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Narayana Prime Cordial Labeling of Grid Graph

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Abstract. Let $G = (V, E)$ be a graph. The binary numbers 0 and 1 have been allotted to the edges of the graph G through the evaluating functions defined on V and E by ensuring the cordiality conditions. This has been obtained through the prime and the Narayana numbers. Any graph G which admits this labeling is known as Narayanaprime cordial graph. In this research paper , we compute the Narayana prime cordial labeling of Grid graphs.

1. INTRODUCTION

Graph labeling is the process of assigning labels to the vertices in the vertex set V and to the edges in the edge set E of the given graph G=(V, E) [1,12]. The applications of graphlabeling can be found in [10] . In the year 1987, Cahit [6] discussed the cordial labeling of graphs. The Narayana prime cordial labeling of graphs is a recent development in graph labeling which was introduced by B.J Murali et. al[11] . The terminologies and concepts used in this paper have been referred to Harary [8]. The developments in graph labeling have been updated by Gallian [7].The study of Narayana prime cordial labeling for various classes of graphs are found in [2,3,4,13] .

In this research article,we discuss the existence of the Narayana prime cordial labeling of grid graphs and triangular grid graphs.

2. PRELIMINARIES

The Narayana numbers , closely related to Catalan numbers [9] are defined as follows.

Definition 2.1

" LetN_0 be the set of non negative integers and let $k, n \in N_0$
N(n,k) is the kth Narayana number for a given n is defined as

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$$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k+1}, 0 \le k \le n \ where \ \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Narayana numbers $N(n,k)$ for each n=1,2,…7 ,k=1,2,…7 are tabulated below for quick reference:

nk	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	3	1				
4	1	6	6	1			
5	1	10	20	10	1		
6	1	15	50	50	15	1	
7	1	21	105	175	105	21	1

The divisibility of Narayana numbers depends on the properties[5]:
(i)Let p be prime and let $n = p^m - 1$ for some $m \in N_0$. Then for all$k$, $1 \le k \le n-1$, $p \nmid N(n,k)$ and
(ii)Let p be prime and let $n = p^m$for some$m \in N_0$. Then for all$k$, $1 \le k \le n-2$, $p \,/\, N(n,k)$."

3. NARAYANA PRIME CORDIAL LABELING OF GRID GRAPH

We recall the definitions of the Narayana prime cordial labeling of a graph and the Narayana prime cordial graph.

Definition 3.1

"Let $G = (V,E)$ be a graph. An injective function $g: V \to N_0$ is said to be a Narayana prime cordial labeling of the Graph G, if the induced edge function$g^*: E \to \{0,1\}$ satisfies the following conditions:

(i) For every$uv \in E$

$$\begin{aligned} g^*(uv) &= 1 \ if \ p/N\big(g(u),g(v)\big), where \ g(u) > g(v) \ and \ g(u) = p^m \\ &for \ some \ m \in N_0 \,; 1 \le g(v) \le g(u) - 2 \ where \ p \ is \ a \ prime \ number \\ &= 1 \ if \ p/N\big(g(v),g(u)\big), where \ g(v) > g(u) \ and \ g(v) = p^m \\ &for \ some \ m \in N_0 \,; 1 \le g(u) \le g(v) - 2 \ where \ p \ is \ a \ prime \ number \\ &= 0 \ if \ p \nmid N\big(g(u),g(v)\big), where \ g(u) > g(v) \ and \ g(u) = p^m - 1 \\ &for \ some \ m \in N_0 \,; 0 \le g(v) \le g(u) - 1 \ where \ p \ is \ a \ prime \ number \\ &= 0 \ if \ p \nmid N\big(g(v),g(u)\big), where \ g(v) > g(u) \ and \ g(v) = p^m - 1 \\ &for \ some \ m \in N_0 \,; 0 \le g(u) \le g(v) - 1 \ where \ p \ is \ a \ prime \ number \end{aligned}$$

(ii) $\left|e_{g^*}(0) - e_{g^*}(1)\right| \le 1$ where$e_{g^*}(0)$ and $e_{g^*}(1)$ denote respectively the number of edges with the label 0 and the number of edges with label 1."

