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Wellposedness and controllability results of stochastic integrodifferential equations with noninstantaneous impulses and Rosenblatt process

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Abstract
The purpose of this work is to investigate a novel class of noninstantaneous impulsive stochastic integrodifferential equations (SIDEs) driven by Brownian motion and Rosenblatt process. We construct a new set of adequate assumptions for the existence and uniqueness of mild solutions using stochastic analysis, analytic semigroup theory, integral equation theory, and a fixed point methodology. Additionally, we study the asymptotic behavior of mild solutions and provide stochastic system controllability results. Finally, we include an example to illustrate the application of our main findings.

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1 Introduction

Many real-world phenomena, such as stock prices, heat conduction in materials with memory, and population growth model, are affected by random influences or noise in recent decades, necessitating the inclusion of randomness into mathematical descriptions of the phenomena. Stochastic differential equations (SDEs) emerge as a powerful tool for describing and analyzing such phenomena. The theory of SDEs can be successfully applied to a wide range of interesting areas, including economics, epidemiology, chemistry, mechanics, and finance. For more details on SDEs, we refer to [1–6]. SDEs driven by fractional Brownian motion (fBm) can be used to describe noise in a variety of fields, including financial mathematics, hydrology, medicine, and telecommunication networks. However, sometimes the Gaussianity is not realistic for the model. The Rosenblatt process is a non-Gaussian process with notable properties such as increment stationary, long-range dependence, and self-similarity. As a result, a new class of SDEs driven by the Rosenblatt process appears to be of interest; see [7–10].

Furthermore, many physical problems are classified in the literature by abrupt changes in their states. These abrupt changes are referred to as impulsive effects in the system. Many physical problems can be modeled using impulsive differential equations. In many fields of applied research, such as physics, engineering, economics, and medicine, impulsive differential equations have been proved to be extremely useful. The impulsive system is divided into two categories based on the length of the impulsive acts:

(i) The instantaneous impulsive system, in which the length of these sudden changes is small in comparison to the entire evolution process. For example, heart pulsates, natural disasters and shocks, etc.

(ii) The noninstantaneous impulsive system, in which the length of these abrupt changes continues over a very finite time interval.

For example, a very well-known application of noninstantaneous impulses is the introduction of insulin into the bloodstream, which is an abrupt change, and the consequent absorption is a gradual process as it remains active over a finite time interval. Many authors have recently looked into noninstantaneous impulsive differential equations [11–16].

Many fundamental control theory problems, such as stability, pole assignment, and optimal control, can be solved under the assumption that the system is controllable. The ability of a system to move around in its entire configuration space with only a few permissible actions is referred to as controllability. Real-world applications of controllability include predator–prey systems, blood sugar levels, rocket launching issues, missiles, antimissile issues, etc. Exact controllability [17–19] and approximate controllability [20–24] are well-developed control ideas associated with control systems. Kalman [25] introduced the work on exact controllability.

Moreover, Naito [26] established approximate controllability results for the semilinear system by using the Schauder fixed point theorem. Many authors have been demonstrated controllability results for SDEs with impulses [27–33]. Huan [34] investigated the controllability of impulsive stochastic integrodifferential equations with infinite delay. Yan and Jia [35] investigated the controllability of a class of impulsive stochastic integrodifferential inclusions with state-dependent delay. Furthermore, asymptotic behavior of mild solutions and controllability results on noninstantaneous impulsive SIDEs driven by mixed fBm are uncommon in the literature, which motivates the research presented in this paper.

Motivated by the above facts, we consider the following noninstantaneous impulsive neutral SIDEs with infinite delay:

$$\begin{aligned} d\big[\vartheta(\mathfrak{t})+\zeta(\mathfrak{t},\vartheta_{\mathfrak{t}})\big] &= \mathscr{A}\big[\vartheta(\mathfrak{t})+\zeta(\mathfrak{t},\vartheta_{\mathfrak{t}})\big]\,d\mathfrak{t}+\int_0^{\mathfrak{t}}\Theta(\mathfrak{t}-s)\big[\vartheta(s)+\zeta(s,\vartheta_s)\,ds\big]\,d\mathfrak{t} \\ &\quad + \mathfrak{l}(\mathfrak{t},\vartheta_{\mathfrak{t}})\,d\mathfrak{t}+\mathfrak{g}(\mathfrak{t},\vartheta_{\mathfrak{t}})\,d\omega(\mathfrak{t})+\sigma(\mathfrak{t})\,d\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}), \quad \mathfrak{t}\in\bigcup_{\mathrm{k}=0}^{m}(s_{\mathrm{k}},\mathfrak{t}_{\mathrm{k}+1}], \quad (1.1) \\ \vartheta(\mathfrak{t}) &= \mathscr{I}_{\mathrm{k}}\big(\mathfrak{t},\vartheta\big(\mathfrak{t}_{\mathrm{k}}^-\big)\big), \quad \mathfrak{t}\in\bigcup_{\mathrm{k}=1}^{m}(\mathfrak{t}_{\mathrm{k}},s_{\mathrm{k}}], \\ \vartheta(0) &= \varphi\in\mathscr{B}, \end{aligned}$$

where $\vartheta(\cdot)$ takes values in $\mathscr{X}$ with the inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, $\mathscr{A}$ is the generator of an analytic semigroup $\{\mathfrak{S}(\mathfrak{t})\}_{\mathfrak{t}\geq 0}$ on $\mathscr{X}$, and $0 = s_0 = \mathfrak{t}_0 < \mathfrak{t}_1 < s_1 < \mathfrak{t}_2 < \cdots < \mathfrak{t}_m < s_m < \mathfrak{t}_{m+1} = \mathfrak{b} < \infty$, $\mathcal{J} = [0,\mathfrak{b}]$; for each $\mathfrak{t}\geq 0$, $\mathscr{A}$ is a closed linear operator with dense domain $\mathscr{D}(\mathscr{A})$ independent of $\mathfrak{t}$, and for $0 \leq s \leq \mathfrak{t}$, $\Theta(\mathfrak{t}-s)$ is a closed linear operator with domain $\mathscr{D}(\Theta) \supset \mathscr{D}(\mathscr{A})$. The function $\mathscr{I}_{\mathfrak{k}}(\mathfrak{t}, \vartheta(\mathfrak{t}_{\mathfrak{k}}^{-}))$ represents the noninstantaneous impulses in the intervals $(\mathfrak{t}_{\mathfrak{k}}, s_{\mathfrak{k}}]$, $i = 1, 2, \ldots, m$. The time history $\vartheta_{\mathfrak{t}} : (-\infty, 0] \to \mathscr{X}$ given by $\vartheta_{\mathfrak{t}}(\theta) = \vartheta(\mathfrak{t}+\theta)$ belongs to the abstract space $\mathscr{B}$ defined axiomatically. Let E_1 be a real separable Hilbert space containing the Wiener process $\{\omega(\mathfrak{t})\}_{\mathfrak{t}\geq 0}$. In the real separable space E_2, $\mathscr{Z}_{\mathscr{H}} = \{\mathscr{Z}_{\mathscr{H}}(\mathfrak{t})\}_{\mathfrak{t}\geq 0}$ is a Rosenblatt process with Hurst index $\mathscr{H} \in (1/2, 1)$, provided that ω and $\mathscr{Z}_{\mathscr{H}}$ are independent. The maps $\zeta : \mathcal{J} \times \mathscr{B} \to \mathscr{X}$, $\mathfrak{l} : \mathcal{J} \times \mathscr{B} \to \mathscr{X}$, $\mathfrak{g} : \mathcal{J} \times \mathscr{B} \to \mathbb{L}_2^2(\mathrm{E}_1, \mathscr{X})$, $\sigma : \mathcal{J} \to \mathbb{L}_2^2(\mathrm{E}_2, \mathscr{X})$, and $\mathscr{I}_{\mathfrak{k}} : (\mathfrak{t}_{\mathfrak{k}}, s_{\mathfrak{k}}] \times \mathscr{B} \to \mathscr{X}$ satisfy certain conditions that are specified later.

The main contributions of our work are summarized as follows:

• a class of stochastic integrodifferential noninstantaneous impulsive systems with state-dependent delay (1.1) is formulated;

• the existence and uniqueness of mild solution of the aforementioned system is investigated using stochastic analysis theory, fixed point technique, and resolvent operator theory;

• the exponential stability and controllability results of the mild solution of (1.1) with state-dependent delay;

• an example to depict the results obtained.

The outline of this paper is as follows. In Sect. 2, we presnet certain prominent preliminary results needed to establish the findings of the study. In Sect. 3, we investigate the existence and uniqueness of mild solutions for the aforementioned system (1.1). In Sects. 4 and 5, we establish the asymptotic behavior and controllability results of system (1.1), respectively. In Sect. 6, we provide an illustration on how to use our findings.

2 Preliminaries

2.1 Stochastic integration

Let $\mathcal{L}(\mathrm{E}_i, \mathscr{X})$ be the space of all bounded linear operators from E_i to $\mathscr{X}$, $i = 1, 2$.

2.1.1 Wiener process

Let $(\Omega, \mathcal{F}, \mathcal{F}_{\mathfrak{t}\geq 0}, \mathbb{P})$ be a complete probability space, where $\mathcal{F}_{\mathfrak{t}}, \mathfrak{t} \in \mathcal{J}$, is a family of right-continuous increasing σ-algebras with $\mathcal{F}_{\mathfrak{t}} \subset \mathcal{F}$. Let $Q_i \in \mathcal{L}(\mathrm{E}_i, \mathrm{E}_i)$ be two operators defined by $Q_i e_i^j = \lambda_i^j e_i^j$ with finite traces $\operatorname{tr}(Q_i) = \sum_{j=1}^{\infty} \lambda_i^j < \infty$, where $e_i^j, j \geq 1$, is a complete orthonormal basis in E_i, and $\{\lambda_i^j\}_{j\geq 1}$ are nonnegative real numbers. We define an E_1-valued Brownian motion $\omega(\mathfrak{t})$ as

$$\omega(\mathfrak{t}) = \sum_{j=1}^{\infty} \sqrt{\lambda_1^j} \beta_j(t) e_1^j,$$

where $\beta_j(t)$ are real independent Brownian motions.

Let $\phi_1 \in \mathcal{L}(\mathrm{E}_1, \mathscr{X})$ and define

$$\|\phi_1\|_{\mathbb{L}_2^1} = \left[\sum_{j=1}^{\infty} \sqrt{\lambda_1^j} \phi_1 e_1^j \right]^{1/2}.$$

If $\|\phi_1\|_{\mathbb{L}_2^1} < \infty$, then ϕ_1 is called Q_1-Hilbert–Schmidt operator. Denote by $\mathbb{L}_2^1(\mathrm{E}_1, \mathscr{X})$ the real separable Hilbert space of all Q-Hilbert–Schmidt operators with inner product

$$\langle\phi_1, \phi_2\rangle_{\mathbb{L}_2^1} = \sum_{j=1}^{\infty}\langle\phi_1 e_1^j, \phi_2 e_1^j\rangle.$$

We introduce the space $P\mathbb{C}(\mathscr{X})$ of all $\mathcal{F}_{\mathfrak{t}}$-adapted measurable $\mathscr{X}$-valued stochastic processes $\{\vartheta(\mathfrak{t}) : \mathfrak{t} \in [0, \mathfrak{b}]\} \ni \vartheta$ is continuous at $\mathfrak{t} \neq \mathfrak{t}_i$, $\vartheta(\mathfrak{t}_i) = \vartheta(\mathfrak{t}_i^-)$ and $\vartheta(\mathfrak{t}_{\mathrm{k}}^+)$ exist for all $\mathrm{k} = 1, 2, \dots, m$, provided that

$$\|\vartheta\|_{P\mathbb{C}} = \Big(\sup_{0\le\mathfrak{t}\le\mathfrak{b}} \mathbb{E}\big\|\vartheta(\mathfrak{t})\big\|^2\Big)^{1/2}.$$

We suppose that the phase space is axiomatically defined and assume, as proposed by Hale and Kato [36], that $(\mathscr{B}, \|\cdot\|_{\mathscr{B}})$ is a seminormed linear space of $\mathcal{F}_0$-measurable functions mapping $(-\infty, 0]$ into $\mathscr{X}$, fulfilling the following fundamental axioms:

(A1) If $\vartheta : (-\infty, \varepsilon + \mathfrak{b}] \to \mathscr{X}$, $\mathfrak{b} > 0$, is such that $\vartheta|[\varepsilon, \varepsilon + \mathfrak{b}] \in \mathbb{C}([\varepsilon, \varepsilon + \mathfrak{b}], \mathscr{X})$ and $\vartheta_\varepsilon \in \mathscr{B}$, then for every $\mathfrak{t} \in [\varepsilon, \varepsilon + \mathfrak{b}]$, the following conditions hold:
  (i) $\vartheta_{\mathfrak{t}}$ is in $\mathscr{B}$;
  (ii) $\|\vartheta(\mathfrak{t})\|_{\mathscr{X}} \le \mathscr{V}^*\|\vartheta_{\mathfrak{t}}\|_{\mathscr{B}}$, where $\mathscr{V}^* > 0$ is a constant.
  (iii) $\|\vartheta_{\mathfrak{t}}\|_{\mathscr{B}} \le \mu_1(\mathfrak{t} - \varepsilon)\sup\{\|\vartheta(s)\|_{\mathscr{X}} : \varepsilon \le s \le \mathfrak{t}\} + \mu_2(\mathfrak{t} - \varepsilon)\|\vartheta_\varepsilon\|_{\mathscr{B}}$, where $\mathscr{V}^* \ge 0$ is a constant; $\mu_1, \mu_2 : [0, +\infty) \to [1, +\infty)$, μ_2 is locally bounded, and μ_1 is continuous with $\mathscr{V}^*, \mu_1, \mu_2$ independent of $\vartheta(\cdot)$.

