.9)]\\
\widetilde{D}_{13} &= [(15,16,21;0.6),(14,16,24;0.9)] & \widetilde{D}_{14} &= [(10,11,12;0.6),(9,11,13;0.9)]\\
\widetilde{D}_{21} &= [(1.5,2,4.5;0.6),(1,2,10;0.9)] & \widetilde{D}_{22} &= [(13,14,15;0.6),(11,14,17;0.9)]\\
\widetilde{D}_{23} &= [(14,15,20;0.6),(13,15,23;0.9)] & \widetilde{D}_{24} &= [(10,11,12;0.6),(8,11,13;0.9)]\\
\widetilde{D}_{31} &= [(1.5,3,4.5;0.6),(1,3,9;0.9)] & \widetilde{D}_{32} &= [(2.5,4,5.5;0.6),(2,4,10;0.9)]\\
\widetilde{D}_{33} &= [(2,3,5;0.6),(1,3,5;0.9)] & \widetilde{D}_{34} &= [(5,6,12;0.6),(3,6,16;0.9)]\\
\widetilde{D}_{41} &= [(3,6,8;0.6),(2,6,12;0.9)] & \widetilde{D}_{42} &= [(6,7,8;0.6),(3,7,9;0.9)]\\
\widetilde{D}_{43} &= [(6,11,16;0.6),(5,11,17;0.9)] & \widetilde{D}_{44} &= [(2,6,7;0.6),(1,6,8;0.9)]\,.
\end{aligned} \tag{6.2}$$

The values of $\widetilde{a}_i$ for $i = 1,2,3,4$ and $\widetilde{b}_j$ for $j = 1,2,3,4$ are given as in (6.1) Proposed method is illustrated here with the help of an example

$$\begin{cases}
\text{Minimize } \widetilde{Z}^1(x) \approx \dfrac{\sum_{i=1}^{4}\sum_{j=1}^{4}\widetilde{C}^1_{ij}x_{ij}+\alpha}{\sum_{i=1}^{4}\sum_{j=1}^{4}\widetilde{D}^1_{ij}x_{ij}+\beta}\\
\text{Minimize } \widetilde{Z}^2(x) \approx \dfrac{\sum_{i=1}^{4}\sum_{j=1}^{4}\widetilde{C}^2_{ij}x_{ij}+\alpha}{\sum_{i=1}^{4}\sum_{j=1}^{4}\widetilde{D}^2_{ij}x_{ij}+\beta}\\
\text{subject to}\\
\qquad \sum_{j=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{a}_i, \quad i = 1,2,3,4.\\
\qquad \sum_{i=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{b}_j, \quad j = 1,2,3,4.\\
\qquad\quad x_{ij} \geq 0 \quad \forall \;\; i,j \;\; \text{with } \sum_{i=1}^{4}\widetilde{a}_i \approx \sum_{j=1}^{4}\widetilde{b}_j
\end{cases}$$

where the values of $\widetilde{C}^q_{ij}$, for $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$ and $q = 1, 2$ are

$$\begin{aligned}
\widetilde{C}^1_{11} &= [(3,6,8;0.6),(2,6,9;0.9)] & \widetilde{C}^1_{12} &= [(1,3,8;0.6),(1,3,9;0.9)]\\
\widetilde{C}^1_{13} &= [(4,6,12;0.6),(2,6,16;0.9)] & \widetilde{C}^1_{14} &= [(6,7,8;0.6),(5,7,9;0.9)]\\
\widetilde{C}^1_{21} &= [(2,3,4;0.6),(1,3,5;0.9)] & \widetilde{C}^1_{22} &= [(6,8,12;0.6),(5,8,18;0.9)]\\
\widetilde{C}^1_{23} &= [(2,3,4;0.6),(1,3,18;0.9)] & \widetilde{C}^1_{24} &= [(0.5,1,5.5;0.6),(0.25,1,7.75;0.9)]\\
\widetilde{C}^1_{31} &= [(5,7,13;0.6),(3,7,17;0.9)] & \widetilde{C}^1_{32} &= [(8.5,9,9.5;0.6),(7,9,11;0.9)]\\
\widetilde{C}^1_{33} &= [(2,3,4;0.6),(1,3,13;0.9)] & \widetilde{C}^1_{34} &= [(5,6,7;0.6),(3,6,9;0.9)]\\
\widetilde{C}^1_{41} &= [(3,6,9;0.6),(2,6,12;0.9)] & \widetilde{C}^1_{42} &= [(5,6,9;0.6),(3,6,12;0.9)]\\
\widetilde{C}^1_{43} &= [(2,5,9;0.6),(1,5,17;0.9)] & \widetilde{C}^1_{44} &= [(2,7,12;0.6),(1,7,19;0.9)]
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}^1_{11} &= [(6,7,12;0.6),(5,7,15;0.9)] & \widetilde{D}^1_{12} &= [(17,20,21;0.6),(11,20,22;0.9)]\\
\widetilde{D}^1_{13} &= [(15,16,21;0.6),(14,16,24;0.9)] & \widetilde{D}^1_{14} &= [(10,11,12;0.6),(9,11,13;0.9)]\\
\widetilde{D}^1_{21} &= [(1.5,2,4.5;0.6),(1,2,10;0.9)] & \widetilde{D}^1_{22} &= [(13,14,15;0.6),(11,14,17;0.9)]\\
\widetilde{D}^1_{23} &= [(14,15,20;0.6),(13,15,23;0.9)] & \widetilde{D}^1_{24} &= [(10,11,12;0.6),(8,11,13;0.9)]\\
\widetilde{D}^1_{31} &= [(1.5,3,4.5;0.6),(1,3,9;0.9)] & \widetilde{D}^1_{32} &= [(2.5,4,5.5;0.6),(2,4,10;0.9)]\\
\widetilde{D}^1_{33} &= [(2,3,5;0.6),(1,3,5;0.9)] & \widetilde{D}^1_{34} &= [(5,6,12;0.6),(3,6,16;0.9)]\\
\widetilde{D}^1_{41} &= [(3,6,8;0.6),(2,6,12;0.9)] & \widetilde{D}^1_{42} &= [(6,7,8;0.6),(3,7,9;0.9)]\\
\widetilde{D}^1_{43} &= [(6,11,16;0.6),(5,11,17;0.9)] & \widetilde{D}^1_{44} &= [(2,6,7;0.6),(1,6,8;0.9)]
\end{aligned}$$