Definition 3.2

"A graph $G = (V,E)$which admits a Narayana prime cordial labeling is called a Narayana prime cordial graph."

Theorem 3.1

The grid graph $P_m X\, P_n$ $(m \le n)$admits a Narayana prime cordial labeling where$m, n \equiv 1(mod\ 2)$.

Proof

Let G be the grid graph $P_m X\, P_n$ $(m \le n)$.

Let $V = \{u_{i,j}; 1 \le i \le m, 1 \le j \le n\}$ be the vertex set and $E = \{u_{i,j}u_{i,j+1}, u_{i,j}u_{i+1,j}, u_{i+1,j}u_{i+1,j+1}, u_{i,j+1}u_{i+1,j+1}\ ; 1 \le i \le m, 1 \le j \le n\}$ be the edge set of G.Then$G$ has mn vertices and $2mn - m - n$ edges.

The labeling of vertices of the grid graph having m rows and n columns can be viewed as$m$ paths each of length $n-1$ which are denoted byP^1 (first row), P^2 (second row), …P^m (mthrow) are done as follows

Since m is odd there will be $\left\lceil \frac{m}{2} \right\rceil$ of odd rows(i.e paths denoted by odd number as superscript say P^1, P^3 ,….. P^m) and $\left\lfloor \frac{m}{2} \right\rfloor$ of even rows (sayP^2, P^4 , … P^{m-1}) and where each P^i denotes a path of length $n-1$.

Define avertex function $g: V \to N_0$ such that

the vertices in each odd row P^{i+1} corresponding to each i, where $i = 0,2,4, \ldots\ m-1$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{(in)+2j}\ ; 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$$

$g(u_{i+1,2j}) = 2^{(in)+(2j+1)} - 1;\ 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$----------(1)

The vertices in each even row P^{i+1} corresponding to i, where $i = 1,3,5, \ldots\ m-2$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{((i+1)n+1)-(2j-2)} - 1; 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$$

$g(u_{i+1,2j}) = 2^{((i+1)n+2)-2j}\ ; 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$----------(2)

By the induced edge function$g^*: E \to \{0,1\}$asinthedefinition 3.1, an edge whose end vertices are assigned the lables2^l and $2^k - 1$ (l,k>1)takes the value 1 if $2^l > 2^k - 1$ else 0. As the path $P^i$is of even length (n-1) in each row, whose $\frac{n-1}{2}$ edges receive 1 and $\frac{n-1}{2}$ edges receive 0 . Also path along each column if of even length (m-1) ,whose $\frac{m-1}{2}$ edges receive 1 and $\frac{m-1}{2}$ edges receive 0 .

Thus $\left\{m\left(\frac{n-1}{2}\right) + n\left(\frac{m-1}{2}\right)\right\} + \left\{m\left(\frac{n-1}{2}\right) + n\left(\frac{m-1}{2}\right)\right\} = 2mn - m - n$

i.e edges labeled 1 + edges labeled 0 = total number of edges

Therefore $\left|e_{g^*}(1) - e_{g^*}(0)\right| \le 1$ for all edges ,where$e_{g^*}(1)$ denote edges labeled 1 and $e_{g^*}(0)$ denotes edges labeled 0 .Hence the grid graph $P_m X\ P_n$is Narayana prime cordial graph when $m, n \equiv 1(mod\ 2)$.

Example 3.1 A Narayana Prime Cordial Labeling of the Grid Graph $P_3 \times P_7$ is shown in figure 1.

