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Finite-time asynchronous fault detection filter design for conic-type nonlinear semi-Markovian jump systems

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ABSTRACT In this work, the problem of finite-time asynchronous fault detection filter design is investigated for conic-type nonlinear semi-Markovian jump systems with time delay, missing measurements and randomly jumping fault signal. In particular, the transition probability of the semi-Markov process is considered as time-varying along with lower and upper bounds of the transition rate. Besides, the asynchronous fault detection filter is developed for semi-Markovian jump systems with specific time-varying transition probability satisfying semi-Markov process. To quantify the effects of missing measurements a stochastic variable that satisfies Bernoulli's distribution is adopted. Furthermore, a set of sufficient conditions is derived in terms of linear matrix inequalities (LMIs) by constructing proper mode-dependent Lyapunov-Krasovskii functional such that the augmented asynchronous fault detection filtering error system is stochastically finite-time bounded with prescribed strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance. Finally, the provided filter designs applicability and usefulness has been verified with two numerical examples.

INDEX TERMS Semi-Markovian jump system; Conic-type nonlinearity; Missing measurements; Asynchronous fault detection filter; Stochastic finite-time boundedness.

I. INTRODUCTION

MARKOVIAN Jump Systems (MJSs) are certain kind of hybrid dynamical systems which can be used to successfully model many practical systems such as in economic systems, aircraft control, robotics, power systems, chemical process and so on [1], [2]. Besides, MJSs are more desirable to specify the dynamical systems with random instantaneous changes in their structure due to parameter shifting, environmental variations, abrupt faults or failures in components and so on. MJSs consist of a group of subsystems with the transitions between the models regulated by a Markov chain with constant transition rates. Recently, MJSs have intensively been investigated and many fruitful results are proposed, see for example [3]–[12]. The dissipativity-based asynchronous filter was designed in [4] for a class of discrete-time uncertain fuzzy nonhomogeneous Markovian jump systems, where the fuzzy asynchronous full-order filter is designed to ensure the dissipative performance of the filtering error system using a triple-parameterized matrix inequality and relaxation technique. Furthermore, in [6], the authors investigated the robustness of the constructed filter with H_∞ performance for Markovian jump systems with quantized output and unknown transfer probabilities. In [7], a novel summation inequality is adapted to design an H_∞ control for Markovian jump systems with time delay.

The Semi-Markovian Jump Systems (SMJSs) are modified MJSs in which the transfer probability rate is determined by the sojourn time which is time-varying, in contrast with fixed transition probability rate in MJSs. It should be mentioned that when dealing with MJSs, the elements of the transition rate matrix are considered as a constant forever. This condition may not be satisfied for certain real-time systems modeled in the framework of MJSs since the transition rate matrix can even be time-variant in practice. As compared to other approaches, SMJSs have a great number of applications due to the unpredictable condition on the probability distribution. Nowadays, a huge deal of consideration has been paid to the study of SMJSs [13]–[15]. The authors in [13] addressed the problem of stochastic stability and stabilization for a class of semi-Markovian jump systems where the jump parameters obey the semi-Markovian process. Moreover, by the virtue of relaxation approach and sojourn-time-dependent matrix inequalities, the sufficient conditions are obtained in the form of LMIs for the stabilization of the considered system. In [14], a reliable filtering problem is investigated for SMJSs subject to time delay, uncertainties, and sensor failures, where the considered filter design ensures mixed passivity and H_∞ filtering for error system performances.

While modeling the system, certain environmental variables such as nonlinearities, uncertainties, time delays, modeling errors, external disruptions and faults may often cause various challenges for the stabilization of dynamic systems. The conic-type nonlinearity is a type of nonlinear factor that occurs within a hypersphere, where the center is a linear system and the radius is a supplement linear system bounded by the norm. In a conic-type nonlinear model, rather than knowing the exact dynamics, it is sufficient to know a dynamic bound of the system nonlinearities. In addition it makes the modeling of practical nonlinear systems simple and better. The study of conic-type nonlinearities for dynamical system are addressed in [16]–[19]. An asynchronous filtering problem for T-S fuzzy Markovian jump system subject to nonhomogeneous transition probability is reported in [18]. On the other hand, faults in any dynamical systems are inevitable and will influence the systems stability. The primary goal of fault detection scheme is to detect a fault signal effectively for getting the desired performance. For this to be achieved, a residual signal is supposed in the filter system to point out the deviation between nominal and faulty system operation. Furthermore, a determined threshold is used to compare the generated residual evaluation function. Further, it may be deduced that the fault has occurred if the residual evaluation function crosses the threshold. In accordance with its significance, great number of findings are provided in the literature regarding fault detection techniques for dynamical systems [20]–[23]. By deploying stochastic analysis methods, optimization techniques and cone complementarity linearization technique, a fault detection filter is designed for underactuated manipulators based on Markovian jump model in [20]. Similarly, fault detection filtering technology is recognized as a valid research topic due to its significant impact on a wide range of applications.