$$\begin{aligned}
\widetilde{C}^2_{11} &= [(3,6,10;0.6),(4,6,9;0.9)] & \widetilde{C}^2_{12} &= [(5,8,12;0.6),(6,8,12;0.9)]\\
\widetilde{C}^2_{13} &= [(4,8,13;0.6),(6,8,15;0.9)] & \widetilde{C}^2_{14} &= [(3,8,14;0.6),(6,8,16;0.9)]\\
\widetilde{C}^2_{21} &= [(6.5,8.5,12;0.6),(10.5,8.5,9.5;0.9)] & \widetilde{C}^2_{22} &= [(6.5,9.5,12.5;0.6),(7,9.5,12.5;0.9)]\\
\widetilde{C}^2_{23} &= [(6,9.5,16;0.6),(6,9.5,14;0.9)] & \widetilde{C}^2_{24} &= [(5,7,13;0.6),(3,7,17;0.9)]\\
\widetilde{C}^2_{31} &= [(2.5,4,5.5;0.6),(2,4,10;0.9)] & \widetilde{C}^2_{32} &= [(6,8,12;0.6),(2,8,18;0.9)]\\
\widetilde{C}^2_{33} &= [(2,7,13;0.6),(6,7,10;0.9)] & \widetilde{C}^2_{34} &= [(4,13,23;0.6),(6,13,29;0.9)]\\
\widetilde{C}^2_{41} &= [(6,8,17;0.6),(5,8,17;0.9)] & \widetilde{C}^2_{42} &= [(12,16,21;0.6),(7,16,26;0.9)]\\
\widetilde{C}^2_{43} &= [(5,9,19;0.6),(6,9,19;0.9)] & \widetilde{C}^2_{44} &= [(9.5,12,19.5;0.6),(7.5,12,20;0.9)]
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}^2_{11} &= [(6,7,12;0.6),(3,7,15;0.9)] & \widetilde{D}^2_{12} &= [(10,13,19;0.6),(5,13,22;0.9)]\\
\widetilde{D}^2_{13} &= [(7,14,22;0.6),(10,14,19;0.9)] & \widetilde{D}^2_{14} &= [(4,11,21;0.6),(3,11,21;0.9)]\\
\widetilde{D}^2_{21} &= [(12,16,21;0.6),(9,16,24;0.9)] & \widetilde{D}^2_{22} &= [(15,19,24;0.6),(17,19,26;0.9)]\\
\widetilde{D}^2_{23} &= [(18,22,28;0.6),(20,22,29;0.9)] & \widetilde{D}^2_{24} &= [(12,18,28;0.6),(16,18,21;0.9)]\\
\widetilde{D}^2_{31} &= [(6,12,19;0.6),(7,12,23;0.9)] & \widetilde{D}^2_{32} &= [(12,17,23;0.6),(10,17,28;0.9)]\\
\widetilde{D}^2_{33} &= [(9,16,27;0.6),(10,16,24;0.9)] & \widetilde{D}^2_{34} &= [(0,16,26;0.6),(8,16,28;0.9)]\\
\widetilde{D}^2_{41} &= [(19,23,29;0.6),(20,23,29;0.9)] & \widetilde{D}^2_{42} &= [(12,19,29;0.6),(14,22,31;0.9)]\\
\widetilde{D}^2_{43} &= [(22,24,31;0.6),(21,24,33;0.9)] & \widetilde{D}^2_{44} &= [(22,27,33;0.6),(21,27,35;0.9)]
\end{aligned}$$

The values of $\widetilde{a}_i$ for $i = 1, 2, 3, 4$ and $\widetilde{b}_j$ for $j = 1, 2, 3, 4$ are given as in (6.1)

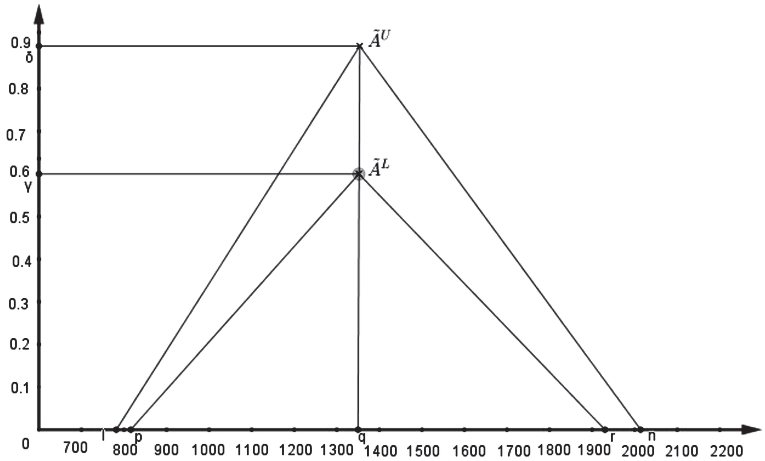

FIGURE 2. Level (γ, δ) interval valued fuzzy number for $Z^1(x)$.