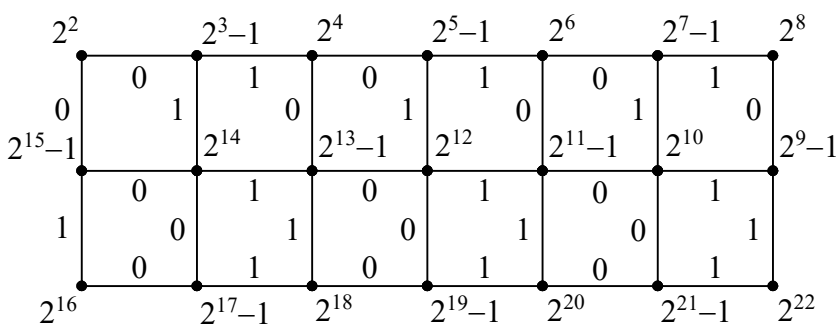

FIGURE 1. The Grid Graph $P_3 \times P_7$ is aNarayana prime cordial graph

Theorem 3.2

The grid graph $P_m X\ P_n (m \le n)$admits a Narayana prime cordial labeling where $m, n \equiv 0(mod\ 2)$.

Proof

Let G be the grid graph $P_m X\ P_n\ (m \le n)$. We define the vertex set V and edge set E as in theorem 3.1

As m and n are even there will be $\frac{m}{2}$ odd rows (sayP^1, P^3 ,….. P^{m-1}) and $\frac{m}{2}$even rows (sayP^2,P^4 , …P^m) where each P^i denotes a path of length n-1.

Define avertex function $g: V \to N_0$ such that the vertices in each odd row P^{i+1} corresponding to i, where $i = 0,2,4, \ldots\ m-2$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{(in)+2j}\,; 1 \le j \le \frac{n}{2}$$

$g(u_{i+1,2j}) = 2^{(in)+(2j+1)} - 1\,; 1 \le j \le \frac{n}{2}$----------(3)

and the vertices in each even row P^{i+1} corresponding to i, where $i = 1,3,5, \ldots\ m-1$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{((i+1)n+1)-(2j-2)}; 1 \le j \le \frac{n}{2}$$

$g(u_{i+1,2j}) = 2^{((i+1)n)-2j} - 1;\ 1 \le j \le \frac{n}{2}$----------(4)

The paths are of odd length in this case along each row and each column.

By the induced edge function g* : E $\rightarrow$ {0, 1} as in definition 3.1, the path along each odd row whose $\left\lfloor\frac{n-1}{2}\right\rfloor$ edges receive 1 and $\left\lceil\frac{n-1}{2}\right\rceil$ edges receive 0 . Also path along each even row ,whose $\left\lceil\frac{n-1}{2}\right\rceil$ edges receive 1 and $\left\lfloor\frac{n-1}{2}\right\rfloor$ edges receive 0 . An odd row and even row together has n-1 edges as 1 and n-1 edges as 0. As m is even ,$\frac{m}{2}(n-1)$ edges receive 1 and $\frac{m}{2}(n-1)$ edges receive 0 .

The path along each column whose all the m-1 edges either receive 1 or receive 0 alternatively. As n is even ,$\frac{n}{2}(m-1)$ edges receive 1 and $\frac{n}{2}(m-1)$ edges receive 0.Thus $2(\frac{m}{2}(n-1) + \frac{n}{2}(m-1)) = 2mn - m - n$.

Therefore $\left|e_{g^*}(1) - e_{g^*}(0)\right| \le 1$ for all edges ,where$e_{g^*}(1)$ denote edges labeled 1 and $e_{g^*}(0)$ denotes edges labeled 0 .Hence the grid graph $P_m X\ P_n$is a Narayana prime cordial graph when $m, n \equiv 0(mod\ 2)$.

Example 3.2. A Narayana Prime Cordial Labeling of the Grid Graph $P_4 \times P_6$ is shown in figure 2.