(A2) For $\vartheta(\cdot)$, the map $\mathfrak{t} \to \vartheta_{\mathfrak{t}}$ is continuous from $[\varepsilon, \varepsilon + \mathfrak{b}]$ into $\mathscr{B}$.

(A3) The space $\mathscr{B}$ is complete.

2.1.2 Rosenblatt process

Let $Q_2 \in \mathcal{L}(\mathrm{E}_2, \mathrm{E}_2)$ represent a nonnegative self-adjoint operator. Let $\mathbb{L}_2^2 = \mathbb{L}_2^2(Q^{1/2}\mathrm{E}_2, \mathscr{X})$ denote the separable Hilbert space of all Hilbert–Schmidt operators from $Q^{1/2}\mathrm{E}_2$ into $\mathscr{X}$, equipped with the norm

$$\|\phi_2\|_{\mathbb{L}_2^2}^2 = \big\|\phi_2 Q^{1/2}\big\|^2 = \mathrm{tr}\big(\phi_2 Q \phi_2^*\big).$$

The Wiener–Itô multiple integral of order k with respect to standard Brownian motion $\beta = \beta(\mathfrak{t})$, $\mathfrak{t} \in \mathbb{R}$, is given by

$$\mathscr{Z}_{\mathscr{H}}^{\mathrm{k}} = p(\mathscr{H}, \mathrm{k})\int_{\mathbb{R}^{\mathrm{k}}}\int_0^{\mathfrak{t}}\left(\prod_{j=1}^{\mathrm{k}}(s - \mathfrak{t}_j)_+^{-(1/2+(1-\mathscr{H})/\mathrm{k})}\right) ds\, d\beta(\mathfrak{t}_1)\dots d\beta(\mathfrak{t}_{\mathrm{k}}), \tag{2.1}$$

where $\mathfrak{a}_+ = \max(\mathfrak{a}, 0)$, and the normalizing constant $p(\mathscr{H}, \mathrm{k})$ ensures $\mathbb{E}(\mathscr{Z}_{\mathscr{H}}^{\mathrm{k}}(1))^2 = 1$. The process $(\mathscr{Z}_{\mathscr{H}}^{\mathrm{k}}(\mathfrak{t}))_{\mathfrak{t}\ge0}$ is called the Hermite process.

- For k = 1, process (2.1) is the fractional Brownian motion with Hurst parameter $\mathscr{H} \in (1/2, 1)$.
- For k = 2, the process given by (2.1) is called the Rosenblatt process, and it is not a Gaussian process.

Let $\{\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}) : \mathfrak{t} \geq 0\}$ represent a one-dimensional Rosenblatt process with Hurst constant $\mathscr{H} \in (1/2, 1)$. Now the Rosenblatt process with parameter $\mathscr{H} > 1/2$ can be written as

$$\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}) = p(\mathscr{H}) \int_0^{\mathfrak{t}} \int_0^{\mathfrak{t}} \left(\int_{\mathfrak{t}_1 \vee \mathfrak{t}_2}^{\mathfrak{t}} \frac{\partial K_{\mathscr{H}'}}{\partial \mathfrak{t}}(\mathfrak{t}, \mathfrak{t}_1) \frac{\partial K_{\mathscr{H}'}}{\partial \mathfrak{t}}(\mathfrak{t}, \mathfrak{t}_2)\, d\mathfrak{t} \right) d\beta(\mathfrak{t}_1)\, d\beta(\mathfrak{t}_2), \tag{2.2}$$

where β is a Wiener process, and $K_{\mathscr{H}}(\mathfrak{t}, s)$ is given by

$$K_{\mathscr{H}}(\mathfrak{t}, s) = \begin{cases} m_{\mathscr{H}} s^{1/2-\mathscr{H}} \int_s^{\mathfrak{t}} (\tau - s)^{\mathscr{H}-3/2} \tau^{\mathscr{H}-1/2}\, d\tau & \text{for } \mathfrak{t} > s, \\ 0 & \text{for } \mathfrak{t} \leq s, \end{cases}$$

where $m_{\mathscr{H}} = [\frac{\mathscr{H}(2\mathscr{H}-1)}{\mathsf{B}(2-2\mathscr{H}, B-1/2)}]^{1/2}$, $\mathsf{B}(\cdot, \cdot)$ is the beta function, $p(\mathscr{H}) = \frac{1}{1+\mathscr{H}} \sqrt{\frac{\mathscr{H}}{2\mathscr{H}-1}}$ is a normalizing constant, and $\mathscr{H}' = \frac{\mathscr{H}+1}{2}$. The covariance of Rosenblatt process $\mathscr{Z}_{\mathscr{H}}(\mathfrak{t})$ is

$$\mathbb{E}\big(\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}), \mathscr{Z}_{\mathscr{H}}(s)\big) = \frac{1}{2}\big(s^{2\mathscr{H}} + \mathfrak{t}^{2\mathscr{H}} - |\mathfrak{t} - s|^{2\mathscr{H}}\big).$$

Consider the E_2-valued stochastic process $\mathscr{Z}_Q(t)$ by the series

$$\mathscr{Z}_Q(\mathfrak{t}) = \sum_{j=1}^{\infty} \zeta_j(\mathfrak{t}) Q^{1/2} e_2^j, \quad \mathfrak{t} \geq 0,$$

where $\zeta_j(\mathfrak{t})$ is a sequence of mutually independent two-sided one-dimensional Rosenblatt processes on $(\Omega, \mathcal{F}, \mathbb{P})$. Let $Q \in \mathcal{L}(\mathrm{E}_2, \mathrm{E}_2)$ be the operator defined by $Qe_2^j = \lambda_2^j e_2^j$ with finite trace $\operatorname{tr}(Q) = \sum_{j=1}^{\infty} \lambda_2^j$, where $\lambda_2^j \geq 0$ for all $j = 1, 2, \ldots$, and $\{e_2^j : j = 1, 2, \ldots\}$ is a complete orthonormal basis in E_2. Let $\mathscr{Z}_{\mathscr{H}}(\mathfrak{t})$ be the E_2-valued Rosenblatt process with covariance Q defined as

$$\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}) = \mathscr{Z}_Q(\mathfrak{t}) = \sum_{j=1}^{\infty} \sqrt{\lambda_2^j} \zeta_j(t) e_2^j.$$

Before announcing the following definition, let us clarify some facts about Rosenblatt processes. Define the linear operator $K_{\mathscr{H}}^*$ from $\mathcal{E}$ to $\mathbb{L}^2([0, \mathfrak{b}])$ by

$$\big(K_{\mathscr{H}}^* \mathfrak{f}\big)(\mathfrak{t}, \mathfrak{t}_2) = \int_{\mathfrak{t}_1 \vee \mathfrak{t}_2}^{\mathfrak{b}} \mathfrak{f}(\mathfrak{t}) \frac{\partial \mathcal{K}}{\partial \mathfrak{t}}(\mathfrak{t}, \mathfrak{t}_1, \mathfrak{t}_2)\, d\mathfrak{t}, \tag{2.3}$$

where $\mathcal{E}$ is the set of step functions on $[0, T]$ of the form

$$\mathfrak{f} = \sum_{i=0}^{n-1} \mathfrak{a}_i 1_{(\mathfrak{t}_i, \mathfrak{t}_{i+1}]}, \quad \mathfrak{t} \in [0, \mathfrak{b}],$$

and $\mathcal{K}$ is the kernel of Rosenblatt process in representation (2.2),

$$\mathcal{K}(\mathfrak{t}, \mathfrak{t}_1, \mathfrak{t}_2) = 1_{[0,\mathfrak{t}]}(\mathfrak{t}_1) 1_{[0,\mathfrak{t}]}(\mathfrak{t}_2) \int_{\mathfrak{t}_1 \vee \mathfrak{t}_2}^{\mathfrak{t}} \frac{\partial K_{\mathscr{H}'}}{\partial \mathfrak{t}}(\mathfrak{t}, \mathfrak{t}_1) \frac{\partial K_{\mathscr{H}'}}{\partial \mathfrak{t}}(\mathfrak{t}, \mathfrak{t}_2)\, d\mathfrak{t}.$$

The operator $K_{\mathscr{H}}^*$ is an isometry between $\mathcal{E}$ and $\mathbb{L}^2([0, T])$, which can be extended to the Hilbert space $\mathscr{X}$.

Definition 2.1 ([6]) Let $\psi : \mathcal{J} \to \mathbb{L}_2^2(Q^{1/2}\mathrm{E}_2, \mathscr{X})$ be such that $\sum_{j\geq 1} \|K^*_{\mathscr{H}}(\psi Q^{1/2}e_2^j)\|_{\mathbb{L}_2^2} < \infty$. Then for $\mathfrak{t} \geq 0$, its stochastic integral with respect to the Rosenblatt process is defined as

$$\begin{aligned}\int_0^{\mathfrak{t}} \psi(s)\, d\mathscr{Z}_{\mathscr{H}}(s) &= \sum_{j=1}^{\infty} \int_0^{\mathfrak{t}} \psi(s) Q^{1/2} e_2^j \, d\zeta_j(s) \\ &= \sum_{j=1}^{\infty} \int_0^{\mathfrak{t}} \int_0^{\mathfrak{t}} \bigl(K^*_{\mathscr{H}}\bigl(\psi Q^{1/2} e_2^j\bigr)\bigr)(\mathfrak{t}_1, \mathfrak{t}_2)\, d\beta(\mathfrak{t}_1)\, d\beta(\mathfrak{t}_2).\end{aligned}$$

Lemma 2.1 ([7]) *Let* $\psi : \mathcal{J} \to \mathbb{L}_2^2(Q^{1/2}\mathrm{E}_2, \mathscr{X})$ *be such that* $\sum_{j\geq 1} \|\psi Q^{1/2} e_2^j\|_{\mathbb{L}^{1/\mathscr{H}}(\mathcal{J},\mathscr{X})} < \infty$. *Then for any* $\mathfrak{a}, \mathfrak{b} \in \mathcal{J}$ *with* $\mathfrak{b} > \mathfrak{a}$, *we have*

$$\mathbb{E}\left\| \int_{\mathfrak{a}}^{\mathfrak{b}} \psi(s)\, d\mathscr{Z}_{\mathscr{H}}(s) \right\|^2 \leq m_{\mathscr{H}} (\mathfrak{b} - \mathfrak{a})^{2\mathscr{H}-1} \sum_{j=1}^{\infty} \int_{\mathfrak{a}}^{\mathfrak{b}} \left\| \psi(s) Q^{1/2} e_2^j \right\|^2 ds.$$

If, in addition, $\sum_{j=1}^{\infty} \|\psi Q^{1/2} e_2^j\|$ *uniformly converges for* $\mathfrak{t} \in J$, *then*

$$\mathbb{E}\left\| \int_{\mathfrak{a}}^{\mathfrak{b}} \psi(s)\, d\mathscr{Z}_{\mathscr{H}}(s) \right\|^2 \leq m_{\mathscr{H}} (\mathfrak{b} - \mathfrak{a})^{2\mathscr{H}-1} \int_{\mathfrak{a}}^{\mathfrak{b}} \left\| \psi(s) \right\|^2_{\mathbb{L}_2^2} ds. \tag{2.4}$$

Lemma 2.2 ([6]) *If* $\mathscr{F} : \mathcal{J} \to \mathbb{L}_2^2(\mathrm{E}_2, \mathscr{X})$ *satisfies* $\int_0^{\mathfrak{b}} \|\mathscr{F}(s)\|^2_{\mathbb{L}_2^2} ds < \infty$, *then* (2.4) *is a well-defined* $\mathscr{X}$*-valued random variable, and*

$$\mathbb{E}\left\| \int_0^{\mathfrak{t}} \mathscr{F}(s)\, d\mathscr{Z}_{\mathscr{H}}(s) \right\|^2 \leq 2\mathscr{H} \mathfrak{t}^{2\mathscr{H}-1} \int_0^{\mathfrak{t}} \left\| \mathscr{F}(s) \right\|^2_{\mathbb{L}_2^2} ds.$$

Remark 2.1 The Rosenblatt process is a generalized one. The Rosenblatt process is important, because it is a "Hermite process", which is the limit of normalized sums of long-range dependent random variables. However, a number of integral representations clarify the nature of the Rosenblatt process and will be described further. The Rosenblatt processes include

(i) The time;
(ii) Spectral;
(iii) Finite time interval, to construct stochastic integrals in which Rosenblatt processes appear as the integrators by means of Malliavin calculus.