Using step 1 and step 2 of proposed method, the crisp model P_4

$$
\begin{cases}
\text{Minimize } Z^1(x) = \dfrac{\begin{array}{r} 11x_{11} + 8x_{12} + 14x_{13} + 19x_{14} + 6x_{21} + 18x_{22} + 9x_{23} \\ +4x_{24} + 16x_{31} + 18x_{32} + 8x_{33} + 12x_{34} \\ + 12x_{41} + 12x_{42} + 12x_{43} + 16x_{44} + \alpha \end{array}}{\begin{array}{r} 16x_{11} + 38x_{12} + 34x_{13} + 22x_{14} + 6x_{21} + 28x_{22} + 32x_{23} \\ +22x_{24} + 2x_{31} + 9x_{32} + 6x_{33} + 6x_{34} \\ + 12x_{41} + 14x_{42} + 22x_{43} + 11x_{44} + \beta \end{array}} \\
\text{Minimize } Z^2(x) = \dfrac{\begin{array}{r} 12x_{11} + 16x_{12} + 16x_{13} + 17x_{14} + 20x_{21} + 19x_{22} + 19x_{23} \\ +15x_{24} + 9x_{31} + 17x_{32} + 13x_{33} + 27x_{34} \\ + 15x_{41} + 40x_{42} + 17x_{43} + 24x_{44} + \alpha \end{array}}{\begin{array}{r} 15x_{11} + 27x_{12} + 27x_{13} + 22x_{14} + 33x_{21} + 38x_{22} + 44x_{23} \\ +34x_{24} + 25x_{31} + 36x_{32} + 32x_{33} + 33x_{34} \\ + 46x_{41} + 38x_{42} + 49x_{43} + 55x_{44} + \beta \end{array}} \\
\text{subject to} \\
\quad \sum_{j=1}^{4} x_{1j} = 9; \sum_{j=1}^{4} x_{2j} = 18; \sum_{j=1}^{4} x_{3j} = 20; \sum_{j=1}^{4} x_{4j} = 12. \\
\quad \sum_{i=1}^{4} x_{i1} = 18; \sum_{i=1}^{4} x_{i2} = 13; \sum_{i=1}^{4} x_{i3} = 12; \sum_{i=1}^{4} x_{i4} = 16.
\end{cases}
$$

Solving the above fuzzy programming technique used in [28], the dominated solution is $x_{11} = 9, x_{21} = 9, x_{24} = 9, x_{32} = 1, x_{33} = 12, x_{34} = 7, x_{42} = 12$ $x_{11} = 6.1, x_{12} = 2.8, x_{22} = 10.1, x_{24} = 7.8, x_{31} = 11.8, x_{33} = 8.18, x_{43} = 3.81, x_{44} = 8.18$ and fuzzy values of objective functions are:

$$
\begin{aligned}
Z^1(x) &= [(369, 525, 690; 0.6), (240, 525, 924.8)] \\
Z^2(x) &= [(797, 1326, 1914.5; 0.6), (897, 1326, 2003)].
\end{aligned}
$$

Level (γ, δ) interval valued fuzzy number for $Z^1(x)$ and $Z^2(x)$ are plotted below.

In Figure 2, interval values are marked in the x axis and level (γ, δ) are marked on the y axis. Lower grade membership function at 0.6 and upper grade membership function at 0.9 are marked.General form is represented

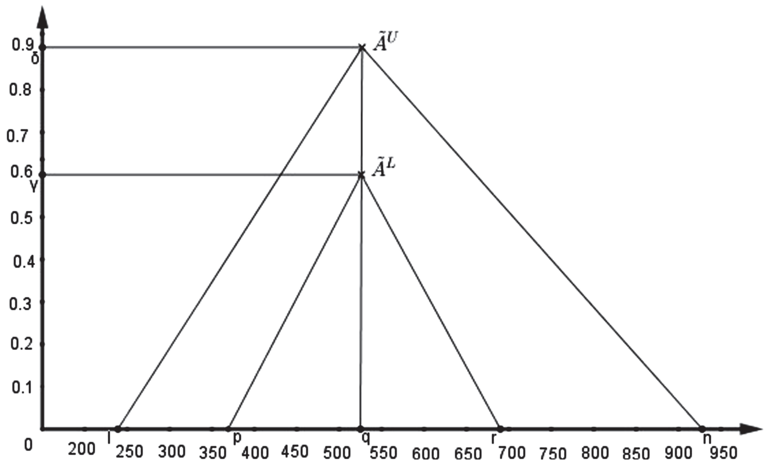

FIGURE 3. Level (γ, δ) interval valued fuzzy number for $Z^2(x)$.

as $[(p, q, r; \gamma), (l, q, n; \delta)] = \left[\widetilde{A}^L, \widetilde{A}^U\right]$, also in this q values are same for both interval valued numbers. In this, the objective values of $Z^1(x)$ and $Z^2(x)$ are plotted.

Advantages of proposed method over existing method is discussed here. It is shown that by using the proposed method all the shortcomings of the existing method [9]discussed are removed and also it is shown that it is better to use proposed method for solving fractional transportation problems occurring in real life situations as compared to the existing method [9]. It is pointed out that existing method [9] cannot be applied to find solution, as the condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied. By using the proposed method it can be solved as,

