ISSN 1541-308X, Physics of Wave Phenomena, 2020, Vol. 28, No. 3, pp. 305–312. © Allerton Press, Inc., 2020.

PHYSICS OF 1D AND 2D MEDIA

Peregrine Soliton Management of Breathers in Two Coupled Gross–Pitaevskii Equations with External Potential

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Received April 3, 2019; revised October 8, 2019; accepted October 10, 2019

Abstract—By solving analytically two 1D coupled Gross–Pitaevskii equations with a time-dependent harmonic trap, we find Peregrine solutions that can be effectively controlled by modulating the external potential frequency. Indeed, one observes the onset of instability in the dynamical system as the frequency is varied. This leads to the possibility of stabilizing the Peregrine solitons.

DOI: 10.3103/S1541308X20030036

1. INTRODUCTION

Freak (or rogue) waves are mainly rare events, but appear in so many different areas of physics, ranging from the large amplitude ocean wave events [1] to optics [2–5], and also as solutions of the nonlinear Schrödinger equation. The main feature of these waves is their ability to suddenly emerge from nowwhere with an amplitude significantly larger than that of the surrounding wave crests and disappear without leaving a noticeable trace. They are in many limiting cases described by Peregrine solitons [6] which represent a spatially localized breather with only one oscillation in time. They constitute one of the group of breathers family along with Akhmediev and Kuznetsov–Ma breather. Generally, they are mathematically expressed by rational polynomials [7].

It is a matter of fact that in order to establish a link with observed rare events, the excitations of these solutions should be as random as possible. Otherwise, one is led to describe them as deterministic rogue waves [8, 9]. In this context, one may use special initial configurations to excite higher order rogue waves as solutions of the nonlinear Schrödinger equation [10, 11], the Hirota equation [12, 13], the Sasa–Satsuma equation, and the coupled nonlinear Schrödinger equations [9, 14, 15].

In the latter cases, and in particular in BEC experiments, Feshbach resonances [16–18] allows for a tunability of the interatomic interactions, thus managing the nonlinearity of the underlying equations at will. The control of the trapping fields also provides a powerful tool for manipulating rogue waves.

In the case of binary mixtures, the situation is rather original. The appearance of rogue waves in these systems bears an interest of its own, both mathematical and physical. From the mathematical point of view, finding exact solutions can lead to a better understanding of the conditions under which the system can sustain Peregrine solitons. On the other hand, nowadays running experiments may determine whether these solutions can indeed be observed by a fine tuning of the various parameters at hand.

In the present work, we are mainly interested in finding and describing analytically the Peregrine soliton solutions of two component BEC described by a set of two coupled Gross–Pitaevskii equations (CGPE) in one dimension or quasi-one dimensional space. By considering a harmonic trap with time-dependent frequency, we will focus on the formation mechanism of these solutions, which may lead to a kind of controllability of these rogue waves.

It is worthwhile noticing that the recently published paper [19] considers only two coupled nonlinear Schrödinger (CNLS) equations with coherent coupling terms, fixed attractive interactions and without external potentials. Here, we analyze the solutions of two coupled NLS equations with external time-dependent harmonic potential. The time-dependence of its frequency will lead to novel phenomena such as the stabilization of the solitons. The presence of the trap breaks the translation invariance of the system and this will have dramatic consequences on the solutions. The various interaction parameters are left free to make the formalism as flexible as possible.

We begin in Sec. 2 by transforming our CGPE into a Manakov system by using a similarity transformation [20–22]. In Sec. 3, we discuss the corresponding Lax-pair and analytical methods which we employ to construct the exact solutions. The Darboux transformation method allows us in Sec. 4 to determine and examine the dynamics of the exact solutions. The results are summarized in Sec. 5.

2. THE MODEL

Consider the following system of two CGPE:

$$i\psi_{1t} + \frac{1}{2}\psi_{1xx} + [V(x,t) + R_{11}|\psi_1|^2 + R_{12}|\psi_2|^2]\psi_1 = 0, \quad (1a)$$

$$i\psi_{2t} + \frac{1}{2}\psi_{2xx} + [V(x,t) + R_{21}|\psi_1|^2 + R_{22}|\psi_2|^2]\psi_2 = 0, \quad (1b)$$

where ψ_{it} and ψ_{ixx} denote, respectively, the first and second derivatives with respect to t and x. The (positive) terms R_{ij} represent the attractive interactions and $V(x,t) = \omega(t)^2 x^2/2$ is the time-dependent trapping field. We first convert the system (1a, 1b) to a Manakov system via a similarity transformation (Appendix):

$$iQ_{1t} = \left[-\frac{1}{2}Q_{1xx} - |Q_1|^2 + |Q_2|^2\right]Q_1, \quad (2a)$$

$$iQ_{2t} = \left[-\frac{1}{2}Q_{1xx} - |Q_1|^2 + |Q_2|^2\right]Q_2. \quad (2b)$$

Then, we look for analytic solutions using the Darboux transformation [23] method.