In general, for MJSs two standard forms of filters are designed such as mode-independent and mode-dependent. The mode-dependent filter is designed under the assumption that, the system mode is accessible to the filter at any instant of time. Nevertheless, for practical systems the above consideration is usually challenging to be assured due to the existence of network-induced imperfections. Hence, it is obvious that the design of mode-dependent filter has several constraints. As a consequence, in recent times the mode-independent filter has been developed to overcome the limitations of mode-dependent filter. It should be mentioned that, the mode-independent filter is appropriate for systems where the information about mode transition is absolutely unavailable. However, these kind of filters have difficulty while handling the asynchronous phenomenon among the system and filter modes since it ignores all the accessible mode information. To tackle this issue, an asynchronous filter is structured and some interesting results have been provided [24]–[28]. The authors in [25] investigated an asynchronous H_∞ filter for a class of discrete-time T-S fuzzy MJSs such that the resulting filtering error system satisfies H_∞ disturbance attenuation performance and finite-time boundedness. It should be noted that, the existence of some unexpected errors during the implementation of designed filter are inevitable. Therefore, the non-fragile filter is assumed to ensure the robustness subject to filter gain fluctuations. In recent years, enormous number of the resilient based filter design have been recorded [29], [30].

Time delay is commonly encountered in various dynamics systems and it is regarded as the main source for instability or poor performance of the systems. To overcome the shortcomings of the time delay, many techniques have been employed and numerous investigation on delay-dependent stability condition have been reported in literature [31]–[33]. Further, the transmitted measurements may be lost or partially communicated because of sudden failures or faults of system components. Therefore, the filtering problems with missing measurements have gained significant research attention in recent years [34], [35]. It is important to point out that, in the study of system stability or stabilization, the dissipative theory plays a crucial role. Moreover, dissipative performance is more general when comparing with H_∞ and passivity performances. Also, it provides a less conservative and more flexible filter design since it manages a better trade-off between the gain and phase performances (see for example [36]–[38]). Moreover, it is practically important to study the state responses within a finite period of time. Therefore, the concepts of finite-time stability and finite-time boundedness have become active research fields in the past few decades [39]. As a consequence of the preceding discussion, in this paper, a finite-time non-fragile asynchronous fault detection filter with dissipative performance for conic-type nonlinear SMJSs with time delay, missing measurements and random jumping fault signal is examined. The main concerns of this work are as follows:

- A finite-time asynchronous fault detection filter design problem is investigated for conic-type nonlinear SMJSs with time delay, missing measurements and random jumping fault signal.
- A stochastic variable is considered for describing the missing measurement phenomena which is assumed to follow Bernoulli distributed white sequence. Further, a jumping mode is provided to describe the random occurrence of fault signal.
- The residual signal is used to solve the asynchronous fault detection filtering problem. In addition, a residual error is produced by measuring the difference between the residual signal and the measured output. Moreover, the fault is detected when the obtained residual evalua-

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tion functional crosses the predefined threshold value.

- By using Lyapunov stability theory along with time-varying transition rate, a new group of sufficient conditions in the form of LMIs is established to ensure the finite-time boundedness and dissipative performance of the asynchronous fault detection filtering error system.

Finally, a Pulse-Width-Modulation-driven boost converter model and R-L-C circuit model are given to show the efficiency of the proposed asynchronous fault detection filter design in the existence and non-existence of time delay, respectively.

Notation: On the whole, the following notations have functioned in this paper. $\mathbb{E}\{\cdot\}$ represents the mathematical expectation. The n- dimensional Euclidean space is denoted by $\mathcal{R}^n$. N^{-1} and N^T indicate the inverse and the transpose of the matrix N, respectively. For a vector, $\|\cdot\|$ indicates its Euclidean norm. $P > 0$ means that P is a positive definite matrix. I and 0 illustrates the identity and zero matrices with suitable dimension, respectively. The symbol $'*'$ is used in matrix terms to represent the transpose elements in the symmetric position.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of discrete-time conic-type nonlinear SMJSs with time delay and random jumping fault signal in the following form:

$$\begin{aligned} x(k+1) =& f(x(k), x(k-d), w(k), \tilde{f}(k)) + B_{\varrho(k)}u(k),\\ y(k) =& \alpha(k)C_{\varrho(k)}x(k) + D_{\varrho(k)}w(k) + H_{\varrho(k)}\tilde{f}(k), \quad (1)\\ x(k) =& x(j),\ -\infty < j \leq 0, \end{aligned}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $y(k) \in \mathcal{R}^b$ is the measured output, $u(k) \in \mathcal{R}^a$ is the controlled input, $w(k) \in \mathcal{R}^w$ is the disturbance input which belongs to $l_2[0,\infty)$, $f(x(k), x(k-d), w(k), \tilde{f}(k))$ is an unknown nonlinear function. $d > 0$ is the constant time delay term, $x(j)$ is the initial condition. $B_{\varrho(k)}$, $C_{\varrho(k)}$, $D_{\varrho(k)}$ and $E_{\varrho(k)}$ are appropriate dimensioned known matrices. $\alpha(k)$ is the random variable that indicates the missing measurement phenomena and designed as Bernoulli distributed white sequence. In addition, it satisfies the distribution law by taking values 0 or 1 with $\Pr\{\alpha(k) = 1\} = \mathbb{E}\{\alpha(k)\} = \bar{\alpha}$, $\Pr\{\alpha(k) = 0\} = \mathbb{E}\{\alpha(k)\} = 1 - \bar{\alpha}$. Let $\{\varrho(k), k > 0\}$ be the discrete-time semi-Markov process taking its values in a finite set $\mathcal{M}_1 = \{1, 2, \cdots, S\}$ along with transition probability as follows:

$$\Pr\{\varrho(k+1) = n, T_{k+1} = \tilde{\tau} | \varrho(k) = m\} = \pi_{mn}(k), \quad (2)$$

where $\pi_{mn}(k) \geq 0$ is the transition probability from mode m at time k to n at time $k+1$ and $\sum_{n=1}^{\mathcal{M}_1} \pi_{mn}(k) = 1$, $\forall\, m \in \mathcal{M}_1$, $T_{k+1} = n - m$ represent the sojourn time begins with k^{th} jump and ends with $k+1^{th}$ jump. Specifically, π_{mn} is the transition with lower and upper bounds as $\pi_{mn}^- \leq \pi_{mn}(k) \leq \pi_{mn}^+$. For the purpose of accessibility, we denote $\varrho(k) = m$. Moreover, we have described the fault signal with the random jump in the following form:

$$\tilde{f}(k) = \varsigma(k)\hat{f}(k), \quad (3)$$

where $\hat{f}(k)$ be the deterministic fault signal and $\varsigma(k)$ is the random jumping signal which consists of two values either 1 or 0.

In this paper, the unknown nonlinear function $f(x(k), x(k-d), w(k), f(k))$ along with conic-type sector is described as

$$\begin{aligned} &\|f(x(k), x(k-d), w(k), f(k)) - (A_m x(k)\\ &+ A_{dm}x(k-d) + E_m w(k) + F_m\tilde{f}(k))\|\\ &\leq \|G_m x(k) + G_{dm}x(k-d) + G_{wm}w(k) + L_m\tilde{f}(k)\|, \end{aligned} \quad (4)$$

where A_m, A_{dm}, E_m, F_m, G_m, G_{dm}, G_{wm} and L_m are known matrices with suitable dimensions. Further, for our convenience we take $\zeta(k) = \|f(x(k), x(k-d), w(k), f(k)) - (A_m x(k) + A_{dm}x(k-d) + E_m w(k) + F_m\tilde{f}(k)\|$ then we obtain

$$\begin{aligned} \zeta^T(k)\zeta(k) \leq& (G_m x(k) + G_{dm}x(k-d) + G_{wm}w(k) + L_m\tilde{f}(k))^T\\ & (G_m x(k) + G_{dm}x(k-d) + G_{wm}w(k) + L_m\tilde{f}(k)). \end{aligned} \quad (5)$$

Remark 1: It is worthy to mention that the non-linear part of SMJSs (1) studied in this paper satisfies conic condition. Thus, the non-linear function $f(x(k), x(k-d), w(k), f(k))$ lies in an n-dimensional hyperspace whose center is a linear system represented by $A_m x(k) + A_{dm}x(k-d) + E_m w(k) + F_m\tilde{f}(k)$ and whose radius is bounded by another linear system $G_m x(k) + G_{dm}x(k-d) + G_{wm}w(k) + L_m\tilde{f}(k)$, which is ensured in inequality (4).

Then, system (1) can be rewritten in the following form:

$$\begin{aligned} x(k+1) =& A_m x(k) + A_{dm}x(k-d) + E_m w(k)\\ &+ F_m\tilde{f}(k) + B_m u(k),\\ y(k) =& \alpha(k)C_m x(k) + D_m w(k) + H_m\tilde{f}(k), \quad (6)\\ x(k) =& x(j),\ -\infty < j \leq 0. \end{aligned}$$

On the other hand, to detect the fault, we are interested in synthesizing the non-fragile asynchronous fault detection filter design with the residual signal as follows:

$$\begin{aligned} x_f(k+1) =& \bar{A}_{f\theta(k)}x_f(k) + \bar{B}_{f\theta(k)}y(k),\\ \tilde{\mu}(k) =& C_{f\theta(k)}x_f(k), \end{aligned} \quad (7)$$

where $\bar{A}_{f\theta(k)} = A_{f\theta(k)} + \Delta A_{f\theta(k)}$ and $\bar{B}_{f\theta(k)} = B_{f\theta(k)} + \Delta B_{f\theta(k)}$; $x_f(k) \in \mathcal{R}^d$ is the filter state; $y(k)$ is the filter input vector; $\tilde{\mu}(k)$ is the residual signal. $A_{f\theta(k)}$, $B_{f\theta(k)}$, $C_{f\theta(k)}$ are the filter gain matrices to be determined later. The terms $\Delta A_{f\theta(k)}(k)$ and $\Delta B_{f\theta(k)}(k)$ represent the filter gain perturbations with the structure

$$\begin{aligned} &\Delta A_{f\theta(k)}(k) = M_{a\theta(k)}\Delta(k)N_{a\theta(k)} \text{ and}\\ &\Delta B_{f\theta(k)}(k) = M_{b\theta(k)}\Delta_m(k)N_{b\theta(k)}, \end{aligned} \quad (8)$$

where $M_{a\theta(k)}$, $M_{b\theta(k)}$, $N_{a\theta(k)}$ and $N_{b\theta(k)}$ are appropriate dimensional known real matrices and $\Delta(k)$ is an unknown