This is also a wavelet representation due to Vladas Pipiras, which extends the wavelet representation of fractional Brownian motion to the Rosenblatt process. The Rosenblatt process admits a version with Hölder-continuous sample paths up to order $\frac{1}{2} < \mathrm{H} < 1$. However, unlike the family of fractional Brownian motions, the family of Rosenblatt processes is not Gaussian. Furthermore, by appealing to a relationship between forward integrals, Skorokhod integrals, and Itô-type formula for functionals of a Rosenblatt process is given under some general applications. A detailed history, construction, and many properties of Rosenblatt processes are given in [7].

Lemma 2.3 ([6]) *For any $\mathfrak{p} \geq 1$ and arbitrary $\mathbb{L}_2^1(K,\mathscr{H})$-valued predictable process $G(\cdot)$,*

$$\sup_{s\in[0,\mathfrak{t}]} \mathbb{E}\left\| \int_0^s G(v)\,d\omega(v)\right\|^{2\mathfrak{p}} \leq \left(\mathfrak{p}(2\mathfrak{p}-1)\right)^{\mathfrak{p}} \left(\int_0^{\mathfrak{t}} \left(\mathbb{E}\|G(s)\|_{\mathbb{L}_2^1}^{2\mathfrak{p}}\right)^{1/\mathfrak{p}} ds\right)^{\mathfrak{p}}, \quad \mathfrak{t}\geq 0.$$

In particular, for $\mathfrak{p} = 1$, we have $\sup_{s\in[0,\mathfrak{t}]} \mathbb{E}\| \int_0^s G(v)\,d\omega(v)\|^2 \leq \int_0^{\mathfrak{t}} \mathbb{E}\|G(s)\|_{\mathbb{L}_2^1}^2\,ds$.

Lemma 2.4 (Bochner's theorem [37]) *A measurable function $\mathcal{V}:\mathcal{J}\to\mathscr{X}$ is Bochner integrable if $\|\mathcal{V}\|$ is Lebesgue integrable.*

2.2 Integrodifferential resolvent operator

In this part, we recall some basic results about the resolvent operators for the following integrodifferential equation:

$$\begin{cases} \vartheta'(\mathfrak{t}) = \mathscr{A}\vartheta(\mathfrak{t}) + \int_0^{\mathfrak{t}} \mathsf{K}(\mathfrak{t}-s)\vartheta(s)\,ds & \text{for } \mathfrak{t}\geq 0, \\ \vartheta(0) = \vartheta_0 \in \mathbb{Y}, \end{cases} \tag{2.5}$$

where $\mathscr{A}$ and $\mathsf{K}(\mathfrak{t})$ are closed linear operators on a Banach space $\mathbb{Y}$.

Let $\mathbb{X}_1$ and $\mathbb{Y}$ be two Banach spaces. By $\mathcal{L}(\mathbb{X}_1,\mathbb{Y})$ we denote the space of all bounded linear operators from $\mathbb{X}_1$ to $\mathbb{Y}$. To simplify, we write $\mathcal{L}(\mathbb{X}_1)$ when $\mathbb{X}_1 = \mathbb{Y}$. Let $\mathbb{X}_1$ be the Banach space $\mathscr{D}(\mathscr{A})$ equipped with the graph norm given by $\|\vartheta\|_{\mathbb{X}_1} = \|\mathscr{A}\vartheta\| + \|\vartheta\|$ for $\vartheta\in\mathbb{X}_1$. The notation $\mathcal{C}(\mathbb{R}^+,\mathbb{X}_1)$ stands for the space of all continuous functions from $\mathbb{R}^+$ into $\mathbb{X}_1$.

Definition 2.2 ([38]) A bounded linear operator-valued function $\mathscr{R}(\mathfrak{t})\in\mathcal{L}(\mathbb{Y})$, $\mathfrak{t}\geq 0$, is called the resolvent operator for system (2.5) if it satisfies the following conditions:

(i) $\mathscr{R}(0) = I$, and $\|\mathscr{R}(\mathfrak{t})\|_{\mathcal{L}(\mathbb{Y})} \leq Me^{\gamma\mathfrak{t}}$ for some constants M and γ;

(ii) For all $\vartheta\in\mathbb{Y}$, $\mathscr{R}(t)$ is strongly continuous for $\mathfrak{t}\geq 0$;

(iii) For $\vartheta\in\mathbb{X}_1$, $\mathscr{R}(\cdot)\vartheta\in\mathcal{C}^1(\mathbb{R}_+,\mathbb{Y})\cap\mathcal{C}(\mathbb{R}_+,\mathbb{X}_1)$, and

$$\begin{aligned} \mathscr{R}'(\mathfrak{t})\vartheta &= \mathscr{A}\mathscr{R}(\mathfrak{t})\vartheta + \int_0^{\mathfrak{t}} \mathsf{K}(\mathfrak{t}-s)\mathscr{R}(s)\vartheta\,ds \\ &= \mathscr{R}(\mathfrak{t})\mathscr{A}\vartheta + \int_0^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\mathsf{K}(s)\vartheta\,ds, \quad \mathfrak{t}\geq 0. \end{aligned}$$

In what follows, we suppose the following assumptions.

(R1) $\mathscr{A}$ is the infinitesimal generator of a C_0-semigroup $\{\mathfrak{S}(\mathfrak{t})\}_{\mathfrak{t}\geq 0}$.

(R2) For all $\mathfrak{t}\geq 0$, $\mathsf{K}(\mathfrak{t})$ is a continuous linear operator from $(\mathbb{X}_1,\|\cdot\|_{\mathbb{X}_1})$ into $(\mathbb{Y},\|\cdot\|_{\mathbb{Y}})$. Moreover, there exists an integrable function $\mu:\mathbb{R}_+\to\mathbb{R}_+$ such that for any $y\in\mathbb{X}_1$, $\mathfrak{t}\mapsto\mathsf{K}(t)y$ belongs to $W^{1,1}(\mathbb{R}_+,\mathbb{Y})$, and

$$\left[\left\|\frac{d}{d\mathfrak{t}}\mathsf{K}(\mathfrak{t})y\right\|_{\mathbb{Y}} \leq \mu(\mathfrak{t})\|y\|_{\mathbb{X}_1} \text{ for } y\in\mathbb{X}_1 \text{ and } t\geq 0.\right]$$

Theorem 2.1 ([38]) *If hypotheses (R1) and (R2) are fulfilled, then equation (2.5) has a unique resolvent operator $(\mathscr{R}(\mathfrak{t}))_{\mathfrak{t}\geq 0}$.*

Remark 2.2 From this definition we deduce that $\vartheta(\mathrm{t}) \in \mathscr{D}(\mathscr{A})$ and the function $s \mapsto \mathrm{K}(\mathrm{t}-s)\vartheta(s)$ is integrable for all $\mathrm{t} > 0$ and $s \geq 0$.

Lemma 2.5 ([38]) *Let assumptions* ($R1$) *and* ($R2$) *be satisfied. The resolvent operator* $(\mathscr{R}(\mathrm{t}))_{\mathrm{t}\geq 0}$ *is compact for* $\mathrm{t} > 0$ *if and only if the* C_0*-semigroup* $(\mathfrak{S}(\mathrm{t}))_{\mathrm{t}\geq 0}$ *is compact for* $\mathrm{t} > 0$.

2.3 Mild solution

Definition 2.3 An $\mathcal{F}_{\mathrm{t}}$-adapted stochastic process $\vartheta : \mathcal{J} \to \mathscr{X}$ is said to be a mild solution of system (1.1) if for every $\mathrm{t} \in \mathcal{J}$, $\vartheta(\mathrm{t})$ satisfies the following conditions:

(i) $\vartheta(0) = \varphi$, $\vartheta(\mathrm{t}) = \mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta(\mathrm{t}_{\mathrm{k}}^{-}))$, $\mathrm{t} \in (\mathrm{t}_{\mathrm{k}}, s_{\mathrm{k}}]$, $\mathrm{k} = 1, 2, \ldots, m$,

(ii) the consecutive equations: for all $\mathrm{t} \in [0, \mathrm{t}_1]$,

$$\begin{aligned}\vartheta(t) = {} & \mathscr{R}(\mathrm{t})\big[\varphi + \zeta\big(0, \vartheta(0)\big)\big] - \zeta(\mathrm{t}, \vartheta_{\mathrm{t}}) + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\,ds \\ & + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s) \\ & + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s),\end{aligned}$$

and

(iii) for $\mathrm{t} \in [s_{\mathrm{k}}, \mathrm{t}_{\mathrm{k}+1}]$, we have

$$\begin{aligned}\vartheta(t) = {} & \mathscr{R}(\mathrm{t}-s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}}, \vartheta\big(\mathrm{t}^{-}\big)\big) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\big] - \zeta(\mathrm{t}, \vartheta_{\mathrm{t}}) + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\,ds \\ & + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s) + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s),\end{aligned}$$

3 Existence and uniqueness of mild solution

To establish the existence and uniqueness of mild solutions for the stochastic system (1.1), we make the following hypotheses:

(H1) The map $\zeta : \mathcal{J} \times \mathscr{B} \to \mathscr{X}$ being continuous, there exist $\mathscr{N}_\zeta, \mathscr{M}_\zeta > 0$ such that

$$\begin{aligned}&\mathbb{E}\big\|\zeta(\mathrm{t}, \vartheta)\big\|^2 \leq \mathscr{N}_\zeta \quad \forall \mathrm{t} \in \mathcal{J}, \vartheta \in \mathscr{B}, \\ &\mathbb{E}\big\|\zeta(\mathrm{t}, \vartheta_1) - \zeta(\mathrm{t}, \vartheta_2)\big\|^2 \leq \mathscr{M}_\zeta \mathbb{E}\|\vartheta_1 - \vartheta_2\|^2, \quad \mathrm{t} \in \mathcal{J}, \vartheta_1, \vartheta_2 \in \mathscr{B}.\end{aligned}$$

(H2) The map $\mathfrak{l} : \mathcal{J} \times \mathscr{B} \to \mathscr{X}$ being continuous, there exist $\mathscr{N}_{\mathfrak{l}}, \mathscr{M}_{\mathfrak{l}} > 0$ such that

$$\begin{aligned}&\mathbb{E}\big\|\mathfrak{l}(\mathrm{t}, \vartheta)\big\|^2 \leq \mathscr{N}_{\mathfrak{l}} \quad \forall \mathrm{t} \in \mathcal{J}, \vartheta \in \mathscr{B}, \\ &\mathbb{E}\big\|\mathfrak{l}(\mathrm{t}, \vartheta_1) - \mathfrak{l}(\mathrm{t}, \vartheta_2)\big\|^2 \leq \mathscr{M}_{\mathfrak{l}} \mathbb{E}\|\vartheta_1 - \vartheta_2\|^2, \quad \mathrm{t} \in \mathcal{J}, \vartheta_1, \vartheta_2 \in \mathscr{B}.\end{aligned}$$

(H3) The map $\mathfrak{g} : \mathcal{J} \times \mathscr{B} \to \mathbb{L}_2^1(\mathrm{E}_1, \mathscr{X})$ being continuous, there exist $\mathscr{N}_{\mathfrak{g}}, \mathscr{M}_{\mathfrak{g}} > 0$ such that

$$\begin{aligned}&\mathbb{E}\big\|\mathfrak{g}(\mathrm{t}, \vartheta)\big\|_{\mathbb{L}_2^1}^2 \leq \mathscr{N}_{\mathfrak{g}} \quad \forall \mathrm{t} \in \mathcal{J}, \vartheta \in \mathscr{B}, \\ &\mathbb{E}\big\|\mathfrak{g}(\mathrm{t}, \vartheta_1) - \mathfrak{g}(\mathrm{t}, \vartheta_2)\big\|_{\mathbb{L}_2^1}^2 \leq \mathscr{M}_{\mathfrak{g}} \mathbb{E}\|\vartheta_1 - \vartheta_2\|^2, \quad \mathrm{t} \in \mathcal{J}, \vartheta_1, \vartheta_2 \in \mathscr{B}.\end{aligned}$$

(H4) The map $\sigma : \mathcal{J} \to \mathbb{L}_2^2(\mathrm{E}_2, \mathscr{X})$ fulfills

$$\int_0^{\mathrm{t}} \|\sigma(s)\|_{\mathbb{L}_2^2}^2\, ds < \infty \quad \forall \mathrm{t} \in \mathcal{J}.$$

There is a constant $\mathscr{M}_\sigma > 0$ such that $\|\sigma(s)\|_{\mathbb{L}_2^2}^2 \le \mathscr{M}_\sigma$ in $\mathcal{J}$.