The generalized CNLS (2a, 2b) equations require finding a linear system of equations for an auxiliary fields $\mathbf{\Phi}(x,t)$. The linear system is usually written in compact form in terms of a pair of matrices as follows:

$$\mathbf{\Phi}_x = \mathbf{U\Phi}, \quad (3a)$$

$$\mathbf{\Phi}_t = \mathbf{V\Phi}, \quad (3b)$$

where $\mathbf{U}$ and $\mathbf{V}$, known as the Lax pair, are functionals of the solutions of the model equations. The consistency condition of the linear system $\mathbf{\Phi}_{xt} = \mathbf{\Phi}_{tx}$ must be equivalent to the model equation under consideration.

We find the following linear system which corresponds to the class of generalized CNLS:

$$\mathbf{\Phi}_x = \mathbf{U}_0\mathbf{\Phi} + \mathbf{U}_1\mathbf{\Phi\Lambda}, \quad (4a)$$

$$\mathbf{\Phi}_t = \mathbf{V}_0\mathbf{\Phi} + \mathbf{V}_1\mathbf{\Phi}\Lambda + \mathbf{V}_2\mathbf{\Phi\Lambda}^2, \quad (4b)$$

where

$$\mathbf{\Phi} = \begin{pmatrix} \psi_1(x,t) & \psi_2(x,t) & \psi_3(x,t) \\ \varphi_1(x,t) & \varphi_2(x,t) & \varphi_3(x,t) \\ \chi_1(x,t) & \chi_2(x,t) & \chi_3(x,t) \end{pmatrix}, \quad \mathbf{U}_0 = \begin{pmatrix} 0 & Q_1(x,t) & Q_2(x,t)(11) \\ -Q_1^*(x,t) & 0 & 0(12) \\ -Q_2^*(x,t) & 0 & 0 \end{pmatrix}, \quad \mathbf{U}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

$$\mathbf{V}_0 = \frac{i}{2}\begin{pmatrix} Q_1(x,t)Q_1^*(x,t) + Q_2(x,t)Q_2^*(x,t) & Q_{1x}(x,t) & Q_{2x}(x,t) \\ Q_{1x}^*(x,t) & -Q_1(x,t)Q_1^*(x,t) & -Q_2(x,t)Q_1^*(x,t) \\ Q_{2x}^*(x,t) & -Q_1(x,t)Q_2^*(x,t) & -Q_2(x,t)Q_2^*(x,t) \end{pmatrix},$$

$$\mathbf{V}_1 = \begin{pmatrix} 0 & -Q_1(x,t) & -Q_2(x,t) \\ Q_1^*(x,t) & 0 & 0 \\ Q_2^*(x,t) & 0 & 0 \end{pmatrix}, \quad \mathbf{V}_2 = i\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

where $\lambda_{1,2,3}$ are the spectral parameters. The consistency condition leads to $\mathbf{U}_t - \mathbf{V}_x + [\mathbf{U},\mathbf{V}] = 0$ which should generate the CNLS equations. The next step is to use the darboux transformation to find the solutions.

3. PEREGRINE SOLITON SOLUTIONS AND THEIR DYNAMICS

3.1. Symmetric Case: Same Seed Solutions

$$Q_1(x,t) = Q_2(x,t) = Ae^{iA^2t}$$

Consider the following version of the Darboux transformation [23]:

$$\mathbf{\Phi}[1] = \mathbf{\Phi\Lambda} - \sigma\mathbf{\Phi},$$

where $\mathbf{\Phi}$ [1] is the transformed field and $\sigma = \Phi_0\Lambda\Phi_0^{-1}$, $\mathbf{\Phi}_0$ being a known solution of the linear system (4). Requiring the transformed linear system to be covariant with the original one yields the condition