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time-varying matrix function satisfying $\Delta^T(k)\Delta(k) \leq I$. The parameter $\theta(k)$ $(k \in \mathbb{Z}^+)$ serves as a discrete-time semi-Markov process which depends on $\varrho(k+1)$ and by taking the values from the finite set $\mathcal{M}_2 = \{1, 2, \cdots, M_2\}$. Moreover, the corresponding transition probability is given as $\Pr\{\theta(k+1) = q, T_{k+1} = \hat{\tau}|\theta(k) = p\} = \phi^n_{pq}(k) \geq 0$ for every $p, q \in \mathcal{M}_2$ with $\sum_{q=1}^{\mathcal{M}_2} \phi^n_{pq}(k) = 1$. Specifically, the process $\varrho(k)$ is pretended to be independent on $\mathcal{F}(k-1) = \sigma\{\theta(1), \theta(2), \cdots, \theta(k-1)\}$, where $\mathcal{F}(k-1)$ is the σ-algebra produced by $\{\theta(1), \theta(2), \cdots, \theta(k-1)\}$. Let us denote the notation $\theta(k) = p$ in the rest of this paper for our convenience.

Here, the residual error $e(k) = \tilde{\mu}(k) - y(k)$ is introduced to improve the sensibility of faults. Then, we consider the augmented asynchronous fault detection filtering error system in the following form

$$\begin{aligned}\eta(k+1) =& \bar{A}_{mp}\eta(k) + \tilde{\alpha}(k)\tilde{A}_m\eta(k) + \bar{A}_{dm}\eta(k-d) \\ &+ \bar{B}_{mp}\nu(k) + \bar{\zeta}(k), \\ e(k) =& \bar{C}_{mp}\eta(k) + \tilde{\alpha}(k)\tilde{C}_m\eta(k) + \bar{D}_m\nu(k), \end{aligned} \tag{9}$$

where

$$\eta^T(k) = \begin{bmatrix} x(k) & x_f(k) \end{bmatrix}^T, \ \nu(k) = \begin{bmatrix} u(k) \ w(k) \ \tilde{f}(k) \end{bmatrix},$$
$$\bar{\zeta}(k) = \begin{bmatrix} \zeta(k) \\ 0 \end{bmatrix}, \ \bar{A}_{mp} = \begin{bmatrix} A_m & 0 \\ \bar{\alpha}\bar{B}_{fp}C_m & \bar{A}_{fp} \end{bmatrix},$$
$$\tilde{A}_m = \begin{bmatrix} 0 & 0 \\ \bar{B}_{fp}C_m & 0 \end{bmatrix}, \ \bar{A}_{dm} = \begin{bmatrix} A_{dm} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{B}_{mp} = \begin{bmatrix} B_m & E_m & F_m \\ 0 & \bar{B}_{fp}D_m & \bar{B}_{fp}H_m \end{bmatrix}, \ \bar{C}_{mp} = \begin{bmatrix} \bar{\alpha}C_m & -C_{fp} \end{bmatrix},$$
$$\tilde{C}_m = \begin{bmatrix} C_m & 0 \end{bmatrix}, \ \bar{D}_m = \begin{bmatrix} 0 & D_m & H_m \end{bmatrix}, \ \tilde{\alpha}(k) = \alpha(k) - \bar{\alpha},$$
$$\mathbb{E}\{\tilde{\alpha}(k)\} = 0, \ \mathbb{E}\{\tilde{\alpha}^2\} = \bar{\alpha}(1-\bar{\alpha}) = \Theta^2_\alpha$$

The residual evaluation function $\mathsf{J}_e(k) = \sqrt{\sum_{k=k_0}^{k_0+\mathsf{L}} e^T(k)e(k)}$ and the threshold $\mathsf{J}_{th} = \sup_{w\neq0,\ u\neq0,\ f=0} \mathsf{J}_e(k)$ are chosen to detect the fault, where L is the finite time length of evaluation. Thus, the detection of the fault can be measured by comparing the evaluation function and the threshold according to the following laws:

$$\mathsf{J}_e(k) > \mathsf{J}_{th} \Longrightarrow with - fault,$$
$$\mathsf{J}_e(k) \leq \mathsf{J}_{th} \Longrightarrow without - fault.$$

To obtain the main results, we now address the following Assumptions.