(H5) The maps $\mathscr{I}_{\mathrm{k}} : (\mathrm{t}_{\mathrm{k}}, s_{\mathrm{k}}] \times \mathscr{X} \to \mathscr{X}$, $\mathrm{k} = 1, 2, \ldots, m$, are continuous, and there exist $\mathscr{N}_{\mathscr{I}_{\mathrm{k}}}, \mathscr{M}_{\mathscr{I}_{\mathrm{k}}}$ such that

$$\mathbb{E}\big\| \mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta)\big\|^2 \le \mathscr{N}_{\mathscr{I}_{\mathrm{k}}} \quad \forall \mathrm{t} \in \mathcal{J}, \vartheta \in \mathscr{B},$$
$$\mathbb{E}\big\| \mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta_1) - \mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta_2)\big\|^2 \le \mathscr{M}_{\mathscr{I}_{\mathrm{k}}} \mathbb{E}\|\vartheta_1 - \vartheta_2\|^2, \quad \mathrm{t} \in \mathcal{J}, \vartheta_1, \vartheta_2 \in \mathscr{B}.$$

Theorem 3.1 *Assume that* (R1), (R2), *and* (H1)–(H5) *hold. Then the stochastic system* (1.1) *has a unique mild solution where*

$$\mathscr{C}_2 = \max_{1 \le \mathrm{k} \le m} \{\mathscr{C}_{2,0}, \mathscr{M}_{\mathscr{I}_{\mathrm{k}}}, \mathscr{C}_{2,\mathrm{k}}\} < 1$$

with

$$\mathscr{C}_{2,0} = 3\big[\mathscr{M}_\zeta + \mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{l}} \mathrm{t}_1^2 + \mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{g}} \mathrm{t}_1\big],$$
$$\mathscr{C}_{2,\mathrm{k}} = 4\big[2\mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathscr{I}_{\mathrm{k}}} + \mathscr{M}_\zeta\big(2\mathscr{M}_{\mathscr{R}}^2 + 1\big) + \mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{l}} \mathrm{t}_{\mathrm{k}+1}^2 + \mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{g}} \mathrm{t}_{\mathrm{k}+1}\big],$$

provided that $\mathscr{M}_{\mathscr{R}} = \sup_{\mathrm{t} \in [0,\mathfrak{b}]} \|\mathscr{R}(\mathrm{t})\|$.

Proof For $r > 0$, we define

$$\mathcal{B}_r = \big\{\vartheta \in P\mathbb{C}(\mathscr{X}) : \|\vartheta\|_{P\mathbb{C}}^2 \le r\big\}.$$

Obviously, $\mathcal{B}_r$ is a bounded and closed subset of $P\mathbb{C}(\mathscr{X})$. Define Ψ on $\mathcal{B}_r$ as follows:

$$(\Psi\vartheta)(\mathrm{t}) = \begin{cases} \mathscr{R}(\mathrm{t})[\varphi + \zeta(0, \vartheta(0))] - \zeta(\mathrm{t}, \vartheta_{\mathrm{t}}) + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\, ds \\ \quad + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s) \\ \quad + \int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s), \quad \mathrm{t} \in [0, \mathrm{t}_1], \\ \mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta(\mathrm{t}_{\mathrm{k}}^-)), \quad \mathrm{t} \in (\mathrm{t}_{\mathrm{k}}, s_{\mathrm{k}}], s \ge 1, \\ \mathscr{R}(\mathrm{t} - s_{\mathrm{k}})[\mathscr{I}_{\mathrm{k}}(s_{\mathrm{k}}, \vartheta(\mathrm{t}^-)) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})] - \zeta(\mathrm{t}, \vartheta_{\mathrm{t}}) \\ \quad + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\, ds + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s) \\ \quad + \int_{s_{\mathrm{k}}}^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s), t \in (s_{\mathrm{k}}, \mathrm{t}_{\mathrm{k}+1}], \quad \mathrm{k} \ge 0. \end{cases}$$

Step 1: To demonstrate that Ψ is well defined, for $\vartheta \in \mathcal{B}_r$ and $\mathrm{t} \in [0, \mathrm{t}_1]$, by Hölder's inequality and (H1)–(H4) we have

$$\mathbb{E}\big\|(\Psi\vartheta)(\mathrm{t})\big\|^2 \le 5\mathbb{E}\big\|\mathscr{R}(\mathrm{t})\big[\varphi + \zeta\big(0, \vartheta(0)\big)\big]\big\|^2$$
$$+ 5\mathbb{E}\big\|\zeta(\mathrm{t}, \vartheta_{\mathrm{t}})\big\|^2 + 5\mathbb{E}\bigg\|\int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\, ds\bigg\|^2$$

$$
\begin{aligned}
&+5\mathbb{E}\left\|\int_0^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\mathfrak{g}(s,\vartheta_s)\,d\omega(s)\right\|^2+5\mathbb{E}\left\|\int_0^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right\|^2\\
&\le 5\left[2\mathscr{M}_{\mathscr{R}}^2\mathbb{E}\|\varphi\|^2+2\mathscr{M}_{\mathscr{R}}^2\mathscr{N}_\zeta\right]+5\mathscr{N}_\zeta+5\mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_1^2\mathscr{N}_{\mathfrak{l}}+5\mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_1^2\mathscr{N}_{\mathfrak{g}}\\
&+5\left(2\mathscr{H}\mathfrak{t}_1^{2\mathscr{H}-1}\right)\mathfrak{t}_1\mathscr{M}_\sigma\\
&:=\mathscr{C}_1.
\end{aligned}
$$

Thus

$$
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2\le\mathscr{C}_1. \tag{3.1}
$$

For $\mathfrak{t}\in(\mathfrak{t}_{\mathrm{k}},s_{\mathrm{k}}]$, $\mathrm{k}=1,2,\dots,m$, we have

$$
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2=\mathbb{E}\left\|\mathscr{I}_{\mathrm{k}}\left(\mathfrak{t},\vartheta\left(\mathfrak{t}_{\mathrm{k}}^-\right)\right)\right\|^2\le\mathscr{N}_{\mathscr{I}_{\mathrm{k}}}. \tag{3.2}
$$

Similarly, for $\mathfrak{t}\in(s_{\mathrm{k}},\mathfrak{t}_{\mathrm{k}+1}]$, $\mathrm{k}=1,2,\dots,m$, by Holder's inequality and (H1)–(H5) we obtain

$$
\begin{aligned}
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2&\le 5\mathbb{E}\left\|\mathscr{R}(\mathfrak{t}-s_{\mathrm{k}})\left[\mathscr{I}_{\mathrm{k}}\left(s_{\mathrm{k}},\vartheta\left(\mathfrak{t}^-\right)\right)\right\|^2+5\mathbb{E}\left\|\zeta(s_{\mathrm{k}},\vartheta_{s_{\mathrm{k}}})\right]\right\|^2+5\mathbb{E}\left\|\zeta(\mathfrak{t},\vartheta_{\mathfrak{t}})\right\|^2\\
&+5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\mathfrak{l}(s,\vartheta_s)\,ds\right\|^2+5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\mathfrak{g}(s,\vartheta_s)\,d\omega(s)\right\|^2\\
&+5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right\|^2\\
&\le 10\mathscr{M}_{\mathscr{R}}^2\mathscr{N}_{\mathscr{I}_{\mathrm{k}}}+10\mathscr{M}_{\mathscr{R}}^2\mathscr{N}_\zeta+5\mathscr{N}_\zeta+5\mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_{\mathrm{k}+1}^2\mathscr{N}_{\mathfrak{l}}\\
&+5\mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_{\mathrm{k}+1}\mathscr{N}_{\mathfrak{g}}+10\mathscr{H}\mathfrak{t}_{\mathrm{k}+1}^{2\mathscr{H}}\mathscr{M}_\sigma\\
&:=\mathscr{C}_{1,\mathrm{k}}.
\end{aligned}
$$

Therefore

$$
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2\le\mathscr{C}_{1,\mathrm{k}}. \tag{3.3}
$$

Using (3.1)–(3.3), for all $\mathfrak{t}\in[0,\mathfrak{b}]$, we have

$$
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2\le\max_{1\le\mathrm{k}\le m}\{\mathscr{C}_1,\mathscr{C}_{\mathscr{I}_{\mathrm{k}}},\mathscr{C}_{1,\mathrm{k}}\}\le r.
$$

By taking the supremum over $\mathfrak{t}$ we get

$$
\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|_{P\mathbb{C}}^2\le r.
$$

Thus $\Psi:\mathcal{B}_r\to\mathcal{B}_r$.

Step 2: Let us show that Ψ is a contraction mapping on $\mathcal{B}_r$. For $\vartheta_1,\vartheta_2\in\mathcal{B}_r$ and $\mathfrak{t}\in[0,\mathfrak{t}_1]$, by Hölder's inequality and (H1)–(H4) we have

$$
\mathbb{E}\left\|(\Psi\vartheta_1)(\mathfrak{t})-(\Psi\vartheta_2)(\mathfrak{t})\right\|^2\le 3\mathbb{E}\left\|\zeta(\mathfrak{t},\vartheta_{1_s})-\zeta(\mathfrak{t},\vartheta_{2_s})\right\|^2
$$

$$
\begin{aligned}
&+ 3\mathbb{E}\left\| \int_0^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\big[\mathfrak{l}(s,\vartheta_{1_s}) - \mathfrak{l}(s,\vartheta_{2_s})\big]\,ds \right\|^2 \\
&+ 3\mathbb{E}\left\| \int_0^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\big[\mathfrak{g}(s,\vartheta_{1_s}) - \mathfrak{g}(s,\vartheta_{2_s})\big]\,d\omega(s) \right\|^2 \\
\le{}& 3\mathscr{M}_\zeta \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 + 3\mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{l}} \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 \mathfrak{t}_1^2 \\
&+ 3\mathscr{M}_{\mathscr{R}}^2 \mathscr{M}_{\mathfrak{g}} \mathfrak{t}_1 \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 \\
\le{}& \mathscr{C}_{2,0} \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2,
\end{aligned}
\tag{3.4}
$$

where, $\mathscr{C}_{2,0} = 3\mathscr{M}_\zeta + 3\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{l}}\mathfrak{t}_1^2 + 3\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{g}}\mathfrak{t}_1$.

For $\mathfrak{t} \in (\mathfrak{t}_{\mathrm{k}}, s_{\mathrm{k}}]$, $\mathrm{k} = 1, 2, \ldots, m$, by (H5) we obtain

$$
\mathbb{E}\big\|(\Psi\vartheta_1)(\mathfrak{t}) - (\Psi\vartheta_2)(\mathfrak{t})\big\|^2 \le \mathscr{M}_{\mathscr{I}_{\mathrm{k}}} \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2. \tag{3.5}
$$

Similarly, for $\mathfrak{t} \in (s_{\mathrm{k}}, \mathfrak{t}_{\mathrm{k}+1}]$, by Hölder's inequality and (H1)–(H5) we have $\mathbb{E}\|(\Psi\vartheta_1)(\mathfrak{t}) - (\Psi\vartheta_2)(\mathfrak{t})\|^2$

$$
\begin{aligned}
&\le 8\mathbb{E}\big\|\mathscr{R}(\mathfrak{t}-s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}},\vartheta_1(\mathfrak{t}^-)\big) - \mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}},\vartheta_2(\mathfrak{t}^-)\big)\big]\big\|^2 \\
&+ 8\mathbb{E}\big\|\mathscr{R}(\mathfrak{t}-s_{\mathrm{k}})\big[\zeta\big(s_{\mathrm{k}},(\vartheta_1)_{s_{\mathrm{k}}}\big) - \zeta\big(s_{\mathrm{k}},(\vartheta_2)_{s_{\mathrm{k}}}\big)\big]\big\|^2 \\
&\quad + 4\mathbb{E}\big\|\zeta(\mathfrak{t},\vartheta_{1_{\mathfrak{t}}}) - \zeta(\mathfrak{t},\vartheta_{2_{\mathfrak{t}}})\big\|^2 + 4\mathbb{E}\left\| \int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\big[\mathfrak{l}\big(s,(\vartheta_1)_s\big) - \mathfrak{l}\big(s,(\vartheta_2)_s\big)\big]\,ds \right\|^2 \\
&\quad + 4\mathbb{E}\left\| \int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\big[\mathfrak{g}\big(s,(\vartheta_1)_s\big) - \mathfrak{g}\big(s,(\vartheta_2)_s\big)\big]\,d\omega(s) \right\|^2 \\
&\le 8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathscr{I}_{\mathrm{k}}}\|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 + 8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_\zeta\|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 + 4\mathscr{M}_\zeta\|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 \\
&\quad + 4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{l}}\mathfrak{t}_{\mathrm{k}+1}^2\|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2 + 4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{g}}\mathfrak{t}_{\mathrm{k}+1}\|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2.
\end{aligned}
$$

Thus

$$
\mathbb{E}\big\|(\Psi\vartheta_1)(\mathfrak{t}) - (\Psi\vartheta_2)(\mathfrak{t})\big\|^2 \le \mathscr{C}_{2,\mathrm{k}} \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2, \tag{3.6}
$$

where

$$
\mathscr{C}_{2,\mathrm{k}} = 8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathscr{I}_{\mathrm{k}}} + 8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_\zeta + 4\mathscr{M}_\zeta + 4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{l}}\mathfrak{t}_{\mathrm{k}+1}^2 + 4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{g}}\mathfrak{t}_{\mathrm{k}+1}.
$$

Gathering (3.4)–(3.6), for all $\mathfrak{t} \in [0, \mathfrak{b}]$, we have

$$
\mathbb{E}\big\|(\Psi\vartheta_1)(\mathfrak{t}) - (\Psi\vartheta_2)(\mathfrak{t})\big\|^2 \le \mathscr{C} \|\vartheta_1 - \vartheta_2\|_{P\mathbb{C}}^2,
$$

where $\mathscr{C}_2 = \max_{1\le \mathrm{k} \le m}\{\mathscr{C}_{2,0}, \mathscr{M}_{\mathscr{I}_{\mathrm{k}}}, \mathscr{C}_{2,\mathrm{k}}\}$ Thus ψ is a contraction mapping on $\mathcal{B}_r$. By the Banach fixed theorem the stochastic integrodifferential system (1.1) has a unique mild solution on $\mathcal{J}$. □

4 Stability analysis of mild solution

In this section, we investigate the exponential stability of stochastic integrodifferential evolution system. We make the following assumptions.