$$\mathbf{U}_0[1] = \mathbf{U}_0 + [\mathbf{U}_1, \sigma]. \quad (5)$$

which in turn gives the solutions

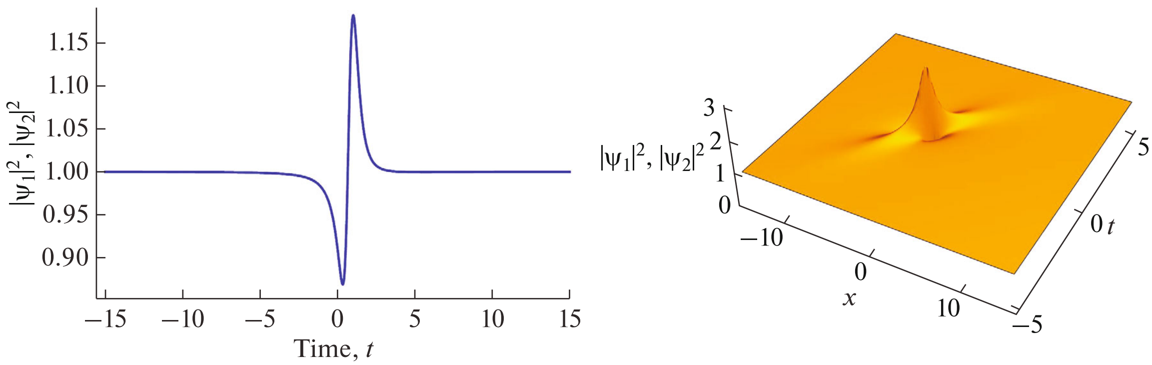

Fig. 1. (Color online) Trapless bright vector solitons. The parameters are $A = 1$, $C_1 = 0.1$ and $C_2 = 0.2$, $h_1(t) = 0$, $h_2(t) = 0$, $h_3(x) = 0$, $\omega(t) = 0$, and $\sigma = 0.5$.

$$Q_1(x,t) = Q_2(x,t) = -\frac{Ae^{2iA^2t}\left(4A^2C_1^2 + C_2^2\left(8A^4t^2 + 4A^2(x^2 - 2it) + 2\sqrt{2}Ax - 1\right) + 2AC_1C_2\left(4Ax + \sqrt{2}\right)\right)}{4A^2C_1^2 + C_2^2\left(8A^4t^2 + 4A^2x^2 + 2\sqrt{2}Ax + 1\right) + 2AC_1C_2\left(4Ax + \sqrt{2}\right)}, \quad (6)$$

with $C_{1,2}$ are arbitrary real constants. The localized solutions are obtained provided $\lambda_1 = -\lambda_2 = \lambda_3 = i\sqrt{2}A$.

Finally, we get the Peregrine solutions

$$\psi_1(x,t) = \psi_2(x,t) = \frac{A\sqrt{h_1(t)}e^{ik_3(x,t)}k_4(x,t)}{k_5(x,t)}, \quad (7)$$

where

$$k_3(x,t) = 2A^2\left(\int h_1(t)^2dt\right) + B_1(x) + B_2(t) - \frac{x^2h_1'(t)}{2h_1(t)} + 2xh_1(t) + h_2(t) + h_3(x),$$

$$k_4(x,t) = 4A^2C_1^2 + 2C_2^2(4A^2x^2h_1(t)^2 + 8A^2(A^2+2)\left(\int h_1(t)^2dt\right)^2 + 2\sqrt{2}Axh_1(t) - 4A\left(4Axh_1(t) + 2iA + \sqrt{2}\right)\int h_1(t)^2dt - 1) + 2AC_1^2C_2^2\left(4Axh_1(t) - 8A\int h_1(t)^2dt + \sqrt{2}\right),$$

$$k_5(x,t) = 4A^2C_1^2 + C_2^2(8A^4\left(\int h_1(t)^2dt\right)^2 + 4A^2\left(xh_1(t) - 2\int h_1(t)^2dt\right)^2 + 2\sqrt{2}A\left(xh_1(t) - 2\int h_1(t)^2dt\right) + 1) + 2AC_1C_2\left(4Axh_1(t) - 8A\int h_1(t)^2dt + \sqrt{2}\right).$$

To understand the dynamics of these solutions, we first consider the homogeneous system by choosing $h_1(t) = 0$ $(\omega(t) = 0)$, which leads to the well-known Manakov model [24]. The behavior of the Peregrine soliton under the above condition is shown in Fig. 1. The upper and lower panels in these figures show, respectively, the projected square moduli of the solutions.

Consider now a static harmonic trap with $(h_1(t) = e^{-t})$ $\omega(t) = 0.5$. The behavior changes dramatically since the densities grow abruptly as shown in the Figs. 5 and 2. The solitons become highly unstable.

In order to overcome this instability and to increase the lifetime of the solitons, a fine tuning of the trap frequency may be helpful [25]. Indeed, choosing arbitrarily $h_1(t) = e^{\int -10tdt}$ yields a significant stabilization of both modes ψ_1 and ψ_2 as shown in the Figs. 6 and 3. This may be well understood since for this set of parameters, the trap is very tight for all times, as its curvature is rapidly growing. One may wonder whether the reduction of the trap frequency will induce instabilities once more. In fact, as we show in Fig. 7, even with a very slowly varying frequency, that is with a very flat trap, the system is still stable.