Assumption 1: The disturbance input vector $\nu(k)$ is time-varying and satisfies $\sum_{k=0}^{\mathcal{N}} \nu^T(k)\nu(k) \leq \varphi$, where $\varphi > 0$.

Assumption 2: The time-varying transition probability $\pi_{mn}(k)$ is bounded and satisfies the following assumption as

$$\pi_{mn}(k) = \sum_{h=1}^{\mathcal{H}} \lambda_h\pi_{mn,h}, \ \sum_{h=1}^{\mathcal{H}} \lambda_h = 1, \ \lambda_h \geq 0, \mathcal{H} \geq 1,$$

where

$$\pi_{mn,h} = \begin{cases} \pi^-_{mn} + (h-1)\frac{\pi^+_{mn} - \pi^-_{mn}}{\mathcal{H}-1}, & m \neq n \\ \pi^-_{mn} - (h-1)\frac{\pi^+_{mn} - \pi^-_{mn}}{\mathcal{H}-1}, & m = n. \end{cases}$$

Lemma 1: [37] Let G, H and $F(k)$ be real matrices of appropriate dimensions with $F^T(k)F(k) \leq I$. Then there exists a constant $\epsilon > 0$ such that the inequality $GF(k)H + H^TF^T(k)G^T \leq \epsilon GG^T + \epsilon^{-1}H^TH$ holds.

Definition 1: [36] Given positive scalars $\mathcal{C}_1, \mathcal{C}_2$ with $\mathcal{C}_2 > \mathcal{C}_1$ and a symmetric matrix $\mathcal{J}$, the augmented fault detection filtering residual error system (9) is said to be finite-time stochastic bounded with respect to $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{N}, \gamma, \mathcal{J}, \varphi)$, if for the disturbance input $\nu(k)$ satisfies Assumption 1, it holds that $\mathbb{E}\{\eta^T(k_1)\mathcal{J}\eta(k_1)\} \leq \mathcal{C}_1 \Longrightarrow \mathbb{E}\{\eta^T(k_2)\mathcal{J}\eta(k_2)\} < \mathcal{C}_2, \forall\, k_1 \in \{-\varsigma_2, -\varsigma_2+1, \ldots, 0\}$, $\varsigma_2 = max\{\bar{\tau}, \bar{d}\}$, $k_2 = \{1, 2, \ldots, \bar{\mathcal{N}}\}$.

Definition 2: [36] For a given scalar $\gamma > 0$ and matrices $\mathbb{Q}, \mathbb{S}$ and $\mathbb{R}$ with $\mathbb{Q}$ and $\mathbb{R}$ are real symmetric, then the system (1) is strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$ dissipative if for every $\bar{\mathcal{N}} > 0$ under zero initial state condition satisfies

$$\mathbb{E}\Big\{\sum_{k=0}^{\mathcal{N}}[e^T(k)\mathbb{Q}e(k) + 2e^T(k)\mathbb{S}\nu(k) + \nu^T(k)[\mathbb{R}]\nu(k)]\Big\} \geq \gamma\sum_{k=0}^{\bar{\mathcal{N}}}\nu^T(k)\nu(k).$$

Also, for convenience, it is assumed that $\mathbb{Q} \leq 0$.

III. MAIN RESULTS

In this section, we aim to establish some sufficient conditions for the existence of non-fragile asynchronous fault detection filter design that ensure the stochastic finite-time boundedness of the augmented asynchronous fault detection filtering error system (9) with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance. Moreover, the desired non-fragile asynchronous fault detection filter system will be established in the form of (7) such that system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance even in the presence of time delay, missing measurements and the random jumping fault signal.

In the following theorem, the stochastic finite-time boundedness along with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance of system (9) with known filter gain parameters is examined.

Theorem 1: Let Assumption 1 holds. Let $d, \mu, a, b, \epsilon_1, \mathcal{C}_1 > 0$ be known positive scalars and $\mathbb{Q} \leq 0$, $\mathbb{S}$, $\mathbb{R} = \mathbb{R}^T$, $\mathcal{J} \geq 0$ be known constant matrix. The augmented asynchronous fault detection filtering error system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance subject to $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{N}, \gamma, \mathcal{J}, \varphi)$ if there exist positive definite matrices P_m, $\bar{P}_m$, Q_1 and positive scalars