$$\begin{cases} \text{Minimize } Z^1(x) = \dfrac{\begin{array}{r} 11x_{11} + 8x_{12} + 14x_{13} + 19x_{14} + 6x_{21} + 18x_{22} + 9x_{23} \\ +4x_{24} + 16x_{31} + 18x_{32} + 8x_{33} + 12x_{34} \\ + 12x_{41} + 12x_{42} + 12x_{43} + 16x_{44} + \alpha \end{array}}{\begin{array}{r} 16x_{11} + 38x_{12} + 34x_{13} + 22x_{14} + 6x_{21} + 28x_{22} + 32x_{23} \\ +22x_{24} + 2x_{31} + 9x_{32} + 6x_{33} + 6x_{34} \\ + 12x_{41} + 14x_{42} + 22x_{43} + 11x_{44} + \beta \end{array}} \\ \text{Minimize } Z^2(x) = \dfrac{\begin{array}{r} 12x_{11} + 16x_{12} + 16x_{13} + 17x_{14} + 20x_{21} + 19x_{22} + 19x_{23} \\ +15x_{24} + 9x_{31} + 17x_{32} + 13x_{33} + 27x_{34} \\ + 15x_{41} + 40x_{42} + 17x_{43} + 24x_{44} + \alpha \end{array}}{\begin{array}{r} 15x_{11} + 27x_{12} + 27x_{13} + 22x_{14} + 33x_{21} + 38x_{22} + 44x_{23} \\ +34x_{24} + 25x_{31} + 36x_{32} + 32x_{33} + 33x_{34} \\ + 46x_{41} + 38x_{42} + 49x_{43} + 55x_{44} + \beta \end{array}} \\ \text{subject to} \\ \qquad \sum_{j=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{a}_i, \quad i = 1, 2, 3, 4, \\ \qquad \sum_{i=1}^{4} x_{ij}\widetilde{1} \approx \widetilde{b}_j, \quad j = 1, 2, 3, 4, \\ \qquad x_{ij} \geq 0 \quad \forall\ i, j, \end{cases}$$

with $\sum_{i=1}^{4} \widetilde{a}_i \approx \sum_{j=1}^{4} \widetilde{b}_j$ where the values of $\widetilde{a}_i$ for $i = 1, 2, 3, 4$ and $\widetilde{b}_j$ for $j = 1, 2, 3, 4$ are given as in (6.1).

Using step 1 and step 2 in the proposed method, we get

$$\begin{cases} \text{Minimize } Z^1(x) = \dfrac{\begin{array}{r} 11x_{11}+8x_{12}+14x_{13}+19x_{14}+6x_{21}+18x_{22}+9x_{23} \\ +4x_{24}+16x_{31}+18x_{32}+8x_{33}+12x_{34} \\ +12x_{41}+12x_{42}+12x_{43}+16x_{44}+\alpha \end{array}}{\begin{array}{r} 16x_{11}+38x_{12}+34x_{13}+22x_{14}+6x_{21}+28x_{22}+32x_{23} \\ +22x_{24}+2x_{31}+9x_{32}+6x_{33}+6x_{34} \\ +12x_{41}+14x_{42}+22x_{43}+11x_{44}+\beta \end{array}} \\ \text{Minimize } Z^2(x) = \dfrac{\begin{array}{r} 12x_{11}+16x_{12}+16x_{13}+17x_{14}+20x_{21}+19x_{22}+19x_{23} \\ +15x_{24}+9x_{31}+17x_{32}+13x_{33}+27x_{34} \\ +15x_{41}+40x_{42}+17x_{43}+24x_{44}+\alpha \end{array}}{\begin{array}{r} 15x_{11}+27x_{12}+27x_{13}+22x_{14}+33x_{21}+38x_{22}+44x_{23} \\ +34x_{24}+25x_{31}+36x_{32}+32x_{33}+33x_{34} \\ +46x_{41}+38x_{42}+49x_{43}+55x_{44}+\beta \end{array}} \\ \text{subject to} \\ \quad \sum_{j=1}^{4} x_{1j}=9; \sum_{j=1}^{4} x_{2j}=18; \sum_{j=1}^{4} x_{3j}=20; \sum_{j=1}^{4} x_{4j}=12. \\ \quad \sum_{i=1}^{4} x_{i1}=18; \sum_{i=1}^{4} x_{i2}=13; \sum_{i=1}^{4} x_{i3}=12; \sum_{i=1}^{4} x_{i4}=16. \end{cases}$$

Solving the fuzzy programming technique we get the objective values as,

$$Z^1(x) = 1074 \quad \text{and} \quad Z^2(x) = 2711.74.$$

The solutions for the above objective values are mentioned below,

$x_{11} = 9, x_{21} = 9, x_{24} = 9, x_{33} = 12, x_{34} = 7, x_{42} = 12$ and $x_{11} = 6.1, x_{12} = 2.8, x_{22} = 10.1, x_{24} = 7.8, x_{31} = 11.8, x_{33} = 8.18, x_{43} = 3.81, x_{44} = 8.18$.

To illustrate the proposed method, we consider the following example of multi-objective fractional transportation problem with interval cost.

$$\begin{cases} \text{Minimize } Z^1(x) = \sum_{i=1}^{4}\sum_{j=1}^{4} \dfrac{\left[C^1_{L_{ij}}, C^1_{R_{ij}}\right]}{\left[D^1_{L_{ij}}, D^1_{R_{ij}}\right]} x_{ij} \\ \text{Minimize } Z^2(x) = \sum_{i=1}^{4}\sum_{j=1}^{4} \dfrac{\left[C^2_{L_{ij}}, C^2_{R_{ij}}\right]}{\left[D^2_{L_{ij}}, D^2_{R_{ij}}\right]} x_{ij} \\ \text{subject to} \\ \quad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{1j} = [8,8] \qquad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{2j} = [18,18] \\ \quad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{3j} = [21,21] \qquad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{4j} = [12,11] \\ \quad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{i1} = [17,17] \qquad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{i2} = [13,13] \\ \quad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{i3} = [12,12] \qquad \sum_{i=1}^{4}\sum_{j=1}^{4} x_{i4} = [16,16] \\ \qquad x_{ij} \geq 0, \quad i,j = 1,2,3,4, \end{cases}$$

where

$$C^1_{R_{ij}} = \begin{bmatrix} (3,6,8)(2,6,9)(1,3,8)(1,3,9)(4,6,12)(2,6,16)(6,7,8)(5,7,29) \\ (2,3,4)(1,3,5)(6,8,12)(5,8,18)(2,3,4)(1,3,18)(0.5,1,5.5)(0.25,1,7.75) \\ (5,7,13)(3,7,17)(8.5,9,9.5)(7,9,11)(2,3,4)(1,3,13)(5,6,7)(3,6,9) \\ (3,6,9)(2,6,12)(5,6,9)(3,6,12)(2,5,9)(1,5,17)(2,7,12)(1,7,19) \end{bmatrix}$$