(H6) A resolvent operator $(\mathscr{R}(\mathrm{t}))_{\mathrm{t}\geq 0}$ is exponentially stable, that is, there exist $\beta, \mathscr{M}_{\mathscr{R}} > 0$ such that

$$\|\mathscr{R}(\mathrm{t})\| \leq \mathscr{M}_{\mathscr{R}} e^{-\beta \mathrm{t}}, \quad \mathrm{t} \geq 0.$$

(H7) The map $\sigma : \mathcal{J} \to \mathbb{L}_2^2(\mathrm{E}_2, \mathscr{X})$ satisfies

$$\int_0^{\mathrm{t}} e^{\beta s} \|\sigma(s)\|_{\mathbb{L}_2^2}^2 \, ds < \infty, \quad \mathrm{t} \in \mathcal{J}.$$

(H8) There exist continuous functions $\aleph_i : \mathbb{R}^+ \to \mathbb{R}^+$, $\mathrm{k} = 1, 2, \dots, m$, and nonnegative real numbers $\nu_1, \nu_3 \geq 0$ such that
 (i) $\mathbb{E}\|\zeta(\mathrm{t}, \vartheta)\|^2 \leq \aleph_1(\mathrm{t})$,
 (ii) $\mathbb{E}\|\mathfrak{l}(\mathrm{t}, \vartheta)\|^2 \leq \nu_1 \mathbb{E}\|\vartheta\|^2 + \aleph_{2_{\mathrm{t}}}$,
 (iii) $\mathbb{E}\|\mathfrak{g}(t, \vartheta)\|^2 \leq \nu_3 \mathbb{E}\|\vartheta\|^2 + \aleph_{3_{\mathrm{t}}}$,
 (iv) $\mathbb{E}\|\mathscr{I}_{\mathrm{k}}(\mathrm{t}, \vartheta)\|^2 \leq \aleph_{4,\mathrm{k}}(\mathrm{t}), \mathrm{k} = 1, 2, \dots, m$.
 Moreover, there exist $\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_{4,\mathrm{k}} \geq 0, \mathrm{k} = 1, 2, \dots, m$, and $\overline{\beta} > \beta > 0$ such that

$$\aleph_i(\mathrm{t}) \leq \mathfrak{a}_i e^{-\overline{\beta}\mathrm{t}}, \quad i = 1, 2, 3, \quad \text{and} \quad \aleph_{4,\mathrm{k}}(\mathrm{t}) \leq \mathfrak{a}_{4,\mathrm{k}} e^{-\overline{\beta}\mathrm{t}}, \quad \mathrm{k} = 1, 2, \dots, m.$$

Theorem 4.1 *Assume that* $(R1)$, $(R2)$, *and* $(H1)$–$(H8)$ *hold and*

$$5\mathscr{M}_{\mathscr{R}}^2 \nu_1 + 5\mathscr{M}_{\mathscr{R}}^2 \nu_3 < \beta.$$

Then the unique mild solution of stochastic system (1.1) *exponentially decays to zero in mean square, that is, there exist* $\mathscr{M}_s, \Lambda > 0$ *such that*

$$\mathbb{E}\|\vartheta(\mathrm{t})\|^2 \leq \mathscr{M}_s e^{-\Lambda \mathrm{t}}, \quad \mathrm{t} \geq 0,$$

where $\mathscr{M}_s = \max\{\mathscr{M}_{s1}, \mathscr{M}_{s2}, \mathscr{M}_{s3}\}$ *and* $\Lambda = \min\{\Lambda_1, \Lambda_2, \Lambda_3\}$ *with*

$$\mathscr{M}_{s1} = \mathfrak{q}_1 + \mathfrak{q}_2 + \mathfrak{q}_5 + 5\mathscr{M}_{\mathscr{R}}^2 \frac{\mathfrak{a}_2 + \mathfrak{a}_3}{2\beta - \overline{\beta}},$$

$$\mathscr{M}_{s2} = \mathfrak{q}_1^1 + \mathfrak{q}_2^1 + \mathfrak{q}_5^1 + 5\mathscr{M}_{\mathscr{R}}^2 \frac{\mathfrak{a}_2 + \mathfrak{a}_3}{2\beta - \overline{\beta}},$$

$$\mathscr{M}_{s3} = \mathfrak{a}_{4,\mathrm{k}},$$

$\Lambda_1 = 2\beta - \mu_1, \Lambda_2 = 2\beta - \mu_2, \Lambda_3 = 2\beta.$

Proof For $\mathrm{t} \in [0, \mathrm{t}_1]$, we have

$$\begin{aligned}\mathbb{E}\|\vartheta(\mathrm{t})\|^2 &\leq 5\mathbb{E}\|\mathscr{R}(\mathrm{t})[\varphi + \zeta(0, \vartheta(0))]\|^2 + 5\mathbb{E}\|\zeta(\mathrm{t}, \vartheta_{\mathrm{t}})\|^2 + 5\mathbb{E}\left\|\int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{l}(s, \vartheta_s)\, ds\right\|^2 \\ &\quad + 5\mathbb{E}\left\|\int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s)\right\|^2 + 5\mathbb{E}\left\|\int_0^{\mathrm{t}} \mathscr{R}(\mathrm{t}-s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s)\right\|^2 \\ &:= \sum_{i=1}^5 \mathscr{P}_i. \end{aligned} \tag{4.1}$$

By (H6) and (H8) we have

$$\begin{aligned}\mathscr{P}_1 &= 5\mathbb{E}\big\|\mathscr{R}(\mathfrak{t})\big[\varphi + \zeta\big(0, \vartheta(0)\big)\big]\big\|^2\\ &\leq \big[10\mathscr{M}_{\mathscr{R}}^2\mathbb{E}\|\varphi\|^2 + 10\mathscr{M}_{\mathscr{R}}^2\aleph_1\big]e^{-2\beta\mathfrak{t}}\\ &:= \mathfrak{q}_1 e^{-2\beta\mathfrak{t}}, \end{aligned}\tag{4.2}$$

$$\begin{aligned}\mathscr{P}_2 &= 5\mathbb{E}\big\|\zeta(\mathfrak{t}, \vartheta_{\mathfrak{t}})\big\|^2\\ &\leq 5\aleph_1(\mathfrak{t}) \leq 5\mathfrak{a}_1 e^{-\overline{\beta}\mathfrak{t}} \leq 5\mathfrak{a}_1 e^{-2\beta\mathfrak{t}}\\ &:= \mathfrak{q}_2 e^{-2\beta\mathfrak{t}}.\end{aligned}\tag{4.3}$$

By Hölder's inequality and (H6) we have

$$\begin{aligned}\mathscr{P}_3 &= 5\mathbb{E}\bigg\|\int_0^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\mathfrak{l}(s, \vartheta_s)\,ds\bigg\|^2\\ &\leq 5\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\big\|\mathfrak{l}(s, \vartheta_s)\big\|^2\,ds.\end{aligned}\tag{4.4}$$

By (H6) and (H7) we have

$$\begin{aligned}\mathscr{P}_4 &= 5\mathbb{E}\bigg\|\int_0^{\mathfrak{t}}\mathscr{R}(\mathfrak{t}-s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s)\bigg\|^2\\ &\leq 5\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\big\|\mathfrak{g}(s, \vartheta_s)\big\|_{\mathbb{L}_2^2}^2,\\ \mathscr{P}_5 &\leq 5\big(2\mathscr{H}\mathfrak{t}_1^{2\mathscr{H}-1}\big)\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\big\|\sigma(s)\big\|_{\mathbb{L}_2^2}^2\,ds.\end{aligned}$$

Let $\mathfrak{q}_5$ be a constant such that $5(2\mathscr{H}\mathfrak{t}_1^{2\mathscr{H}-1})\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\|\sigma(s)\|_{\mathbb{L}_2^2}^2\,ds \leq \mathfrak{q}_5$ for $\mathfrak{t} \geq 0$. Consequently,

$$\mathscr{P}_5 \leq \mathfrak{q}_5 e^{-2\beta\mathfrak{t}}.\tag{4.5}$$

By (4.2)–(4.5),

$$\begin{aligned}&e^{2\beta\mathfrak{t}}\mathbb{E}\big\|\vartheta(\mathfrak{t})\big\|^2\\ &\quad\leq \mathfrak{q}_1 + \mathfrak{q}_2 + \mathfrak{q}_5 + 5\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\big\|\mathfrak{l}(s, \vartheta_s)\big\|^2\,ds + 5\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\big\|\mathfrak{g}(s, \vartheta_s)\big\|_{\mathbb{L}_2^2}^2\,ds\\ &\quad\leq \mathfrak{q}_1 + \mathfrak{q}_2 + \mathfrak{q}_5 + 5\mathscr{M}_{\mathscr{R}}^2(\nu_1 + \nu_3)\int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2\,ds + 5\mathscr{M}_{\mathscr{R}}^2\int_0^{\mathfrak{t}} e^{2\beta s}(\aleph_{2_s} + \aleph_{3_s})\,ds\\ &\quad\leq \mathfrak{q}_1 + \mathfrak{q}_2 + \mathfrak{q}_5 + 5\mathscr{M}_{\mathscr{R}}^2(\nu_1 + \nu_3)\int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2\,ds + 5\mathscr{M}_{\mathscr{R}}^2\frac{\mathfrak{a}_2 + \mathfrak{a}_3}{2\beta - \overline{\beta}}.\end{aligned}$$

Let $\mathscr{M}_{s1} = \mathfrak{q}_1 + \mathfrak{q}_2 + \mathfrak{q}_5 + 5\mathscr{M}_{\mathscr{R}}^2\frac{\mathfrak{a}_2+\mathfrak{a}_3}{2\beta-\overline{\beta}}$ and $\mu_1 = 5\mathscr{M}_{\mathscr{R}}^2(\nu_1 + \nu_3)$.

Then we have

$$e^{2\beta\mathfrak{t}}\mathbb{E}\big\|\vartheta(\mathfrak{t})\big\|^2 \leq \mathscr{M}_{s1} + \mu_1\int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2\,ds.$$

Using Gronwall's inequality, we get

$$\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \leq \mathscr{M}_{s1}e^{-\Lambda_1 \mathfrak{t}}, \tag{4.6}$$

where $\Lambda_1 = 2\beta - \mu_1$.