3.2. *Non-Symmetric Case: Distinct Seed Solutions* $Q_1(x,t) = e^{it}$, $Q_2(x,t) = 0$

In order to confirm our findings, the question is whether they depend on the seed solutions. We therefore consider different seed solutions. Following the same procedure as in the previous section, we get

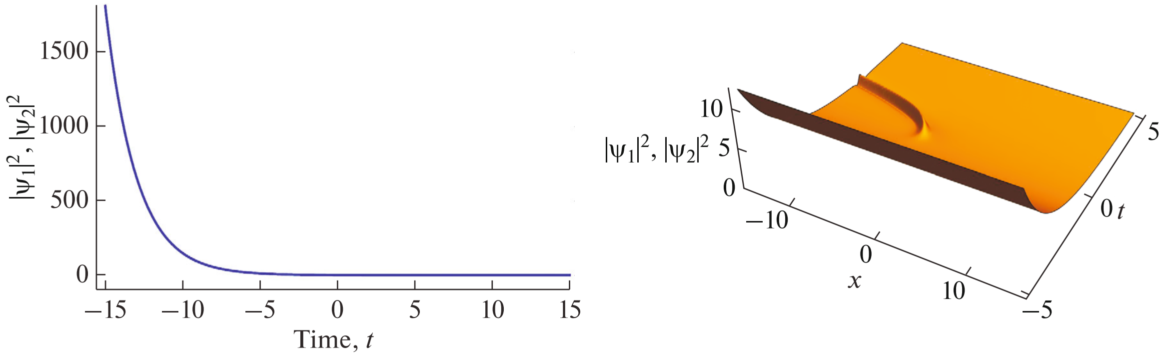

Fig. 2. (Color online) The onset of instability in the densities for a static harmonic trap with $A = 1$, $h_1(t) = e^{-t}$, $h_2(t) = 0.1$, $h_3(x) = 0.5$, $\omega(t) = 0.5$, $C_1 = 0.1$, $C_2 = 0.2$, and $\sigma = 0.5$.

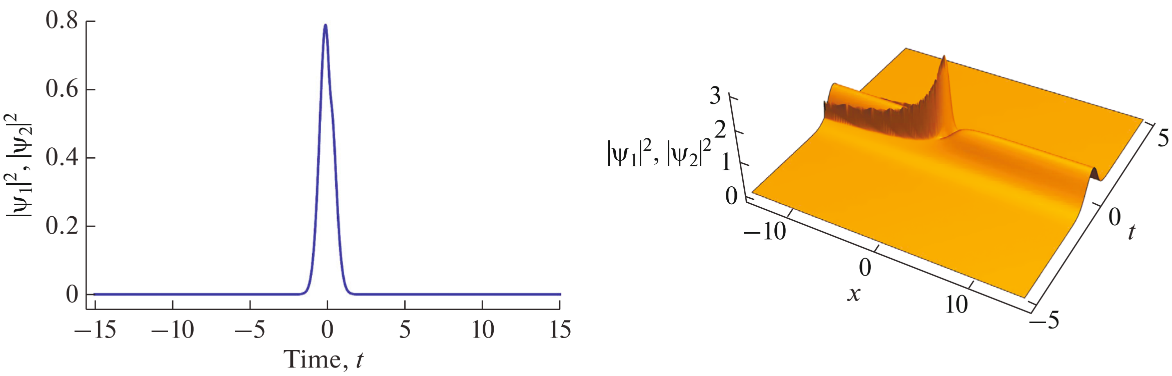

Fig. 3. (Color online) The density profiles of the bright vector solitons in the time dependent trap with $A = 1$, $h_1(t) = e^{\int -10t dt}$, $h_2(t) = 0.9t$, $h_3(x) = 0.1$, $\omega(t) = 5 + 50t^2$, $C_1 = 0.2$, $C_2 = 0.1$, and $\sigma = 0.5$.

$$Q_1(x,t) = \frac{e^{it}[-1 + 2C_1^2 e^{2x} + 2C_1C_2e^{2x}(-1+2x) + C_2^2 e^{2x}(-1 - 4it + 2t^2 - 2x + 2x^2)]}{1 + 2C_1^2 e^{2x} + 2C_1C_2 e^{2x}(-1+2x) + C_2^2 e^{2x}(1 + 2t^2 - 2x + 2x^2)}, \tag{8a}$$

$$Q_2(x,t) = \frac{4e^{3it/2+x}[C_1 + C_2(it + x)]}{1 + 2C_1^2 e^{2x} + 2C_1C_2 e^{2x}(-1+2x) + C_2^2 e^{2x}(1 + 2t^2 - 2x + 2x^2)}, \tag{8b}$$

where $C_{1,2}$ are arbitrary real constants. The spectral parameters have been chosen such that $\lambda_1 = -\lambda_2 = \lambda_3 = i$.