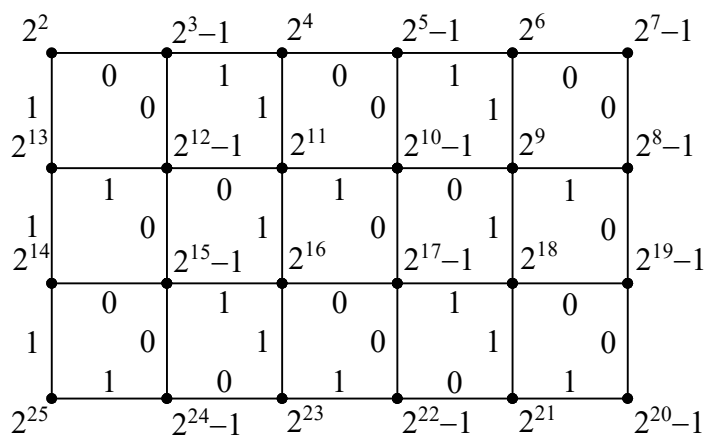

FIGURE 2. The Grid Graph $P_4 \times P_6$is a Narayana prime cordial graph

Theorem 3.3

A grid graph $P_m X\ P_n\ (m \le n)$ is aNarayana prime cordial graph when$m \equiv 1(mod\ 2)$ and $m \equiv 0(mod\ 2)$.

Proof

Let G be the grid graph $P_m X\ P_n\ (m \le n)$. We define the vertex set V and edge set E as in theorem 3.1.

Define avertex function $g: V \rightarrow N_0$ such that

The vertices in each odd row P^{i+1} corresponding to i, where $i = 0,2,4, \ldots\ m-1$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{(in)+2j}\,; 1 \le j \le \frac{n}{2}$$

$$g(u_{i+1,2j}) = 2^{(in)+(2j+1)} - 1\,; 1 \le j \le \frac{n}{2}$$

The vertices in each even row P^{i+1} corresponding to i, where $i = 1,3,5, \ldots\ m-2$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{((i+1)n+1)-(2j-2)}; 1 \le j \le \frac{n}{2}$$

$$g(u_{i+1,2j}) = 2^{((i+1)n)-2j} - 1;\ 1 \le j \le \frac{n}{2}$$

The path are of odd length along each row and of even length in each column in this case

By the induced edge function g* : E $\rightarrow$ {0, 1} as in definition 3.1, the path along each odd row whose $\left\lceil\frac{n-1}{2}\right\rceil$ edges receive 1 and $\left\lceil\frac{n-1}{2}\right\rceil$ edges receive 0 . Also path along each even row ,whose $\left\lceil\frac{n-1}{2}\right\rceil$ edges receive 1 and $\left\lfloor\frac{n-1}{2}\right\rfloor$ edges receive 0 . An odd row and even row together has n-1 edges as 1 and n-1 edges as 0. As m is odd ,$\frac{m-1}{2}(n-1)$ edges receive 1 and

$\frac{m-1}{2}(n-1)$edges receive 0 .The $\left\lceil\frac{n-1}{2}\right\rceil$ edges in last odd row receive 0 and $\left\lfloor\frac{n-1}{2}\right\rfloor$ edges receive 1.

The path along each column whose all the m-1 edges either receive 1 or receive 0 alternatively. As n is even ,$\frac{n}{2}(m-1)$ edges receive 1 and $\frac{n}{2}(m-1)$ edges receive 0.

Thus $2\frac{m-1}{2}(n-1)+\left\lceil\frac{n-1}{2}\right\rceil+\left\lfloor\frac{n-1}{2}\right\rfloor+2\frac{n}{2}(m-1)=2mn-m-n$.

Therefore $\left|e_{g^*}(1)-e_{g^*}(0)\right|\leq 1$ for all edges ,where $e_{g^*}(1)$ denote edges labeled 1 and $e_{g^*}(0)$ denotes edges labeled 0 .Hence the grid graph $P_m X\ P_n$is a Narayana prime cordial graph when $m\equiv 1(mod\ 2)$ and $n\equiv 0(mod\ 2)$.

Example 3.3 A Narayana Prime Cordial Labeling of the Grid Graph $P_5\times P_6$ is given in the figure 3.