For $\mathfrak{t} \in (s_{\mathrm{k}}, \mathfrak{t}_{\mathrm{k}+1}]$, $\mathrm{k} = 1, 2, \ldots, m$, we obtain

$$\begin{aligned}\mathbb{E}\left\|(\Psi\vartheta)(\mathfrak{t})\right\|^2 &\leq 5\mathbb{E}\left\|\mathscr{R}(\mathfrak{t}-s_{\mathrm{k}})\left[\mathscr{I}_{\mathrm{k}}\left(s_{\mathrm{k}}, \vartheta\left(\mathfrak{t}^-\right)\right)\right\|^2 + 5\mathbb{E}\left\|\zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\right]\right\|^2 + 5\mathbb{E}\left\|\zeta(\mathfrak{t}, \vartheta_{\mathfrak{t}})\right\|^2 \\ &\quad + 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\mathfrak{l}(s, \vartheta_s)\,ds\right\|^2 + 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s)\right\|^2 \\ &\quad + 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right\|^2 \\ &:= \sum_{i=1}^{5} \mathscr{P}_i^1. \end{aligned} \tag{4.7}$$

By (H6)–(H8) and Hölder's inequality,

$$\begin{aligned}\mathscr{P}_1^1 &= 5\mathbb{E}\left\|\mathscr{R}(\mathfrak{t}-s_{\mathrm{k}})\left[\mathscr{I}_{\mathrm{k}}\left(s_{\mathrm{k}}, \vartheta\left(\mathfrak{t}^-\right)\right)\right\|^2 \\ &\leq 10\mathscr{M}_{\mathscr{R}}^2\left[(\aleph_{4,\mathrm{k}})_{\mathfrak{t}} + (\aleph_1)_{\mathfrak{t}}\right] \\ &:= \mathfrak{q}_1^1 e^{-2\beta\mathfrak{t}}, \end{aligned} \tag{4.8}$$

$$\begin{aligned}\mathscr{P}_2^1 &= 5\mathbb{E}\left\|\zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\right]\right\|^2 \\ &\leq 5\aleph_1(\mathfrak{t}) \leq 5\mathfrak{a}_1 e^{-\overline{\beta}\mathfrak{t}} \leq 5\mathfrak{a}_1 e^{-2\beta\mathfrak{t}} \\ &:= \mathfrak{q}_2^1 e^{-2\beta\mathfrak{t}}, \end{aligned} \tag{4.9}$$

$$\begin{aligned}\mathscr{P}_3^1 &= 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\mathfrak{l}(s, \vartheta_s)\,ds\right\|^2 \\ &\leq 5\mathscr{M}_{\mathscr{R}}^2 \int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\left\|\mathfrak{l}(s, \vartheta_s)\right\|^2 ds, \end{aligned} \tag{4.10}$$

$$\begin{aligned}\mathscr{P}_4^1 &= 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s)\right\|^2 \\ &\leq 5\mathscr{M}_{\mathscr{R}}^2 \int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\left\|\mathfrak{g}(s, \vartheta_s)\right\|_{\mathbb{L}_2^1}^2 ds \end{aligned} \tag{4.11}$$

$$\begin{aligned}\mathscr{P}_5^1 &= 5\mathbb{E}\left\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}} \mathscr{R}(\mathfrak{t}-s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right\|^2 \\ &\leq 5\mathscr{M}_{\mathscr{R}}^2\left(2\mathscr{H}\mathfrak{t}_1^{2\mathscr{H}-1}\right) \int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\left\|\sigma(s)\right\|_{\mathbb{L}_2^2}^2 ds. \end{aligned} \tag{4.12}$$

Let $\mathfrak{q}_5^1 > 0$ be a constant such that

$$\begin{aligned} &5\mathscr{M}_{\mathscr{R}}^2\left(2\mathscr{H}\mathfrak{t}_1^{2\mathscr{H}-1}\right) \int_0^{\mathfrak{t}} e^{-2\beta(\mathfrak{t}-s)}\mathbb{E}\left\|\sigma(s)\right\|_{\mathbb{L}_2^2}^2 ds \leq \mathfrak{q}_5^1, \quad \mathfrak{t} \geq 0, \\ &\mathscr{P}_5^1 \leq \mathfrak{q}_5^1 e^{-2\beta\mathfrak{t}}. \end{aligned} \tag{4.13}$$

Using (4.8)–(4.13), we obtain

$$\begin{aligned}
&e^{2\beta \mathfrak{t}}\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \\
&\quad \le \mathfrak{q}_1^1 + \mathfrak{q}_2^1 + \mathfrak{q}_5^1 + 5\mathscr{M}_{\mathscr{R}}^2 \int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\left\|\mathfrak{l}(s,\vartheta_s)\right\|^2 ds + 5\mathscr{M}_{\mathscr{R}}^2 \int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\left\|\mathfrak{g}(s,\vartheta_s)\right\|_{\mathbb{L}_2^1}^2 ds \\
&\quad \le \mathfrak{q}_1^1 + \mathfrak{q}_2^1 + \mathfrak{q}_5^1 + 5\mathscr{M}_{\mathscr{R}}^2(\nu_1+\nu_3) \int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2 ds + 5\mathscr{M}_{\mathscr{R}}^2 \int_0^{\mathfrak{t}} e^{2\beta s}\big((\aleph_2)_s + (\aleph_3)_s\big) ds \\
&\quad \le \mathfrak{q}_1^1 + \mathfrak{q}_2^1 + \mathfrak{q}_5^1 + 5\mathscr{M}_{\mathscr{R}}^2 \frac{\mathfrak{a}_2+\mathfrak{a}_3}{2\beta-\overline{\beta}} + 5\mathscr{M}_{\mathscr{R}}^2(\nu_1+\nu_3) \int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2 ds.
\end{aligned}$$

Substituting $\mathscr{M}_{s2} = \mathfrak{q}_1^1 + \mathfrak{q}_2^1 + \mathfrak{q}_5^1 + 5\mathscr{M}_{\mathscr{R}}^2 \frac{\mathfrak{a}_2+\mathfrak{a}_3}{2\beta-\overline{\beta}}$ and $\mu_2 = 5\mathscr{M}_{\mathscr{R}}^2(\nu_1+\nu_3)$, we get

$$e^{2\beta \mathfrak{t}}\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \le \mathscr{M}_{s2} + \mu_2 \int_0^{\mathfrak{t}} e^{2\beta s}\mathbb{E}\|\vartheta_s\|^2 ds.$$

By Gronwall's inequality,

$$\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \le \mathscr{M}_{s2}e^{-\Lambda_2 \mathfrak{t}}. \tag{4.14}$$

Now, for $\mathfrak{t} \in (\mathfrak{t}_{\mathrm{k}}, s_{\mathrm{k}}]$, we get

$$\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \le \mathscr{M}_{s3}e^{-\Lambda \mathfrak{t}}, \tag{4.15}$$

where $\mathscr{M}_{s3} = \mathfrak{a}_{4,\mathrm{k}}$ and $\Lambda_3 = 2\beta$.

For $\mathfrak{t} \in [0, \mathfrak{b}]$, we have

$$\mathbb{E}\left\|\vartheta(\mathfrak{t})\right\|^2 \le \mathscr{M}_s e^{-\Lambda \mathfrak{t}}, \quad \mathfrak{t} \ge 0 \mathscr{M}_s, \Lambda > 0,$$

where $\mathscr{M}_s = \max\{\mathscr{M}_{s1}, \mathscr{M}_{s2}, \mathscr{M}_{s3}\}$ and $\Lambda = \min\{\Lambda_1, \Lambda_2, \Lambda_3\}$.

This completes the proof. □

5 Controllability results

This section is devoted to the study of a class of noninstantaneous impulsive stochastic differential equations driven by mixed Rosenblatt process of the form

$$\begin{aligned}
&d\big[\vartheta(\mathfrak{t}) + \zeta(\mathfrak{t},\vartheta_{\mathfrak{t}})\big] = \mathscr{A}\big[\vartheta(\mathfrak{t}) + \zeta(\mathfrak{t},\vartheta_{\mathfrak{t}})\big]d\mathfrak{t} + \int_0^{\mathfrak{t}} \Theta(\mathfrak{t}-s)\big[\vartheta(s) + \zeta(s,\vartheta_s)\,ds\big]d\mathfrak{t} + \mathcal{C}\mathfrak{u}(\mathfrak{t})\,d\mathfrak{t} \\
&\qquad + \mathfrak{l}(\mathfrak{t},\vartheta_{\mathfrak{t}})\,d\mathfrak{t} + \mathfrak{g}(\mathfrak{t},\vartheta_{\mathfrak{t}})\,d\omega(\mathfrak{t}) + \sigma(\mathfrak{t})\,d\mathscr{Z}_{\mathscr{H}}(\mathfrak{t}), \quad \mathfrak{t} \in \bigcup_{\mathrm{k}=0}^{m}(s_{\mathrm{k}}, \mathfrak{t}_{\mathrm{k}+1}], \qquad (5.1) \\
&\vartheta(\mathfrak{t}) = \mathscr{I}_{\mathrm{k}}\big(\mathfrak{t}, \vartheta\big(\mathfrak{t}_{\mathrm{k}}^-\big)\big), \quad \mathfrak{t} \in \bigcup_{\mathrm{k}=1}^{m}(\mathfrak{t}_{\mathrm{k}}, s_{\mathrm{k}}], \\
&\vartheta(0) = \varphi \in \mathscr{B},
\end{aligned}$$

where the control function $\mathfrak{u} \in \mathbb{L}^2(\mathcal{J}, \mathscr{A})$, the Hilbert space of all $\mathscr{A}$-valued $\mathcal{F}$-adapted measurable square-integrable process on $\mathcal{J}$, and $\mathcal{C}$ is a bounded linear operator from $\mathscr{A}$ into $\mathscr{X}$. We make the following assumptions.

(H9) The linear operator $\mathscr{Y}_{s_k}^{t_{k+1}} : \mathbb{L}^2((s_k, t_{k+1}], \mathscr{A}) \to \mathbb{L}^2(\Omega, \mathscr{X})$, $k = 0, 1, 2, \ldots, m$, defined by

$$\mathscr{Y}_{s_k}^{t_{k+1}} u = \int_{s_k}^{t_{k+1}} \mathscr{R}(t_{k+1} - s)\mathcal{C}u(s)\, ds$$

has bounded invertible operators $(\mathscr{Y}_{s_k}^{t_{k+1}})^{-1}$ taking values in $\mathbb{L}^2((s_k, t_{k+1}], \mathscr{A})/\ker(\mathscr{Y}_{s_k}^{t_{k+1}})$, and there exist $\mathscr{M}_{\mathcal{C}} > 0$ such that

$$\left\| \left(\mathscr{Y}_{s_k}^{t_{k+1}}\right)^{-1} \right\| \le \mathscr{M}_{\mathcal{C}}.$$

Definition 5.1 An $\mathcal{F}$-adapted stochastic process $\vartheta : \mathcal{J} \to \mathscr{X}$ is said to be a mild solution of the stochastic system (5.1) if for all $t \in \mathcal{J}$, $\vartheta(t)$ satisfies the following conditions:

(i) $\vartheta(0) = \varphi \in \mathscr{B}$;

(ii) $\vartheta(t) = \mathscr{I}_k(t, \vartheta(t_k^-))$, $t \in (t_k, s_k]$, $k = 1, 2, \ldots, m$;

(iii) the following integral equations are satisfied:

$$\begin{aligned}\vartheta(t) &= \mathscr{R}(t)\big[\varphi + \zeta\big(0, \vartheta(0)\big)\big] - \zeta(t, \vartheta_t) + \int_0^t \mathscr{R}(t - s)\mathfrak{l}(s, \vartheta_s)\, ds \\ &\quad + \int_0^t \mathscr{R}(t - s)\mathcal{C}u(s)\, ds + \int_0^t \mathscr{R}(t - s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s) \\ &\quad + \int_0^t \mathscr{R}(t - s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s), \quad t \in [0, t_1],\end{aligned}$$

and

$$\begin{aligned}\vartheta(t) &= \mathscr{R}(t - s_k)\big[\mathscr{I}_k\big(s_k, \vartheta\big(t^-\big)\big) + \zeta(s_k, \vartheta_{s_k})\big] - \zeta(t, \vartheta_t) + \int_{s_k}^t \mathscr{R}(t - s)\mathfrak{l}(s, \vartheta_s)\, ds \\ &\quad + \int_{s_k}^t \mathscr{R}(t - s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s) + \int_{s_k}^t \mathscr{R}(t - s)\mathcal{C}u(s)\, ds \\ &\quad + \int_{s_k}^t \mathscr{R}(t - s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s), \quad t \in [s_k, t_{k+1}].\end{aligned}$$

Definition 5.2 The stochastic control system (5.1) is said to be controllable on $\mathcal{J}$ if for all $\vartheta_0, \vartheta_1 \in \mathscr{X}$, there exists a suitable control $u \in \mathbb{L}^2(\mathcal{J}, \mathscr{A})$ such that the mild solution of the stochastic control system (5.1) with respect to u satisfies $\vartheta(0) = \vartheta_0$ and $\vartheta(\mathfrak{b}) = \vartheta_1$, where $\mathfrak{b}$ and ϑ_1 are preassigned time and terminal state, respectively.

We may choose the feedback control $u(t)$ as follows:

$$\begin{aligned}u(t) &= \left(\mathscr{Y}_{s_k}^{t_{k+1}}\right)^{-1}\Bigg[\vartheta_{t_{k+1}} - \mathscr{R}(t_{k+1} - s_k)\big[\mathscr{I}_k\big(s_k, \vartheta\big(t^-\big)\big) + \zeta(s_k, \vartheta_{s_k})\big] + \zeta(t_{k+1}, \vartheta_{t_{k+1}}) \\ &\quad - \int_{s_k}^{t_{k+1}} \mathscr{R}(t_{k+1} - s)\mathfrak{l}(s, \vartheta_s)\, ds - \int_{s_k}^{t_{k+1}} \mathscr{R}(t_{k+1} - s)\mathfrak{g}(s, \vartheta_s)\, d\omega(s) \\ &\quad - \int_{s_k}^{t_{k+1}} \mathscr{R}(t_{k+1} - s)\sigma(s)\, d\mathscr{Z}_{\mathscr{H}}(s)\Bigg], t \in (s_k, t_{k+1}], \quad k = 0, 1, 2, \ldots, m, \qquad (5.2)\end{aligned}$$

where $\mathscr{I}_0(0, \cdot) = \vartheta_0$.