$$D^1_{R_{ij}} = \begin{bmatrix} (6,7,12)(5,7,15)(17,20,21)(11,20,22)(15,16,21)(14,16,24)(10,11,12)(9,11,13) \\ (1.5,2,4.5)(1,2,10)(13,14,15)(11,14,17)(14,15,20)(13,15,23)(10,11,12)(8,11,13) \\ (1.5,3,4.5)(1,3,9)(2.5,4,5.5)(2,4,10)(2,3,5)(1,3,5)(5,6,12)(3,6,16) \\ (3,6,8)(2,6,12)(6,7,8)(3,7,9)(6,11,16)(5,11,17)(2,6,7)(1,6,8) \end{bmatrix}$$

$$C^2_{R_{ij}} = \begin{bmatrix} (3,6,10)(4,6,9)(5,8,12)(6,8,12)(4,8,13)(6,8,15)(3,8,14)(6,8,16) \\ (6.5,8.5,12)(10.5,8.5,9.5)(6.5,9.5,12.5)(7,9.5,12.5)(6,9.5,16)(6,9.5,14)(5,7,13)(3,7,17) \\ (2.5,4,5.5)(2,4,10)(6,8,12)(2,8,18)(2,7,13)(6,7,10)(4,13,23)(6,13,29) \\ (6,8,17)(5,8,17)(12,16,21)(7,16,26)(5,9,19)(6,9,19)(9.5,12,19.5)(7.5,12,20) \end{bmatrix}$$

$$D^2_{R_{ij}} = \begin{bmatrix} (6,7,12)(3,7,15)(10,13,19)(5,13,22)(7,14,22)(10,14,19)(4,11,21)(3,11,21) \\ (12,16,21)(9,6,24)(15,19,24)(17,19,26)(18,22,28)(20,22,29)(12,18,28)(16,18,21) \\ (6,12,19)(7,12,23)(12,17,23)(10,17,28)(9,16,27)(10,16,24)(0,16,26)(8,16,28) \\ (19,23,29)(20,23,29)(12,19,29)(14,22,31)(22,24,31)(21,24,33)(22,27,33)(21,27,35) \end{bmatrix}.$$

By Definition 3.10, the equivalent multi-objective deterministic problem as,

$$\begin{cases} \text{Minimize } Z^1_R(x) = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\left[C^1_{R_{ij}}\right]}{\left[D^1_{R_{ij}}\right]} x_{ij} \quad \text{Minimize } Z^2_R(x) = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\left[C^2_{R_{ij}}\right]}{\left[D^2_{R_{ij}}\right]} x_{ij} \\ \text{Minimize } Z^1_C(x) = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\left[C^1_{R_{ij}}\right]}{\left[D^1_{R_{ij}}\right]} x_{ij} \quad \text{Minimize } Z^2_C(x) = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\left[C^2_{R_{ij}}\right]}{\left[D^2_{R_{ij}}\right]} x_{ij} \\ \text{subject to} \\ \qquad \displaystyle\sum_{j=1}^{4} x_{1j} \le 2 \qquad \displaystyle\sum_{j=1}^{4} x_{1j} \ge 10 \\ \qquad \displaystyle\sum_{j=1}^{4} x_{2j} \le 18 \qquad \displaystyle\sum_{j=1}^{4} x_{2j} \ge 18 \\ \qquad \displaystyle\sum_{j=1}^{4} x_{3j} \le 21 \qquad \displaystyle\sum_{j=1}^{4} x_{3j} \ge 21 \\ \qquad \displaystyle\sum_{j=1}^{4} x_{4j} \le 11 \qquad \displaystyle\sum_{j=1}^{4} x_{4j} \ge 12 \\ \qquad \displaystyle\sum_{i=1}^{4} x_{i1} \le 17 \qquad \displaystyle\sum_{i=1}^{4} x_{i1} \ge 17 \\ \qquad \displaystyle\sum_{i=1}^{4} x_{i2} \le 13 \qquad \displaystyle\sum_{i=1}^{4} x_{i2} \ge 13 \\ \qquad \displaystyle\sum_{i=1}^{4} x_{i3} \le 12 \qquad \displaystyle\sum_{i=1}^{4} x_{i3} \ge 12 \\ \qquad \displaystyle\sum_{i=1}^{4} x_{i4} \le 16 \qquad \displaystyle\sum_{i=1}^{4} x_{i4} \ge 16 \end{cases}$$

as well as the condition stated in

$$C^1_{R_{ij}} = \begin{bmatrix} 8.5 & 8.5 & 14 & 18.5 \\ 4.5 & 15 & 11 & 6.6 \\ 15 & 10.2 & 8.5 & 8 \\ 10.5 & 10.5 & 13 & 15.5 \end{bmatrix} \qquad C^2_{R_{ij}} = \begin{bmatrix} 9.5 & 12 & 14 & 15 \\ 10.75 & 12.5 & 15 & 15 \\ 7.75 & 18 & 11.5 & 26 \\ 17 & 26 & 19 & 19.75 \end{bmatrix}$$

$$D^1_{R_{ij}} = \begin{bmatrix} 13.5 & 21.5 & 22.5 & 12.5 \\ 7.25 & 16 & 21.5 & 12.5 \\ 6.75 & 7.75 & 5 & 14 \\ 10 & 8.5 & 16.5 & 7.5 \end{bmatrix} \qquad D^2_{R_{ij}} = \begin{bmatrix} 13.5 & 20.5 & 20.5 & 21 \\ 22.5 & 25 & 28.5 & 24.5 \\ 21 & 25.5 & 25.5 & 27 \\ 29 & 30 & 32 & 34 \end{bmatrix}$$