The relations between the Q_i's and the ψ_i's (see Appendix) yield the Peregrine solutions:

$$\psi_1(x,t) = \sqrt{h_1(t)}e^{iK_1(x,t)} \times \left(-1 + \frac{2 + C_2^2(2 + 4i\int h_1(t)^2 dt)e^{2xh_1(t) - 4\int h_1(t)^2 dt}}{G(xh_1(t) - 2\int h_1(t)^2 dt \int h_1(t)^2 dt)}\right), \tag{9a}$$

$$\psi_2(x,t) = \sqrt{h_1(t)}e^{iK_2(x,t)} \times \left(-1 + \frac{2 + C_2^2(2 + 4i\int h_1(t)^2 dt)e^{2xh_1(t) - 4\int h_1(t)^2 dt}}{G(xh_1(t) - 2\int h_1(t)^2 dt \int h_1(t)^2 dt)}\right), \tag{9b}$$

where

$$K_1(x,t) = B_1(x) + B_2(t) - \frac{x^2 h_1'(t)}{2h_1(t)} + 2xh_1(t) + h_2(t) + \int h_1(t)^2 dt + h_3(x),$$

$$K_2(x,t) = B_1(x) + B_2(t) - \frac{x^2 h_1'(t)}{2h_1(t)} + (2 - i)xh_1(t) + h_2(t) + \left(\frac{3}{2} + 2i\right)\int h_1(t)^2 dt + h_3(x),$$

$$B_1(x) = -h_3(x), \quad B_2(t) = \int_1^t \frac{1}{2}\left(-2h_2'(t) - 4h_1(t)^2\right)dt,$$

$$G(X_1, T_1) = 2C_2^2 C_1 e^{2X_1}(2X_1 - 1) + 2C_1^{2X_1} + 1 + C_2^2 e^{2X_1}(2X_1^2 - X_1 + 2T_1^2 + 1),$$

with $X_1 = xh_1(t) - 2\int h_1(t)^2 dt$, $T_1 = \int h_1(t)^2 dt$.