χ_a $(a = 1, 2, 3)$ such that the following LMIs hold for any $m, n \in \mathcal{M}_1,\ p, q \in \mathcal{M}_2$:

$$[\Psi]_{8\times 8} < 0, \tag{10}$$

$$\begin{bmatrix} -\bar{P}_m & \bar{P}_m \\ * & -\frac{a}{3}I \end{bmatrix} < 0, \tag{11}$$

$$\upsilon \mathcal{C}_1 + \chi_W \varphi < \mathcal{C}_2 \chi_1 \mu^{-k}, \tag{12}$$

$$\chi_1 I \leq \hat{P}_m \leq \chi_2 I,\ 0 < \hat{Q}_1 < \chi_3 I, \tag{13}$$

where

$$\Psi_{1,1} = -P_m + Q_1,\ \Psi_{1,4} = \sqrt{3}\bar{A}_{mp}^T, \Psi_{1,5} = \Theta_\alpha \tilde{A}_{mp}^T,$$
$$\Psi_{1,6} = \sqrt{-\epsilon_1^{-1}}\bar{A}_{dm}^T, \Psi_{1,7} = b\bar{G}_m^T + \sqrt{\epsilon_1}\bar{G}_m^T,$$
$$\Psi_{1,8} = \bar{C}_{mp}^T\sqrt{-\mathbb{Q}} + \Theta_\alpha \tilde{C}_m^T \sqrt{-\mathbb{Q}},\ \Psi_{2,2} = -\mu^d Q_1,$$
$$\Psi_{2,4} = 2\bar{A}_{dm}^T,\ \Psi_{2,7} = (\sqrt{\epsilon_1} + b)\bar{G}_{dm}^T,\ \Psi_{3,3} = -\mathbb{R} + \gamma I$$
$$- 2\tilde{D}_m^T\mathbb{S},\ \Psi_{3,4} = 2\bar{B}_{mp}^T,\ \Psi_{3,7} = (\sqrt{\epsilon_1} + b)\bar{L}_m^T,$$
$$\Psi_{4,4} = -\bar{P}_m^{-1},\ \Psi_{5,5} = -\bar{P}_m^{-1},\ \Psi_{6,6} = -I,\ \Psi_{7,7} = -I,$$
$$\Psi_{8,8} = -I, \bar{P}_m = \sum_{n\in\mathcal{M}_1}\sum_{q\in\mathcal{M}_q} \pi_{mn}(k)\phi_{pq}^n(k)P_n,$$
$$\upsilon = \chi_2 + d\chi_3.$$

Proof: Let us first define the Lyapunov-Krasovskii functional to prove the required results as follows:

$$\mathbb{V}(k) = \eta^T(k)\mathcal{P}_m\eta(k) + \sum_{s=k-d}^{k-1}\mu^{k-s-1}\eta^T(k)Q_1\eta(k). \tag{14}$$

Then, by calculating the differences of $\mathbb{V}(k)$ along with the trajectories of system (9) and taking mathematical expectation, we obtain

$$\begin{aligned}
\mathbb{E}\{\Delta\mathbb{V}(k) - (\mu-1)\mathbb{V}(k)\} =& \mathbb{E}\Big\{\eta^T(k+1)(\sum_{n\in\mathcal{M}_1}\sum_{q\in\mathcal{M}_q} \\
&\times \pi_{mn}(k)\phi_{pq}^n(k)P_n)\eta(k+1) - \mu\eta^T(k)P_m\eta(k) \\
&+ \sum_{s=k+1-d}^{k}\mu^{k-s}\eta^T(s)Q_1\eta(s) - \sum_{s=k-d}^{k-1}\mu^{k-s-1}\eta^T(s)Q_1\eta(s)\Big\} \\
=&\mathbb{E}\Big\{[\bar{A}_{mp}\eta(k) + \tilde{\alpha}(k)\tilde{A}_m\eta(k) + \bar{A}_{dm}\eta(k-d) + \bar{B}_{mp}\nu(k) \\
&+ \bar{\zeta}(k)]^T\bar{P}_m[\bar{A}_{mp}\eta(k) + \tilde{\alpha}(k)\tilde{A}_m\eta(k) + \bar{A}_{dm}\eta(k-d) \\
&+ \bar{B}_{mp}\nu(k) + \bar{\zeta}(k)] - \mu\eta^T(k)P_m\eta(k) \\
&+ \eta^T(k)Q_1\eta(k) - \mu^d\eta^T(k-d)Q_1\eta(k-d)\Big\} \\
\leq&\mathbb{E}\Big\{\eta^T(k)(\bar{A}_{mp} + \tilde{\alpha}(k)\tilde{A}_m)^T\bar{P}_m(\bar{A}_{mp} + \tilde{\alpha}(k)\tilde{A}_m)\eta(k) \\
&+ 2\eta^T(k)(\bar{A}_{mp} + \tilde{\alpha}(k)\tilde{A}_m)^T\bar{P}_m\bar{A}_{dm}\eta(k-d) + 2\eta^T(k) \\
&\times (\bar{A}_{mp} + \tilde{\alpha}(k)\tilde{A}_m)^T\bar{P}_m\bar{B}_{mp}\nu(k) + 2\eta^T(k)(\bar{A}_{mp} \\
&+ \tilde{\alpha}(k)\tilde{A}_m)^T\bar{P}_m\bar{\zeta}(k) + \eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{A}_{dm}\eta(k-d) \\
&+ 2\eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{B}_{mp}\nu(k) + 2\eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{\zeta}(k) \\
&+ \nu^T(k)\bar{B}_{mp}^T\bar{P}_m\bar{B}_{mp}\nu(k) + 2\nu^T(k)\bar{B}_{mp}^T\bar{P}_m\bar{\zeta}(k) + \bar{\zeta}^T(k) \\
&\times \bar{P}_m\bar{\zeta}(k) - \mu\eta^T(k)P_m\eta(k) + \eta^T(k)Q_1\eta(k) \\
&- \mu^d\eta^T(k-d)Q_1\eta(k-d)\Big\}.
\end{aligned} \tag{15}$$