$$C^1_{c_{ij}} = \begin{bmatrix} 6 & 3 & 6 & 7 \\ 3 & 8 & 3 & 1 \\ 7 & 9 & 3 & 6 \\ 6 & 6 & 5 & 7 \end{bmatrix} \qquad C^2_{c_{ij}} = \begin{bmatrix} 6 & 8 & 8 & 8 \\ 8.5 & 9.5 & 9.5 & 7 \\ 4 & 8 & 9.5 & 13 \\ 8 & 16 & 7 & 12 \end{bmatrix}$$

$$D^1_{c_{ij}} = \begin{bmatrix} 7 & 20 & 16 & 11 \\ 2 & 14 & 15 & 11 \\ 3 & 15 & 3 & 6 \\ 6 & 11 & 11 & 6 \end{bmatrix} \qquad D^2_{c_{ij}} = \begin{bmatrix} 7 & 13 & 14 & 11 \\ 16 & 19 & 22 & 18 \\ 12 & 17 & 16 & 16 \\ 23 & 19 & 24 & 27 \end{bmatrix}.$$

The optimal solution to the problem is obtained by using the following proposed steps of the previous section, those steps are presented as well.

$$\begin{cases} \text{Minimize } Z^1_R(x) = \dfrac{\begin{array}{r} 8.5 * x_{11} + 8.5 * x_{12} + 14 * x_{13} + 18.5 * x_{14} + 4.5 * x_{21} + 15 * x_{22} + 11 * x_{23} \\ +6.6 * x_{24} + 15 * x_{31} + 10.2 * x_{32} + 8.5 * x_{33} + 8 * x_{34} \\ + 10.5 * x_{41} + 10.5 * x_{42} + 13 * x_{43} + 15.5 * x_{44} + \alpha \end{array}}{\begin{array}{r} 13.5 * x_{11} + 21.5 * x_{12} + 22.5 * x_{13} + 12.5 * x_{14} + 7.25 * x_{21} + 16 * x_{22} + 21.5 * x_{23} \\ +12.5 * x_{24} + 6.75 * x_{31} + 7.75 * x_{32} + 5 * x_{33} + 14 * x_{34} \\ + 10 * x_{41} + 8.5 * x_{42} + 16.5 * x_{43} + 7.5 * x_{44} + \beta \end{array}} \\ \text{Minimize } Z^2_R(x) = \dfrac{\begin{array}{r} 9.5 * x_{11} + 12 * x_{12} + 14 * x_{13} + 15 * x_{14} + 10.75 * x_{21} + 12.5 * x_{22} + 15 * x_{23} \\ +15 * x_{24} + 7.75 * x_{31} + 18 * x_{32} + 11.5 * x_{33} + 26 * x_{34} \\ + 17 * x_{41} + 26 * x_{42} + 19 * x_{43} + 19.75 * x_{44} + \alpha \end{array}}{\begin{array}{r} 13.5 * x_{11} + 20.5 * x_{12} + 20.5 * x_{13} + 21 * x_{14} + 22.5 * x_{21} + 25 * x_{22} + 28.5 * x_{23} \\ +24.5 * x_{24} + 21 * x_{31} + 25.5 * x_{32} + 25.5 * x_{33} + 27 * x_{34} \\ + 29 * x_{41} + 30 * x_{42} + 32 * x_{43} + 34 * x_{44} + \beta \end{array}} \\ \text{subject to} \\ \quad x_{11} + x_{12} + x_{13} + x_{14} = 9 \\ \quad x_{21} + x_{22} + x_{23} + x_{24} = 18 \\ \quad x_{31} + x_{32} + x_{33} + x_{34} = 20 \\ \quad x_{41} + x_{42} + x_{43} + x_{44} = 12 \\ \quad x_{11} + x_{21} + x_{31} + x_{41} = 18 \\ \quad x_{12} + x_{22} + x_{32} + x_{42} = 13 \\ \quad x_{13} + x_{23} + x_{33} + x_{43} = 12 \\ \quad x_{14} + x_{24} + x_{34} + x_{44} = 16 \\ \qquad x_{i,j} \geq 0. \end{cases}$$

(1) The optimal solution to each single objective transportation problem is

$$[x_{12} = 9, x_{21} = 6, x_{23} = 12, x_{32} = 4, x_{34} = 16, x_{41} = 12]$$
$$[x_{12} = 5, x_{14} = 4, x_{22} = 8, x_{23} = 10, x_{31} = 18, x_{33} = 2, x_{44} = 12].$$

(2) The objective values are

$$Z_R^1(x) = 0.6095 \quad Z_R^2(x) = 0.510$$
$$Z_C^1(x) = 0.4045 \quad Z_C^2(x) = 0.3822$$

(3) The upper and lower bounds of each objective function can be written as follows:
$0.311 \leq 0.40 \leq 0.6, \quad 0.32 \leq 0.38 \leq 0.5$
Hence, $L_1 = 0.311 \quad U_1 = 0.6 \quad L_2 = 0.32 \quad U_2 = 0.5$