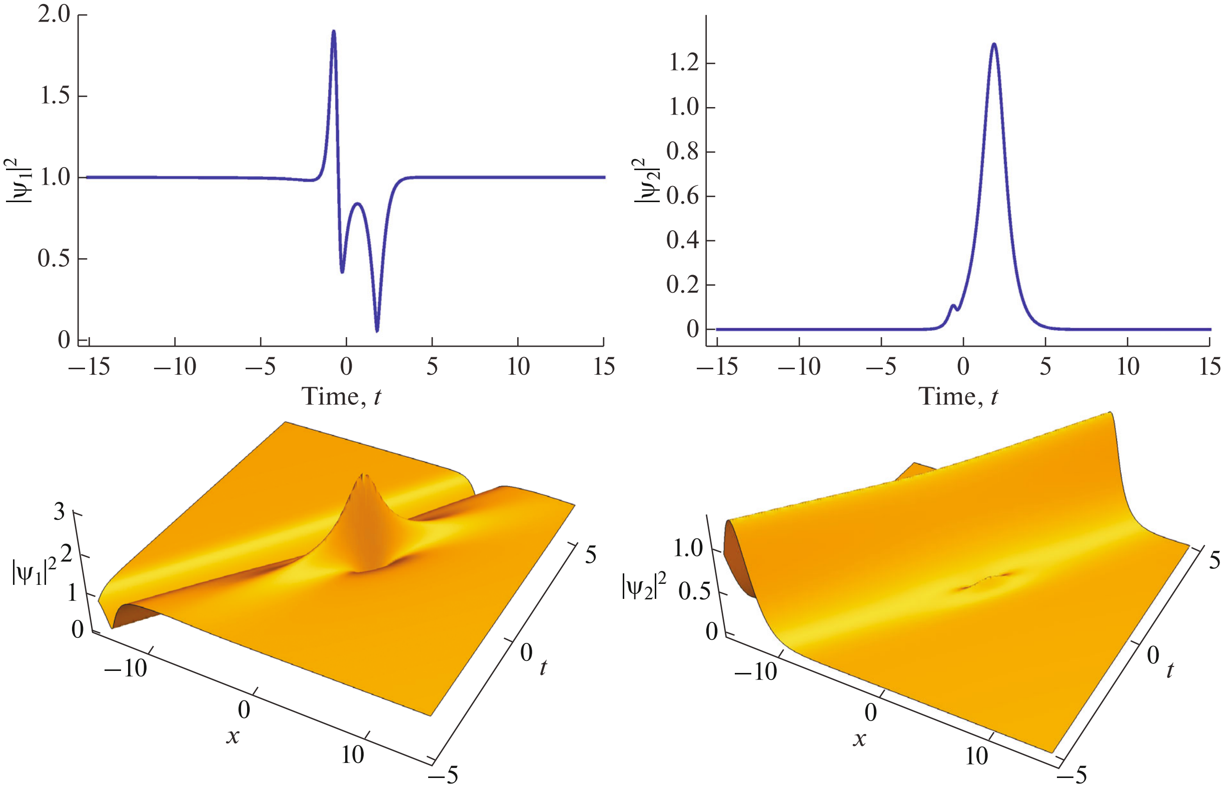

Fig. 4. (Color online) Trapless bright vector solitons. The parameters are $C_1 = -5$, $C_2 = 5$, $h_1(t) = h_2(t) = h_3(x) = 0$, $\omega(t) = 0$, and $\sigma = 0.5$ (see text for details).

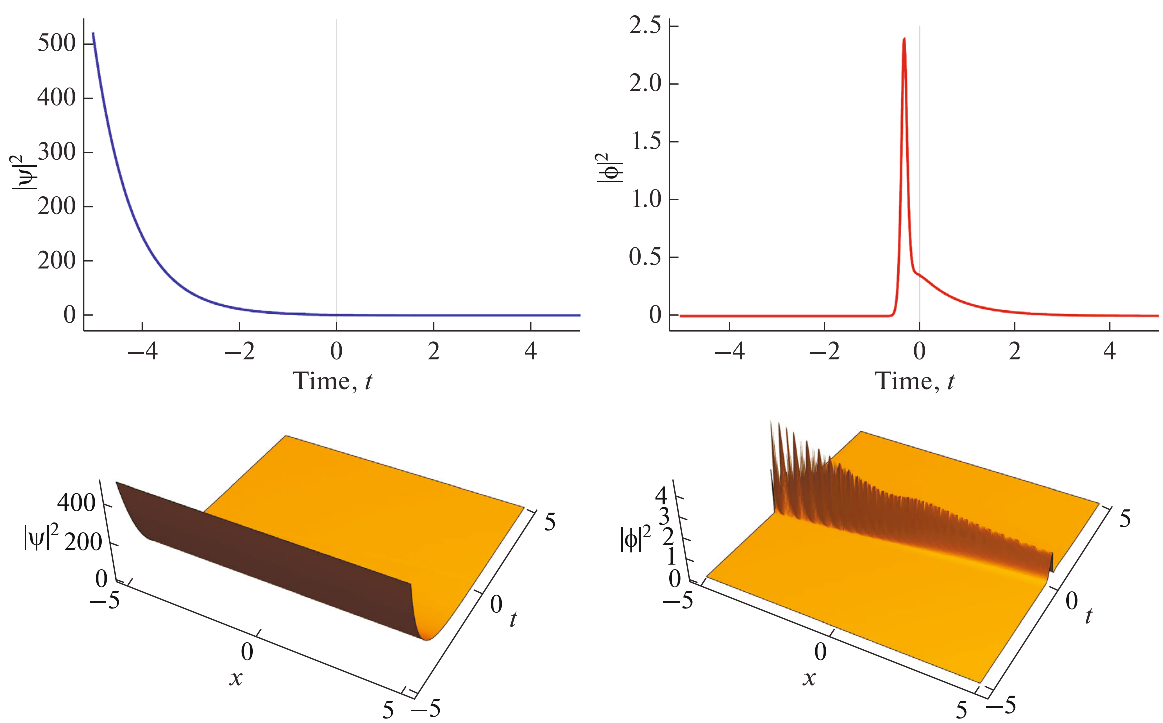

Fig. 5. (Color online) The onset of instability in the densities for a static harmonic trap with $h_1(t) = e^{-t}$, $h_2(t) = 0.1$, $h_3(x) = 0.5$, $\omega(t) = 0.5$, $C_1 = -0.1$, $C_2 = 0.1$, and $\sigma = 0.5$.