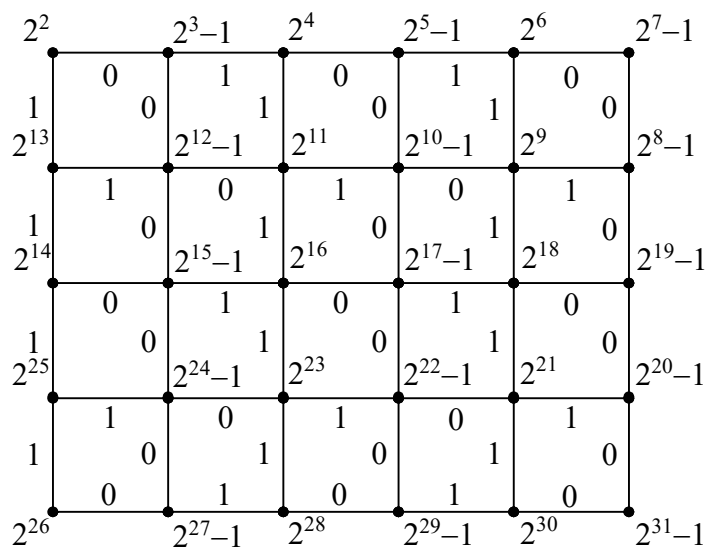

FIGURE 3.The Grid Graph $P_5\times P_6$is a Narayana prime cordial graph

Theorem 3.4

A grid graph $P_m X\ P_n$ $(m\leq n)$ is aNarayana prime cordial graphwhen $m\equiv 0(mod\ 2)$ and $n\equiv 1(mod\ 2)$.

Proof

Let G be the grid graph $P_m X\ P_n$ $(m\leq n)$. We define the vertex set V and edge set E as in theorem 3.1.

Define avertex function $g: V\rightarrow N_0$ such that

The vertices in each odd row P^{i+1} corresponding to i, where $i=0,2,4,\ldots\ m-2$ are labeled as

$$g(u_{i+1,2j-1})=2^{(in)+2j}\ ;1\leq j\leq\left\lceil\frac{n}{2}\right\rceil$$

$$g(u_{i+1,2j})=2^{(in)+(2j+1)}-1\ ;1\leq j\leq\left\lfloor\frac{n}{2}\right\rfloor$$

The vertices in each even row P^{i+1} corresponding to i, where $i=1,3,5,\ldots\ m-1$ are labeled as

$$g(u_{i+1,2j-1}) = 2^{((i+1)n+1)-(2j-2)} - 1 \ ; 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$$
$$g(u_{i+1,2j}) = 2^{((i+1)n+2)-2j} \qquad ; 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$$

By the induced edge function g* : E $\rightarrow$ {0, 1} as in the definition 3.1, as the path $P^i$is of even length in each row, whose $\frac{n-1}{2}$ edges receive 1 and $\frac{n-1}{2}$ edges receive 0 .The path along each column is of odd length whose m-1 edges along odd column , $\left\lceil \frac{m-1}{2} \right\rceil$ edges receive 0 and $\left\lfloor \frac{m-1}{2} \right\rfloor$ edges receive 1 and even column, $\left\lceil \frac{m-1}{2} \right\rceil$ edges receive 1 and $\left\lfloor \frac{m-1}{2} \right\rfloor$ edges receive 0.

Thus $2m\left(\frac{n-1}{2}\right) + n\left\lceil \frac{m-1}{2} \right\rceil + n\left\lfloor \frac{m-1}{2} \right\rfloor = 2mn - m - n$

Therefore $\left|e_{g^*}(1) - e_{g^*}(0)\right| \le 1$ for all edges ,where $e_{g^*}(1)$ denote edges labeled 1 and $e_{g^*}(0)$ denotes edges labeled 0 .Hence the grid graph $P_m X\ P_n$is a Narayana prime cordial graph when $m \equiv 0(mod\ 2)$ and $n \equiv 1(mod\ 2)$.

Example 3.4. A Narayana Prime Cordial Labeling of the Grid Graph $P_4 \times P_7$ is given in figure 4.