Now, by implementing Lemma 1 in [16] and [17], we obtain

$$\begin{aligned}
2\eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{\zeta}(k) \leq& \epsilon_1^{-1}\eta^T(k-d)\bar{A}_{dm}\eta(k-d) \\
&+ \epsilon_1\bar{\zeta}^T(k)\bar{\zeta}(k), \\
\eta^T(k)\bar{A}_{mp}^T\bar{P}_m\bar{A}_{dm}\eta(k-d) \leq& \eta^T(k)\bar{A}_{mp}^T\bar{P}_m\bar{A}_{mp}\eta(k) \\
&+ \eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{A}_{dm}\eta(k-d).
\end{aligned}$$

Further, by using similar analysis for the remaining term in (15), we obtain

$$\begin{aligned}
\mathbb{E}\{\Delta V(k) - (\mu-1)V(k)\} \leq& \eta^T(k)[4\bar{A}_{mp}^T\bar{P}_m\bar{A}_{mp} + \Theta_\alpha^2\tilde{A}_m^T\bar{P}_m \\
&\times \tilde{A}_m - \mu P_m + \epsilon_1\bar{G}_m^T\bar{G}_m]\eta(k) \\
&+ \eta^T(k-d)[3\bar{A}_{dm}^T\bar{P}_m\bar{A}_{dm} + \bar{G}_{dm}^T \\
&\times \bar{G}_{dm}]\eta(k-d) + \nu^T(k)[4\bar{B}_{mp}^T\bar{P}_m \\
&\times \bar{B}_{mp} + \epsilon_1\bar{L}_m^T\bar{L}_m]\nu(k) \\
&+ 3\bar{\zeta}^T(k)\bar{P}_m\bar{\zeta}(k).
\end{aligned} \tag{16}$$

Besides, to demonstrate strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance of the system (9), we define performance index in the following form

$$\begin{aligned}
J =&\mathbb{E}\Big\{\sum_{k=0}^{\mathcal{N}}[e^T(k)\mathbb{Q}e(k) + 2e^T(k)\mathbb{S}\nu(k) \\
&+ \nu^T(k)[\mathbb{R} - \gamma I]\nu(k)]\Big\}.
\end{aligned} \tag{17}$$

Then, by combining the inequalities (16) and (17) together with Schur complement, we have

$$\mathbb{E}\{\Delta V(k) - (\mu-1)V(k) - J\} \leq \tilde{J} = \begin{bmatrix} \Xi & -3\bar{\zeta}^T(k)\bar{P}_m \\ * & -3\bar{P}_m \end{bmatrix}, \tag{18}$$

where

$$\begin{aligned}
\Xi =&4\eta^T(k)\bar{A}_{mp}^T\bar{P}_m\bar{A}_{mp}\eta(k) + \Theta_\alpha^2\eta^T(k)\tilde{A}_m^T\bar{P}_m\tilde{A}_m\eta(k) \\
&- \mu\eta^T(k)P_m\eta(k) + \epsilon_1\eta^T(k)\bar{G}_m^T\bar{G}_m\eta(k) \\
&- \eta^T(k)\bar{C}_{mp}^T\mathbb{Q}\bar{C}_{mp}\eta(k) - \Theta_\alpha^2\eta^T(k)\tilde{C}_m^T\mathbb{Q}\tilde{C}_m\eta(k) \\
&- 2\eta^T(k)\bar{C}_{mp}^T\mathbb{S}\nu(k) + 3\eta^T(k-d)\bar{A}_{dm}^T\bar{P}_m\bar{A}_{dm}\eta(k-d) \\
&+ \eta^T(k-d)\bar{G}_{dm}^T\bar{G}_{dm}\eta(k-d) + \eta^T(k)Q\eta(k) \\
&- \mu^d\eta^T(k-d)Q\eta(k-d) + 4\nu^T(k)\bar{B}_{mp}^T\bar{P}_m\bar{B}_{mp}\nu(k) \\
&- \nu^T(k)[\mathbb{R} - I]\nu(k) + \epsilon_1\nu^T(k)\bar{L}_m^T\bar{L}_m\nu(k) \\
&+ 2\nu^T(k)\bar{D}_m^T\mathbb{S}\nu(k).
\end{aligned}$$

On the other hand, by considering two non-negative scalars a and b with $a - b < 0$, the following formula is established

$$-2b\bar{\zeta}^T(k)\bar{\zeta}(k) + a\bar{\zeta}^T(k)\bar{\zeta}(k) < 0. \tag{19}$$