(4) The model is

$$\begin{cases} \text{Minimize } \phi \\ \text{subject to} \\ \frac{1}{2}\tanh\left[\left[\frac{U_q+L_q}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{C_R^q, C_C^q}{D_R^q, D_C^q}x_{ij}\right]\alpha_q\right] + \frac{1}{2} + D_q^- - D_q^+ = 1. \\ \text{where,} \\ Z_R^1 = \dfrac{\begin{array}{r} 8.5*x_{11}+8.5*x_{12}+14*x_{13}+18.5*x_{14}+4.5*x_{21}+15*x_{22}+11*x_{23} \\ +6.6*x_{24}+15*x_{31}+10.2*x_{32}+8.5*x_{33}+8*x_{34} \\ +10.5*x_{41}+10.5*x_{42}+13*x_{43}+15.5*x_{44}+\alpha \end{array}}{\begin{array}{r} 13.5*x_{11}+21.5*x_{12}+22.5*x_{13}+12.5*x_{14}+7.25*x_{21}+16*x_{22}+21.5*x_{23} \\ +12.5*x_{24}+6.75*x_{31}+7.75*x_{32}+5*x_{33}+14*x_{34} \\ +10*x_{41}+8.5*x_{42}+16.5*x_{43}+7.5*x_{44}+\beta \end{array}} \\ Z_R^2 = \dfrac{\begin{array}{r} 9.5*x_{11}+12*x_{12}+14*x_{13}+15*x_{14}+10.75*x_{21}+12.5*x_{22}+15*x_{23} \\ +15*x_{24}+7.75*x_{31}+18*x_{32}+11.5*x_{33}+26*x_{34} \\ +17*x_{41}+26*x_{42}+19*x_{43}+19.75*x_{44}+\alpha \end{array}}{\begin{array}{r} 13.5*x_{11}+20.5*x_{12}+20.5*x_{13}+21*x_{14}+22.5*x_{21}+25*x_{22}+28.5*x_{23} \\ +24.5*x_{24}+21*x_{31}+25.5*x_{32}+25.5*x_{33}+27*x_{34} \\ +29*x_{41}+30*x_{42}+32*x_{43}+34*x_{44}+\beta \end{array}} \\ Z_C^1 = \dfrac{\begin{array}{r} 6*x_{11}+3*x_{12}+6*x_{13}+7*x_{14}+3*x_{21}+8*x_{22}+3*x_{23} \\ +1*x_{24}+7*x_{31}+9*x_{32}+3*x_{33}+6*x_{34} \\ +6*x_{41}+6*x_{42}+5*x_{43}+7*x_{44}+\alpha \end{array}}{\begin{array}{r} 7*x_{11}+20*x_{12}+16*x_{13}+11*x_{14}+2*x_{21}+14*x_{22}+15*x_{23} \\ +11*x_{24}+3*x_{31}+15*x_{32}+3*x_{33}+6*x_{34} \\ +6*x_{41}+11*x_{42}+11*x_{43}+6*x_{44}+\beta \end{array}} \\ Z_C^2 = \dfrac{\begin{array}{r} 6*x_{11}+8*x_{12}+8*x_{13}+8*x_{14}+8.5*x_{21}+9.5*x_{22}+9.5*x_{23} \\ +7*x_{24}+4*x_{31}+8*x_{32}+9.5*x_{33}+13*x_{34} \\ +8*x_{41}+16*x_{42}+7*x_{43}+12*x_{44}+\alpha \end{array}}{\begin{array}{r} 7*x_{11}+13*x_{12}+14*x_{13}+11*x_{14}+16*x_{21}+19*x_{22}+22*x_{23} \\ +18*x_{24}+12*x_{31}+17*x_{32}+16*x_{33}+16*x_{34} \\ +23*x_{41}+19*x_{42}+24*x_{43}+27*x_{44}+\beta \end{array}}. \end{cases}$$

The problem was solved by the linear interactive global optimization (LINGO) [16] software and the optimal compromise solution is presented as follows:

$$Z^1 = \left[Z_R^1, Z_C^1\right] = [0.6, 0.404] \qquad Z^2 = \left[Z_R^2, Z_C^2\right] = [0.510, 0.3822]$$
$$D_1^+ = [2.49, 1.07] \quad D_2^+ = [2.49, 0.95] \quad D_{1,2}^- = [0] \text{ and } \phi = 0.1.$$

7. Comparison

In [9], the linear multi-objective transportation problem representing all parameters as (λ, ρ) interval valued fuzzy numbers, and it is applied to all single and multi-objective transportation problem occuring in real life problems. Hence it is better to use the proposed method. While comparing with the existing paper, the proposed method has dominated solution and existing paper has non-dominated solution.

In [21], a special type of hyperbolic membership function is used and a non-linear optimization model is developed to solve a multi-objective solid transportation problem with interval cost, when compared the proposed method has dominated solution and existing paper has non-dominated solution.

In this paper multi-objective fractional transportation problem representing all parameters as (λ, ρ) interval valued fuzzy numbers, and it is applied to single and multi-objective transportation problem occuring in real life problems. Hence fractional transportation problem are very useful in our daily lives and learning them is a very useful and important skill in our daily lives, and helps with many daily tasks and jobs. Also in this paper a goal programmic approach is used with a special type of hyperbolic membership function, and a non-linear optimization also developed to solve a mult-objective solid transportation problem with interval cost. As compared to the above papers this paper shows greater domination.

The comparison of the existing papers and proposed method are tabulated below.

EXISTING PAPER A. Gupta A. Kumar	Linear multi-objective transportation problem $Z^1(x) = [(116.69, 152.025, 218.611; 0.6)(66.71, 152.02, 292.23; 0.9)]$ $Z^2(x) = [(134.76, 201.95, 245.55; 0.6)(43.21, 201.95, 308.48; 0.9)]$	$Z^1(x) = 286$ $Z^2(x) = 334$	
EXISTING PAPER B. Radhakrishnan P. Anukokila	Fractional Goal Programming Approach $Z^1(x) = [0.25, 1.041]$ $Z^2(x) = [0.062, 0.138]$		Deviational variables $D_1^+ = [0.47, 0.52]$ $D_2^+ = [0.52, 2.5]$ $D_{1,2}^- = [0,0]$
PROPOSED PAPER P. Anukokila B. Radhakrishnan A. Anju	Fractional multi-objective transportation problem $Z^1(x) = [(369, 525, 690; 0.6)(240, 525.02, 924; 0.9)]$ $Z^2(x) = [(803, 1326, 1914.5; 0.6)(797, 1326, 2003; 0.9)]$ Fractional Goal Programming Approach $Z^1(x) = [0.6, 0.404]$ $Z^2(x) = [0.510, 0.382]$	$Z^1(x) = 1074$ $Z^2(x) = 2711.74$	Deviational variables $D_1^+ = [2.49, 1.09]$ $D_2^+ = [0.47, 0.95]$ $D_{1,2}^- = [0,0]$

FIGURE 4. Comparison Table.