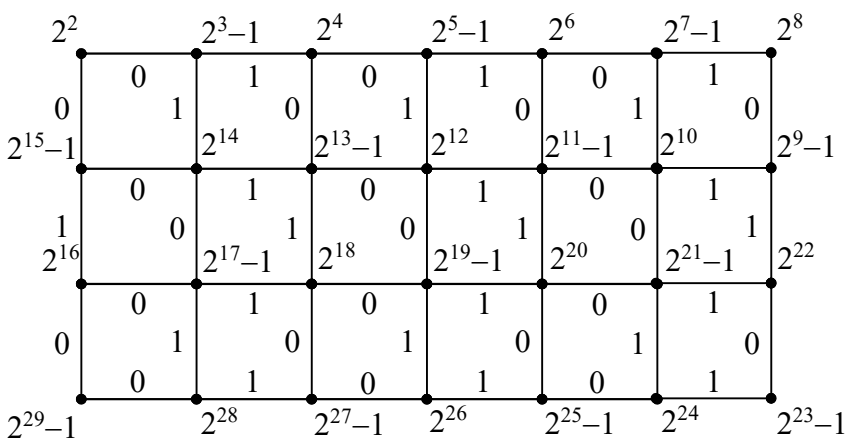

FIGURE 4. The Grid Graph $P_4 \times P_7$ is a Narayana prime cordial graph

4. NARAYANA PRIME CORDIAL LABELING OF TRIANGULAR GRID GRAPH

In this section we find the Narayana prime cordial labeling of a graph which is the union of few copies of triangular grid graph T_2.

Theorem 4.1

The graph *G* which is the union of four copies of triangular grid graph T_2 join through three edges is a Narayana prime cordial graph.

Proof

The triangular grid graph $T_n$is a graph such that (i) $T_n$is a lattice (ii) The lattice is obtained by interpreting the order-(n+1) triangular grids, with the intersection of grid lines as the vertices and the line segments between vertices as the edges.

Example 4.1

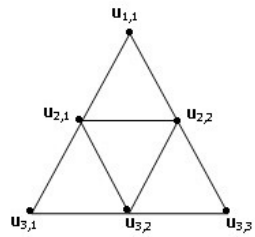

FIGURE 5. Triangular grid graph T_2

Let the graph G be the union of four copies of triangular grid graph T_2 join by three edgesas shown in figure 6.

Let the vertex set of the graph G be denoted by V= $\{u_{1,1}^{(l)}, u_{2,1}^{(l)}, u_{2,2}^{(l)}, u_{3,1}^{(l)}, u_{3,2}^{(l)}, u_{3,3}^{(l)}$, where $l = 0$ denotes vertices of triangular grid graph T_2 in the middle; $l = 1,2,3$ denotes vertices of triangular grid graph T_2 along (i, j, k) directions of T_2 in the middle$\}$. The edge set of the graph be denoted by $E = \{u_{i,1}^{l}u_{i+1,1}^{l}, u_{i,2}^{l}u_{i+1,2}^{l}, u_{3,j}^{l}u_{3,j+1}^{l}, u_{2,1}^{l}u_{2,2}^{l}, u_{2,1}^{l}u_{3,2}^{l}, u_{2,2}^{l}u_{3,2}^{l}, u_{1,1}^{0}u_{1,1}^{3}, u_{3,1}^{0}u_{1,1}^{2}, u_{3,3}^{0}u_{1,1}^{1}, i = 1,2; j = 1,2; l = 0,1,2,3\}$.

For example $u_{3,2}^{0}$ denotes the 2nd vertex in the third row of T_2 in the middle which is assigned the value 2^6in the figure 6.

Define a vertex function $g: V \to N_0$ such that

Let us label the vertices as follows

$$g(u_{1,1}^{l}) = 2^{k+6l}, k > 1, l = 0,1 \ \& \ g(u_{1,1}^{l}) = 2^{k+6l} - 1, l = 2,3$$
$$g(u_{2,1}^{l}) = 2^{(k+1)+6l}, l = 0,1,2,3$$
$$g(u_{2,2}^{l}) = 2^{(k+2)+6l} - 1\ , l = 0,1,2,3$$
$$g(u_{3,1}^{l}) = 2^{(k+3)+6l}, l = 0,1,2 \ \& g(u_{3,1}^{l}) = 2^{(k+3)+6l} - 1, l = 3$$
$$g(u_{3,2}^{l}) = 2^{(k+4)+6l}, l = 0,1,2 \ \& g(u_{3,2}^{l}) = 2^{(k+4)+6l} - 1, l = 3$$
$$g(u_{3,3}^{l}) = 2^{(k+5)+6l} - 1, l = 0,1,2 \ \& \ g(u_{3,3}^{l}) = 2^{(k+5)+6l}, l = 3$$

There are 9 edges in each triangular grid T_2 and 3 copies of T_2 are joined by an edge to T_2 in the middle. So there are 39 edges in total.