Lemma 5.1 *Let assumptions* $(R1)$,$(R2)$, $(H1)$–$(H5)$ *be satisfied. Then the control function of the stochastic control system* (5.1) *can be estimated as* $\|\mathfrak{u}(\mathfrak{t})\| \leq \mathscr{M}_{\mathcal{C}\mathrm{k}}$ *for all* $\mathfrak{t} \in (s_{\mathrm{k}}, \mathfrak{t}_{\mathrm{k}+1}]$, $\mathrm{k} = 0, 1, 2, \ldots, m$ *where*

$$\begin{aligned}\mathscr{M}_{\mathcal{C}\mathrm{k}} &= 6\mathscr{M}_{\mathcal{C}}^2\big[\mathbb{E}\|\vartheta_{\mathfrak{t}_{\mathrm{k}+1}}\|^2 + 2\mathscr{M}_{\mathscr{R}}^2\mathscr{N}_{\mathscr{I}_{\mathrm{k}}} + \big(1 + 2\mathscr{M}_{\mathscr{R}}^2\big)\mathscr{N}_{\zeta} \\ &\quad + \mathscr{M}_{\mathscr{R}}^2\mathscr{N}_{\mathfrak{l}}\mathfrak{t}_{\mathrm{k}+1}^2 + \mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_{\mathrm{k}+1}\mathscr{N}_{\mathfrak{g}} + 2\mathscr{M}_{\mathscr{R}}^2\mathscr{H}\,\mathfrak{t}_{\mathrm{k}+1}^{2\mathscr{H}}\,\mathscr{N}_{\sigma}\big]\end{aligned}$$

with $\mathscr{N}_{\mathscr{I}_0} = \mathbb{E}\|\vartheta_0\|$.

Proof Replacing $\mathfrak{t}$ with $\mathfrak{t}_{\mathrm{k}+1}$, $\mathrm{k} = 0, 1, 2, \ldots, m$, and $\mathscr{Y}(\mathfrak{t}_{m+1}) = \vartheta_{\mathfrak{t}_{m+1}} = \mathfrak{w}_1$, the value of the control function $\mathfrak{u}(\mathfrak{t})$ from (5.2), we obtain

$$\begin{aligned}\vartheta(\mathfrak{t}_{\mathrm{k}+1}) &= \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}}, \vartheta\big(\mathfrak{t}^-\big)\big) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\big] - \zeta(\mathfrak{t}_{\mathrm{k}+1}, \vartheta_{\mathfrak{t}_{\mathrm{k}+1}}) \\ &\quad + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathcal{C}\mathfrak{u}(s)\,ds + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{l}(s, \vartheta_s)\,ds \\ &\quad + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s) + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s) \\ &= \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}}, \vartheta\big(\mathfrak{t}^-\big)\big) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\big] - \zeta(\mathfrak{t}_{\mathrm{k}+1}, \vartheta_{\mathfrak{t}_{\mathrm{k}+1}}) \\ &\quad + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{l}(s, \vartheta_s)\,ds + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s) \\ &\quad + \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s) + \big(\mathscr{Y}_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}}\big)\big(\mathscr{Y}_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}}\big)^{-1} \\ &\quad \times \Big[\vartheta_{\mathfrak{t}_{\mathrm{k}+1}} - \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}}, \vartheta\big(\mathfrak{t}^-\big)\big) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\big] + \zeta(\mathfrak{t}_{\mathrm{k}+1}, \vartheta_{\mathfrak{t}_{\mathrm{k}+1}}) \\ &\quad - \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{l}(\tau, \vartheta_\tau)\,d\tau - \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{g}(\tau, \vartheta_\tau)\,d\omega(\tau) \\ &\quad - \int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\sigma(\tau)\,d\mathscr{Z}_{\mathscr{H}}(\tau)\Big] \\ &= \vartheta_{\mathfrak{t}_{\mathrm{k}+1}}.\end{aligned}$$

Thus $\mathfrak{u}(\mathfrak{t})$ directs the behavior from the initial state to the target state:

$$\begin{aligned}\mathbb{E}\big\|\mathfrak{u}(\mathfrak{t})\big\|^2 &\leq 6\big\|\big(\mathscr{Y}_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}}\big)^{-1}\big\|^2\Big[\mathbb{E}\|\vartheta_{\mathfrak{t}_{\mathrm{k}+1}}\|^2 + \mathbb{E}\big\|\mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s_{\mathrm{k}})\big[\mathscr{I}_{\mathrm{k}}\big(s_{\mathrm{k}}, \vartheta\big(\mathfrak{t}^-\big)\big) + \zeta(s_{\mathrm{k}}, \vartheta_{s_{\mathrm{k}}})\big]\big\|^2 \\ &\quad + \mathbb{E}\bigg\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{g}(s, \vartheta_s)\,d\omega(s)\bigg\|^2 + \mathbb{E}\bigg\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\mathfrak{l}(s, \vartheta_s)\,ds\bigg\|^2 \\ &\quad + \mathbb{E}\big\|\zeta(\mathfrak{t}_{\mathrm{k}+1}, \vartheta_{\mathfrak{t}_{\mathrm{k}+1}})\big\|^2 + \mathbb{E}\bigg\|\int_{s_{\mathrm{k}}}^{\mathfrak{t}_{\mathrm{k}+1}} \mathscr{R}(\mathfrak{t}_{\mathrm{k}+1} - s)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\bigg\|^2\Big] \\ &\leq 6\mathscr{M}_{\mathcal{C}}^2\big[\mathbb{E}\|\vartheta_{\mathfrak{t}_{\mathrm{k}+1}}\|^2 + 2\mathscr{M}_{\mathscr{R}}^2\mathscr{N}_{\mathscr{I}_{\mathrm{k}}} + \big(1 + 2\mathscr{M}_{\mathscr{R}}^2\big)\mathscr{N}_{\zeta} + \mathscr{M}_{\mathscr{R}}^2\mathscr{N}_{\mathfrak{l}}\mathfrak{t}_{\mathrm{k}+1}^2 \\ &\quad + \mathscr{M}_{\mathscr{R}}^2\mathfrak{t}_{\mathrm{k}+1}\mathscr{N}_{\mathfrak{g}} + 2\mathscr{M}_{\mathscr{R}}^2\mathscr{H}\,\mathfrak{t}_{\mathrm{k}+1}^{2\mathscr{H}}\,\mathscr{N}_{\sigma}\big] \\ &= \mathscr{M}_{\mathcal{C}\mathrm{k}}\end{aligned}$$

□

Theorem 5.1 *Let* (*R*1), (*R*2), *and* (*H*1)–(*H*5) *hold. Then the stochastic integrodifferential system* (5.1) *is controllable on* $\mathcal{J}$.

Proof By Lemma 5.1, employing methods similar to those in Theorem 3.1, we can show that the stochastic control system (5.1) is controllable on $\mathcal{J}$. □

6 Illustration

Consider the following noninstantaneous impulsive stochastic integrodifferential system of the form

$$\begin{aligned}
&d\left[\vartheta(\mathrm{t},\rho)+\frac{1}{10}\int_0^1\int_0^1\rho\sin\bigl(\vartheta(s,\rho)\bigr)\,d\rho\,ds\right]\\
&\quad=\frac{\partial^2}{\partial\rho^2}\left[\vartheta(\mathrm{t},\rho)+\frac{1}{10}\int_0^1\int_0^1\rho\sin\bigl(\vartheta(s,\rho)\bigr)\,d\rho\,ds\right]d\mathrm{t}\\
&\qquad+\int_0^{\mathrm{t}}\widetilde{\Theta}(\mathrm{t}-s)\left[\vartheta(\mathrm{t},\rho)+\frac{1}{10}\int_0^1\int_0^1\rho\sin\bigl(\vartheta(s,\rho)\bigr)\,d\rho\,ds\right]d\mathrm{t}\\
&\qquad+\frac{\sqrt{2}}{10e^{\mathrm{t}}(1+\mathrm{t}^2)}\sin\bigl(\vartheta(\mathrm{t},\rho)\bigr)+e^{-\mathrm{t}}\,d\mathscr{Z}_{\mathscr{H}}(\mathrm{t})\\
&\qquad+\frac{\mathrm{t}}{9e^{\mathrm{t}}}\sin\bigl(\vartheta(\mathrm{t},\rho)\bigr)\,d\mathscr{Z}(\mathrm{t}),\quad \rho\in[0,1],\mathrm{t}\in\left(0,\frac{3}{10}\right]\cup\left(\frac{7}{10},1\right],\\
&\vartheta(\mathrm{t},\rho)=\frac{1}{5}\sin\left(\vartheta\left(\frac{3}{10}^{-},\rho\right)\right),\quad \mathrm{t}\in\left(\frac{3}{10},\frac{7}{10}\right],\\
&\vartheta(\mathrm{t},0)=0=\vartheta(\mathrm{t},1),\\
&\vartheta(0,\rho)=\vartheta_0,
\end{aligned}\tag{6.1}$$

thereby $\mathscr{Z}_{\mathscr{H}}$ is a Rosenblatt process defined on a probability space $(\Omega,\mathcal{F},\mathbb{P})$, and $\mathscr{Z}(\mathrm{t})$ a Wiener process, $0=s_0=\mathrm{t}_0<\mathrm{t}_1<s_1<\mathrm{t}_2=\mathfrak{b}<\infty$, with $s_0=0$, $\mathrm{t}_1=\frac{3}{10}$, $s_1=\frac{7}{10}\mathrm{t}_2=\mathfrak{b}=1$.

Let $\mathrm{E}_1=\mathrm{E}_2=\mathbb{R}$, $\varpi_{\mathrm{k}}^1=1$, $\varpi_{\mathrm{k}}^j=1$, $\mathrm{k}=1,2$. Let $\mathscr{X}=\mathscr{U}=\mathbb{L}^2([0,1])$. Define $\mathscr{A}:\mathscr{X}\to\mathscr{X}$ as $\mathscr{A}=\frac{\partial^2}{\partial\rho^2}$ with domain $\mathscr{D}(\mathscr{A})=\mathscr{X}^2([0,\pi])\cap\mathscr{X}_0^2([0,\pi])$, and

$$\begin{aligned}
&\mathscr{A}\vartheta(\rho)=\int_0^1\min(\rho,\tau)\vartheta(\rho)\,d\rho,\quad \rho\in[0,1],\\
&\mathscr{D}(\mathscr{A})=\bigl\{\vartheta\in\mathbb{L}^2\bigl([0,1]\bigr):\vartheta(0)=0;\vartheta'(1)=0\bigr\},
\end{aligned}$$

where we have taken $\vartheta(\rho)=\vartheta(\cdot,\rho)$. We may show that the operator $\mathscr{A}$ is an infinitesimal generator of a C_0-semigroup.

Initially,

$$\mathscr{A}\vartheta(\rho)=\int_0^1\min(\mu,\tau)\vartheta(\mu)\,d\mu=\int_0^\rho\mu\vartheta(\mu)\,d\mu+\rho\int_\rho^1\vartheta(\mu)\,d\mu.$$

Let υ be an eigenvalue of $\mathscr{A}$, and let ϑ be an associated eigenvector. Let $\Theta:\mathscr{D}(\mathscr{A})\subset\mathscr{X}\to\mathscr{X}$ be the operator defined by $\Theta(\mathrm{t})(z)=\widetilde{\Theta}(\mathrm{t})\mathscr{A}z$ for $\mathrm{t}\geq 0$ and $z\in\mathscr{D}(\mathscr{A})$. Considering $\upsilon=0$, as $\mathscr{A}\vartheta$ is of class $\mathbb{C}^1$, we have $(\mathscr{A}\vartheta)'=0$, and $\int_\rho^1\vartheta(\tau)\,d\tau=0$ for all $\rho\in[0,1]$. By differentiating we deduce that ϑ is zero. Thus 0 is not an eigenvalue of $\mathscr{A}$.