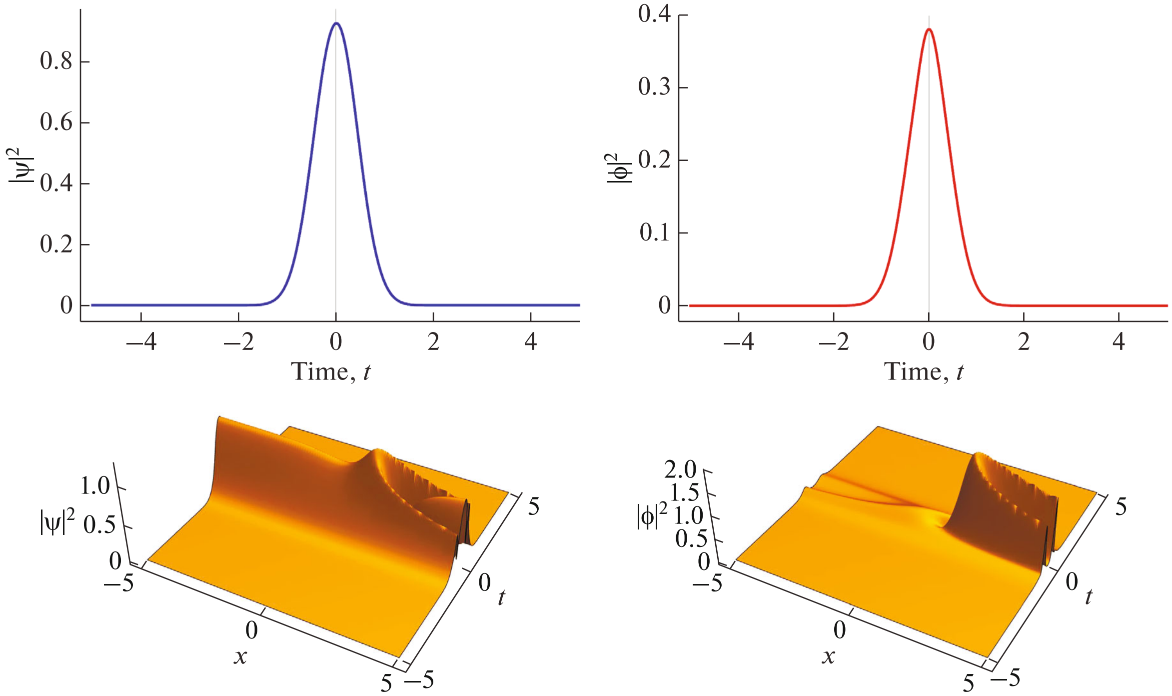

Fig. 6. (Color online) Trapped bright vector solitons in a time dependent trap, $h_1(t) = e^{\int -10t dt}$, $h_2(t) = 0.9t$, $h_3(x) = 0.5$, $\omega(t) = 5 + 50t^2$, $C_1 = -0.1$, $C_2 = 0.1$, and $\sigma = 0.5$.