8. CONCLUSION

Transportation problem with fractional objective function are widely used as performance measure in many real life situations. Goal programming is a useful technique in operational research and decision theory, which permit us to find solutions which fulfill some goals. The advantage of a fuzzy programming technique is that, for a goal programming, approach is utilizes in order to allow for the optimization of conflicting goals while permitting an explicit consideration of an existing decision environment. A special type of hyperbolic membership function is used and a non linear optimization model is developed to solve a multi-objective fractional transportation problem with fuzzy parameters, which can be solved by using one of the software LINGO. These results shows great promise in developing an efficient solution for multi-objective fractional transportation problems and this can be extended for all engineering applications in the future to achieve a global solution.

REFERENCES

[1] W.A. Wahed and S.M. Lee, Interactive fuzzy goal programming for multi-objective transportation problems. *Int. J. Mangt. Sci.* **34** (2006) 158–166.

[2] A.K. Bit, M.P. Biswal and S.S. Alam, Multi-objective transportation problem with interval cost source and destination parameters. *Eur. J. Oper. Res.* **117** (1999) 100–112.

[3] A.K. Bhurjee and G. Panda, Multi-objective optimization problem with bounded parameters. *RAIRO: OR* **48** (2014) 545–558.
[4] J. Chiang, The optimal solution of the transportation problem with fuzzy demand and fuzzy product. *J. Inf. Sci. Eng. Oper. Res.* **21** (2005) 439–451.
[5] C.T. Chang, A goal programming approach for fuzzy multi-objective fractional programming problems. *Int. J. Syst. Sci.* **8** (2009) 867–874.
[6] S.K. Das, T. Mandal and S.A. Edalatpanah, A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO: OR* **51** (2017) 285–297.
[7] A. Ebrahimnejad, A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. *Appl. Soft. Comput.* **19** (2014) 171–176.
[8] M.R. Fegade, V.A. Jadhav and A.A. Muley, Solving Fuzzy transportation problem using zero suffix and robust ranking methodology. *IOSR J. Eng.* **2** (2012) 36–39.
[9] A. Gupta and A. Kumar, A new method for solving linear multi-objective transportation problems with fuzzy parameters. *Appl. Math. Model.* **36** (2012) 1421–1430.
[10] F.L. Hitchcock, The distribution of a product from several sources to numerous localities. *J. Math. Phys.* **20** (1941) 224–230.
[11] H. Isermann, The enumeration of all efficient solutions for a linear multi-objective transportation problem. *Naval. Res. Logis. Quart.* **26** (1979) 123–139.
[12] G.R. Jahanshahloo, B. Talebian, I.H. Lotfi and J. Sadeghi, Finding a solution for multi-objective linear fractional programming problem based on goal programming and data envelopment analysis. *RAIRO: OR* **51** (2017) 199–210.
[13] A. Kaur and A. Kumar, A new method for solving fuzzy transportation problems using ranking function. *Appl. Math. model.* **35** (2011) 5652–5661.
[14] H.G. Kocken and M. Sivri, Multi-objective transportation model with fuzzy parameters: priority based fuzzy goal programming approach. *J. Trans. Sys. Eng. Inf. Tech.* **8** (2008) 40–48.
[15] B. Liu and X. Chen, Uncertain multi-objective programming and uncertain goal programming. *J. Uncer. Anal. Appl.* **10** (2015) 43–58.
[16] Lingo user's guide, LINDO Syst. Inc. Chicago (1999).
[17] S.T. Liu, Fractional transportation problem with fuzzy parameters. *Soft. Comput.* **10** (2010) 111–119.
[18] R.H. Mohamed, The relationship between goal programming and fuzzy programming. *Fuzzy. Sets. Syst.* **89** (1997) 215–222.
[19] S. Narayanamoorthy and P. Anukokila, Goal programming approach for solving transportation problem with interval cost. *J. Intel. Fuzzy Syst.* **26** (2014) 1143–1154.
[20] A.K. Ojha and R. Ranjan, Multi-objective geometric programming problem with Karush–Kuhn–Tucker condition using $\in$ constraint method. *Rairo-Oper. Res.* **48** (2014) 429–453.
[21] B. Radhakrishnan and P. Anukokila, Fractional goal programming for fuzzy solid transportation problem with interval cost. *Fuzzy Inf. Eng.* **6** (2014) 359–377.
[22] B. Radhakrishnan and P. Anukokila, A compensatory approach to fuzzy fractional transportation problem. *Int. J. Math. Oper. Res.* **6** (2014) 2.
[23] J.L. Ringuest and D.B. Rinks, Interactive solutions for the linear multi-objective transportation problem. *Eur. J. Oper. Res.* **32** (1987) 123–139.
[24] S. Sen and B.B. Pal, Interval goal programming approach to multi-objective fuzzy goal programming problem with interval weights. *Proc. Tech.* **10** (2013) 87–595.
[25] P. Surapati and T.K. Roy, A simple parametric method to generate all optimal solutions of Fuzzy Solid Transportation Problem. *Appl. Math. J.* **23** (2015) 56–71.
[26] M. Upmanyu and R.R. Saxena, On solving multi-objective set covering problem with imprecise linear fractional objectives. *RAIRO: OR* **49** (2015) 495–510.
[27] M. Zangiabadi and H.R. Maleki, Fuzzy Goal programming Technique to solve multi-objective transportation problems with some non linear membership functions. *Iran. J. Fuzzy Syst.* **10** (2013) 61–74.
[28] H.J. Zimmermann, Fuzzy Programming and linear programming with several objective functions. *Fuzzy Sets. Syst.* **1** (1978) 45–55.