So we may suppose that $\upsilon \neq 0$. The relation $\mathscr{A}\vartheta = \upsilon\vartheta$ shows that ϑ is necessarily of class $\mathbb{C}^1$, so by differentiation we get

$$\forall \rho \in [0,1], \quad \upsilon\vartheta'(\rho) = \rho\vartheta(\rho) = \rho\vartheta(\rho) + \int_\rho^1 \vartheta(\rho)\,d\mathrm{t} - \rho\vartheta(\rho) = \int_\rho^1 \vartheta(\rho)\,d\tau.$$

By taking the second derivative we get

$$\upsilon\vartheta''(\rho) = -\vartheta(\rho), \quad \rho \in [0,1].$$

Therefore

$$\vartheta'' + \varpi\vartheta = 0 \quad \text{with } \varpi = 1/\upsilon,$$

and, consequently, there exist $\mathscr{K}_1, \mathscr{K}_2 \in \mathbb{R}$ such that for all $\rho \in [0,1]$,

$$\vartheta(\rho) = \mathscr{K}_1 \sin(\sqrt{\varpi}\rho) + \mathscr{K}_2 \cos(\sqrt{\varpi}\rho) \quad \text{if } \varpi > 0,$$
$$\vartheta(\rho) = \mathscr{K}_1 \sinh(\sqrt{-\varpi}\rho) + \mathscr{K}_2 \cosh(\sqrt{-\varpi}\rho) \quad \text{if } \varpi < 0.$$

The condition $\vartheta(0) = 0$ implies that $\mathscr{K}_2 = 0$. Since $\upsilon\vartheta'(1) = 0$ and $\mathscr{K}_2 \neq 0$, we have

$$\cos(\sqrt{\varpi}) = 0 \quad \text{if } \varpi > 0 \quad \text{and} \quad \cosh(\sqrt{-\varpi}) = 0 \quad \text{if } \varpi < 0.$$

The second condition is impossible. Therefore we have $\upsilon > 0$ and $\varphi = (n\pi + \frac{\pi}{2})^2$ for $n \in \mathbb{N}$. For $\vartheta \in \mathscr{D}(\mathscr{A})$, $\vartheta : \rho \mapsto \mathscr{K}_1 \sin((n\pi + \frac{\pi}{2})\rho)$ and $\upsilon = \frac{1}{(n\pi+\frac{\pi}{2})^2}$ for all $\vartheta \in \mathscr{D}(\mathscr{A})$. In summary, the eigenvalues of A are the real numbers $\upsilon_n = \frac{1}{(n\pi+\frac{\pi}{2})^2}$, $n \in \mathbb{N}$, and each eigenvalue γ_n corresponds to a subspace of dimension 1 generated by $x_n : \rho \mapsto K_1 \sin((n\pi + \frac{\pi}{2})\rho)$.

We have $\langle\vartheta_n, \vartheta_n\rangle = 0$ for all $\mathrm{k} \neq n$ and $\langle\vartheta_n, \vartheta_n\rangle = 1$. Hence $\mathscr{K}_1 = \sqrt{2}$, and

$$\vartheta_n(\rho) = \sqrt{2}\sin\left(\left(n\pi + \frac{\pi}{2}\right)\rho\right), \quad n \in \mathbb{N},$$

is the orthogonal set of eigenvectors of $\mathscr{A}$.

Thus we obtain that for $\vartheta \in \mathscr{D}(\mathscr{A})$, the following expression for the C_0-semigroup $(\mathcal{T}(\mathrm{t}))_{\mathrm{t}\geq 0}$ on $\mathscr{X}$ generated by the operator $\mathscr{A}$:

$$\big(\mathcal{T}(\mathrm{t})\big)\vartheta = \sum_{n\geq 1} e^{\upsilon_n \mathrm{t}}\langle\vartheta, \vartheta_n\rangle\vartheta_n, \quad \text{and} \quad \mathscr{A}\vartheta = \sum_{n\geq 1} \upsilon_n\langle\vartheta, \vartheta_n\rangle\vartheta_n, \quad \vartheta \in \mathscr{D}(\mathscr{A}),$$

where

$$\upsilon_n = \frac{1}{(n\pi + \frac{\pi}{2})^2} \quad \text{and} \quad \vartheta_n(\rho) = \sqrt{2}\sin\left(\left(n\pi + \frac{\pi}{2}\right)\rho\right), \quad n \in \mathbb{N}.$$

Let $\vartheta(t)(\rho) = \vartheta(\mathrm{t}, \rho)$. Define the functions ζ, $\mathfrak{l}$, $\mathfrak{g}$, $\mathscr{I}_\mathrm{k}$, and σ as

$$\zeta(\mathrm{t}, \vartheta)(\rho) = \frac{1}{10}\int_0^1\int_0^1 \rho\sin\big(\vartheta(s,\rho)\big)\,d\rho\,ds, \qquad \mathfrak{l}(\mathrm{t}, \vartheta)(\rho) = \frac{\sqrt{2}}{10e^{\mathrm{t}}(1+\mathrm{t}^2)}\sin\big(\vartheta(\mathrm{t}, \rho)\big),$$

$$\mathfrak{g}(\mathrm{t},\vartheta)(\rho)=\frac{\mathrm{t}}{9e^{\mathrm{t}}}\sin\bigl(\vartheta(\mathrm{t},\rho)\bigr),\qquad \mathscr{I}_{\mathrm{k}}(\mathrm{t},\vartheta)(\rho)=\frac{1}{5}\sin\left(\vartheta\left(\frac{3}{10}^{-},\rho\right)\right),\qquad \sigma(\mathrm{t})=e^{-\mathrm{t}},$$

and the operator $\mathscr{B}$ as follows:

$$\bigl(\mathscr{K}(\mathrm{t})\vartheta\bigr)(\rho)=\cos(\mathrm{t})e^{\mathrm{t}}\mathscr{A}\vartheta(\rho),\quad \mathrm{t}\in[0,1],\vartheta\in\mathscr{D}(\mathscr{A}),\rho\in[0,1].$$

Then system (6.1) is the abstract formulation of system (1.1),

$$\begin{aligned} &d\bigl[\vartheta(\mathrm{t})+\zeta(\mathrm{t},\vartheta_{\mathrm{t}})\bigr]\\ &\quad=\mathscr{A}\bigl[\vartheta(\mathrm{t})+\zeta(\mathrm{t},\vartheta_{\mathrm{t}})\bigr]d\mathrm{t}+\int_0^{\mathrm{t}}\Theta(\mathrm{t}-s)\bigl[\vartheta(s)+\zeta(s,\vartheta_s)\,ds\bigr]d\mathrm{t}\\ &\qquad+\mathfrak{l}(\mathrm{t},\vartheta_{\mathrm{t}})\,d\mathrm{t}+\mathfrak{g}(\mathrm{t},\vartheta_{\mathrm{t}})\,d\omega(\mathrm{t})+\sigma(\mathrm{t})\,d\mathscr{Z}_{\mathscr{H}}(\mathrm{t}),\quad \text{for } \mathrm{t}\in\bigcup_{\mathrm{k}=0}^{m}(s_{\mathrm{k}},\mathrm{t}_{\mathrm{k}+1}],\\ &\vartheta(\mathrm{t})=\mathscr{I}_{\mathrm{k}}\bigl(\mathrm{t},\vartheta\bigl(\mathrm{t}_{\mathrm{k}}^{-}\bigr)\bigr),\quad \text{for } \mathrm{t}\in\bigcup_{\mathrm{k}=1}^{m}(\mathrm{t}_{\mathrm{k}},s_{\mathrm{k}}],\\ &\vartheta(0)=\varphi\in\mathscr{B}. \end{aligned}\tag{6.2}$$

Moreover, $\eta(\mathrm{t}) := \cos \mathrm{t}e^{\mathrm{t}}$ is a bounded $\mathcal{C}^1$ function such that η' is bounded and uniformly continuous, so that (R1) and (R2) are satisfied.

We obtain $\mathscr{N}_{\zeta}=\mathscr{M}_{\zeta}=\frac{1}{100}$, $\mathscr{N}_{\mathfrak{l}}=\mathscr{M}_{\mathfrak{l}}=\frac{1}{50}$, $\mathscr{N}_{\mathfrak{g}}=\mathscr{M}_{\mathfrak{g}}=\frac{1}{81}$, $\mathscr{N}_{\mathscr{I}_{\mathrm{k}}}=\mathscr{M}_{\mathscr{I}_{\mathrm{k}}}=\frac{1}{25}$. Set $\mathscr{M}_{\mathscr{R}}=1$. Now we obtain

$$\begin{aligned} \mathscr{C}_{2,0}&=3\mathscr{M}_{\zeta}+3\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{l}}\mathrm{t}_1^2+3\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{g}}\mathrm{t}_1=0.044,\\ \mathscr{C}_{2,\mathrm{k}}&=8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathscr{I}_{\mathrm{k}}}+8\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\zeta}+4\mathscr{M}_{\zeta}+4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{l}}\mathrm{t}_{\mathrm{k}+1}^2+4\mathscr{M}_{\mathscr{R}}^2\mathscr{M}_{\mathfrak{g}}\mathrm{t}_{\mathrm{k}+1}=0.59,\\ \mathscr{C}_2&=\max\{\mathscr{C}_{2,0},\mathscr{M}_{\mathscr{I}_{\mathrm{k}}},\mathscr{C}_{2,\mathrm{k}}\}=\max\left\{0.044,\frac{1}{25},0.59\right\}=0.59<1. \end{aligned}$$

This implies that the assumptions of Theorem 3.1 are satisfied. We may conclude that the stochastic integrodifferential system (6.1) has a unique mild solution on $\mathcal{J}$.

Let the final state be denoted by ϑ_{t_2}, and let the feedback control $\mathfrak{u}(\mathrm{t})(\rho)$ be defined as follows:

$$\mathfrak{u}(\mathrm{t})(\rho)=\mathfrak{u}(\mathrm{t},\rho)=\begin{cases}\mathfrak{u}_1(\mathrm{t},\rho), & \mathrm{t}\in(0,\frac{3}{10}],\\ \mathfrak{u}_2(\mathrm{t},\rho), & \mathrm{t}\in(\frac{7}{10},1],\end{cases}$$

where

$$\begin{aligned} \mathfrak{u}_1(\mathrm{t},\rho)&=\bigl(\mathscr{Y}_0^{\frac{3}{10}}\bigr)^{-1}\left[\vartheta_{\mathrm{t}_1}-\mathscr{R}\left(\frac{3}{10}-0\right)\bigl[\vartheta_0+\zeta(0,\varphi)(\rho)\bigr]+\zeta\left(\frac{3}{10},\vartheta\left(\frac{3}{10}\right)\right)(\rho)\right.\\ &\quad-\int_0^{\frac{3}{10}}\mathscr{R}\left(\frac{3}{10}-s\right)\mathfrak{l}(s,\vartheta_s)(\rho)\,ds-\int_0^{\frac{3}{10}}\mathscr{R}\left(\frac{3}{10}-s\right)\mathfrak{g}(s,\vartheta_s)(\rho)\,d\omega(s)\\ &\quad\left.-\int_0^{\frac{3}{10}}\mathscr{R}\left(\frac{3}{10}-s\right)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right], \end{aligned}$$

$$\begin{aligned}\mathfrak{u}_2(\mathfrak{t},\rho) &= \left(\mathscr{Y}_{\frac{7}{10}}^{1}\right)^{-1}\left[\vartheta_1 - \mathscr{R}\left(1-\frac{7}{10}\right)\left[\mathscr{I}_{\mathrm{k}}\left(\frac{7}{10},\vartheta\left(\frac{3}{10}^{-}\right)\right)(\rho) + \zeta\left(\frac{7}{10},\vartheta\left(\frac{7}{10}\right)\right)(\rho)\right]\right.\\ &\quad + \zeta\big(1,\vartheta(1)\big)(\rho) - \int_{\frac{7}{10}}^{1}\mathscr{R}(1-s)\mathfrak{f}(s,\vartheta_s)(\rho)\,ds - \int_{\frac{7}{10}}^{1}\mathscr{R}(1-s)\mathfrak{g}(s,\vartheta_s)(\rho)\,d\omega(s)\\ &\quad \left. - \int_{\frac{7}{10}}^{1}\mathscr{R}\left(\frac{3}{10}-s\right)\sigma(s)\,d\mathscr{Z}_{\mathscr{H}}(s)\right],\end{aligned}$$

and

$$\mathscr{Y}_{0}^{\frac{3}{10}}\mathfrak{u}_1(\mathfrak{t},\rho) = \int_0^{\frac{3}{10}}\mathscr{R}\left(\frac{3}{10}-s\right)\mathfrak{u}_1(s,\rho)\,ds, \qquad \mathscr{Y}_{\frac{7}{10}}^{1}\mathfrak{u}_2(\mathfrak{t},\rho) = \int_{\frac{7}{10}}^{1}\mathscr{R}(1-s)\mathfrak{u}_2(s,\rho)\,ds.$$

Hence, the control function steers from initial state ϑ_0 to final state $\vartheta_{\mathfrak{t}_2}$, and all the assumptions of Theorem 5.1 are fulfilled. Thus we can conclude that the (6.1) is controllable on $\mathcal{J}$.

7 Conclusions

A novel class of noninstantaneous impulsive SIDEs driven by Brownian motion and Rosenblatt process has been studied. Initially, the existence and uniqueness of mild solutions are obtained using stochastic analysis, analytic semigroup theory, integral equation theory, and fixed point methodology. In the later part, the asymptotic behavior of mild solutions for the aforementioned system has been established along with the controllability results. Finally, our main findings are verified through an example. There are two direct issues that require a further study. First, we investigate the optimal problems for the noninstantaneous impulsive stochastic integrodifferential equations driven by Levy processes. Second, we study the approximate controllability for the stochastic integrodifferential system with Markov switched stochastic system. For future work, we can present noninstantaneous impulsive SIDEs inclusions with Clarke subdifferential.

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All authors contributed equally in writing this article. KR,KR VS and JJN investigated the problem, proposed regularity assumptions, proved the results, and gave an illustrative example. All authors read and approved the final manuscript.

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