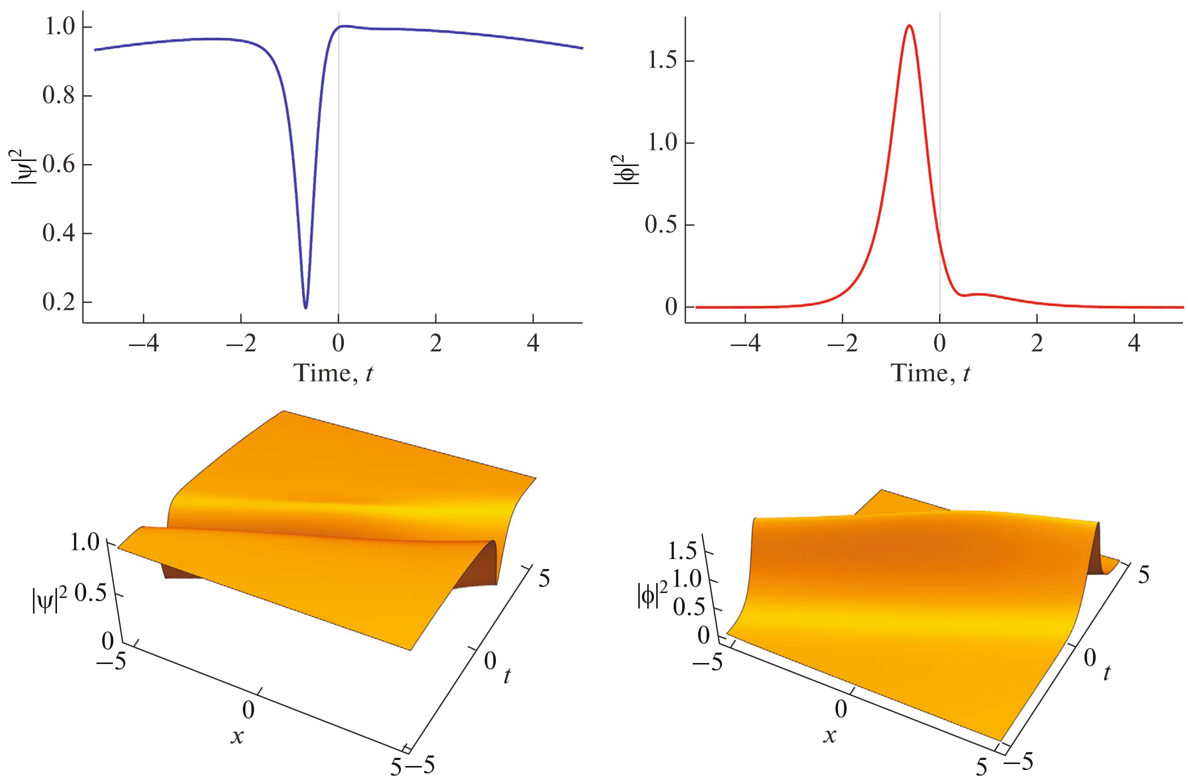

Fig. 7. (Color online) Trapped dark and bright solitons for $h_1(t) = e^{\int -0.01t dt}$, $h_2(t) = 0.9t$, $h_3(x) = 0.5$, $\omega(t) = 0.05 + 0.01t^2$, $C_1 = -0.1$, $C_2 = -0.1$, and $\sigma = 0.5$.

The results are discussed below. Figure 4 depicts the densities for a homogeneous system ($h_1(t) = 0$, $\omega(t) = 0$). For a static harmonic trap ($h_1(t) = e^{-t}$, $\omega(t) = 0.5$), Fig. 5, the solitons are destabilized. Upon tuning the trap frequency with a rapidly growing curvature ($h_1(t) = e^{\int -10t dt}$, $\omega(t) = 5 + 50t^2$) significantly stabilizes the two modes ψ_1 and ψ_2 as shown in Fig. 6. Finally, even with a very slowly varying frequency, the system is not only still stable, but a novel phenomenon takes place. The system now sustains the coexistence of both a bright and a dark soliton (Fig. 7). This result is quite interesting by itself since, to the best of our knowledge, the existence of a stable bright-dark soliton pair in a binary mixture has attracted very little attention.

4. CONCLUSION

The coupled Gross–Pitaevskii equations with a time-dependent harmonic trap are solved analytically by transforming to a Manakov system (via a similarity transformation) and using the Lax pair method. The Darboux transformation is used in two cases. The symmetric case with the same seed solutions yields in the homogeneous case bright vector solitons which are destabilized by the introduction of a static harmonic confinement. Upon modulating the frequency of the trap, the solitons are stabilized being a pair of bright solitons for a growing tight confinement.

These results are almost independent of the seed solutions. Indeed, if one begins with nonsymmetric seed solutions, the overall behavior does not dramatically change. The system still sustains bright vector solitons which are destabilized (for a static trap), then stabilized by a rapidly growing tight confinement. For an almost flat confinement, the solutions consist of a dark-bright soliton pair. The latter situation is quite original and requires much more attention.

5. *APPENDIX*: FROM GPE TO MANAKOV SYSTEM

Similarity transformation and analytical setup from GPE to Manakov system: we apply the following transformation to Eqs. (1):

$$\psi_1 = Q_1(X,T)e^{\int h(t)dt + ia(x,t)},$$

$$\psi_2 = Q_2(X,T)e^{\int h(t)dt + ia(x,t)},$$

where

$$X = xe^{2\int h(t)dt} - 2\int h_1(t)e^{2\int h(t)dt}dt,$$

$$T = \int e^{4\int h(t)dt}dt,$$

$$a(x,t) = -\frac{1}{2}x^2 h(t) + x h_1(t) + h_2(t),$$

$$R_{11} = R_{12} = R_{21} = R_{22} = 2\sigma\gamma(t),$$

$$\gamma(t)e^{2\int h(t)dt}, \quad \sigma = 1/2.$$

Substituting the above transformation given by ψ_1 and ψ_2 in Eqs. (1) and reinforcing the following constraints:

$$\omega^2(t) = h(t)^2 - \frac{h'(t)}{2}, \quad h(t) = \frac{h_1'(t)}{2h_1(t)}, \quad h_2'(t) = -h_1(t)^2,$$

will reduce the coupled GP equations to the coupled NLS equations (2a, 2b) [20–22]

$$iQ_{1t} = \left[-\frac{1}{2}Q_{1xx} - \left(Q_1^2 + Q_2^2\right)\right]Q_1,$$

$$iQ_{2t} = \left[-\frac{1}{2}Q_{1xx} - \left(Q_1^2 + Q_2^2\right)\right]Q_2.$$

ACKNOWLEDGMENTS

The authors HCS, MB and HB acknowledge the support provided by Hassiba Benbouali University of Chlef and the Ministry of Higher Education and Scientific Research of Algeria through the grants HB00L02UN020120180002/2018-2019. The authors PSV and UAK acknowledge the support of UAE University through the grants UPAR(7)-2015, UPAR(4)-2016, and UPAR(6)-2017. PSV wishes to extend his gratitute to the Principal of PSG College of Arts and Sciences, Coimbatore, Tamilnadu, India for her continuous support, motivation and facilities.

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