By the induced edge function g* : E$\rightarrow$ {0, 1} as in definition 3.1 , an edge whose end vertices are assigned the labels$2^l and 2^k - 1$ (l,k>1)takes the value 1 if $2^l > 2^k - 1$ else 0.

Therefore , in the graph G, out of 9 edges in each of three triangular gridT_2 , 5 edges receives 1 & 4 edges receives 0.One triangular grid T_2 of G has 6 edges as 0 and 3 edges as 1.Also out of 3 edges as join , 2 edges receive 0 and 1 edge receive 1

i.e 3* 5+1*3+1=19 edges are labeled 1 & 3*4+ 1*6+2=20 edges are labeled 0

Therefore $\left|e_{g^*}(1) - e_{g^*}(0)\right| \leq 1$ for all edges ,where$e_{g^*}(1)$ denote edges labeled 1 and $e_{g^*}(0)$ denotes edges labeled 0 .Hence the given graph G is a Narayana prime cordial graph.

Example 4.2 A Narayana Prime Cordial Labeling of the Graph *G* is shown in figure 6.

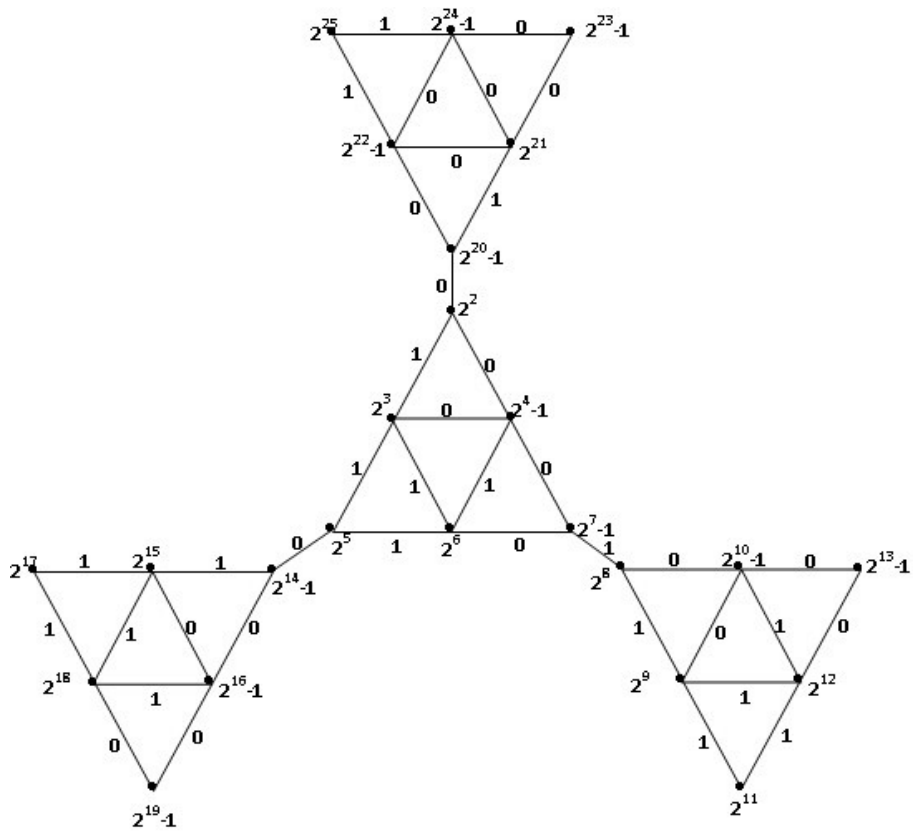

FIGURE 6. A Narayana Prime Cordial Labeling of the graph *G*

5. CONCLUSION

In this paper, we have proved that the grid graphs and the triangular grid graphs admit the Narayana prime cordial labeling.This labeling concept can be applied to other classes of graphs and it is a potential area of research.

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