Using Schur complement lemma in (19), we have

$$\begin{bmatrix} -2b\bar{\zeta}^T(k)\bar{\zeta}(k) & 0 \\ * & -9a^{-1}\bar{P}_m^2 \end{bmatrix} < \begin{bmatrix} 0 & 3\bar{\zeta}^T(k)\bar{p}_m \\ * & 0 \end{bmatrix}. \tag{20}$$

From the inequality (18), we have $\tilde{J} < 0$, that is

$$\begin{bmatrix} \Xi & 0 \\ * & -3\bar{P}_m \end{bmatrix} < \begin{bmatrix} 0 & -3\bar{\zeta}^T(k)\bar{P}_m \\ * & 0 \end{bmatrix}, \tag{21}$$

which is assured by

$$\begin{bmatrix} \Xi & 0 \\ * & -3\bar{P}_m \end{bmatrix} < \begin{bmatrix} -2b\bar{\zeta}^T(k)\bar{\zeta}(k) & 0 \\ * & -9a^{-1}\bar{P}_m^2 \end{bmatrix}. \quad (22)$$

Then, inequality (22) can be modified as

$$\begin{bmatrix} \Xi_1 & 0 \\ * & \Xi_2 \end{bmatrix} < 0, \quad (23)$$

where

$$\Xi_1 = \Xi + 2b\bar{\zeta}^T(k)\bar{\zeta}(k) < 0, \quad (24)$$

$$\Xi_2 = -3\bar{P}_m + 9a^{-1}\bar{P}_m^2 < 0. \quad (25)$$

Thus, LMI (11) holds, by implementing Schur complement to inequality (25). Moreover, from inequality (24) along with Schur complement lemma, we get

$$\mathbb{E}\{\Delta V(k) - (\mu-1)V(k) - J\} \leq \psi^T(k)\Psi\psi(k), \quad (26)$$

where $\psi^T(k) = \begin{bmatrix}\eta^T(k) & \eta^T(k-d) & \nu^T(k)\end{bmatrix}^T$ and the components of Ψ are defined in theorem statement. Therefore, the matrix terms in (26) are equivalent to inequality (10). Hence, if the inequality (10) holds, it is clear that

$$\mathbb{E}\{\Delta V(k) - (\mu-1)V(k) - \nu^T(k)W\nu(k)\} \leq 0,$$
$$\mathbb{E}\{V(k+1) - V(k)\} \leq (\mu-1)\mathbb{E}\{V(k)\} + \mathbb{E}\{\nu^T(k)W\nu(k)\},$$
$$\mathbb{E}\{V(k+1)\} \leq \mu\mathbb{E}\{V(k)\} + \chi_W\mathbb{E}\{\nu^T(k)\nu(k)\}, \quad (27)$$

where $\chi_W = \chi_{\max}(W)$. In addition, there exists $\mu \geq 1$ which would be a non negative scalar and from Assumption 1, it can be observed that

$$\mathbb{E}\{\Delta V(k)\} \leq \mu^k\mathbb{E}\{V(0)\} + \chi_W\mathbb{E}\Big\{\sum_{s=0}^{k-1}\mu^{k-s-1}\nu^T(s)\nu(s)\Big\}$$
$$\leq \mu^k\mathbb{E}\{V(0)\} + \mu^k\chi_W\varphi. \quad (28)$$

Moreover, from the Lyapunov-Krasovskii functional (14), we obtain

$$V(0) = \mathbb{E}\{\eta^T(0)\mathcal{P}_m\eta(0)\} + \mathbb{E}\Big\{\sum_{s=-d}^{-1}\mu^{-s-1}\eta^T(0)Q_1\eta(0)\Big\}.$$

By letting $\hat{P}_m = \mathcal{J}^{-1/2}\mathcal{P}_m\mathcal{J}^{-1/2}$ and $\hat{Q} = \mathcal{J}^{-1/2}Q_1\mathcal{J}^{-1/2}$, we have

$$V(0) \leq \mathbb{E}\{\eta^T(0)\mathcal{J}^{1/2}\hat{P}_m\mathcal{J}^{1/2}\eta(0)\}$$
$$+ \mathbb{E}\Big\{\sum_{s=-d}^{-1}\mu^{-s-1}\eta^T(0)\mathcal{J}^{1/2}\hat{Q}_1\mathcal{J}^{1/2}\eta(0)\Big\}$$
$$\leq \{\chi_2 + \chi_3 d\}\mathcal{C}_1 \leq \upsilon\mathcal{C}_1, \quad (29)$$

where $\chi_1 = \chi_{min}\{P_m\}$, $\chi_2 = \chi_{max}\{P_m\}$ and $\chi_2 = \chi_{max}\{Q\}$. Moreover, it follows from (14) that $\mathbb{E}\{V(k)\} \geq \mathbb{E}\{\eta^T(k)P_m\eta(k)\} \geq \mathbb{E}\{\eta^T(k)\mathcal{J}^{1/2}\hat{P}_m\mathcal{J}^{1/2}\eta(k)\} \geq \chi_1\mathbb{E}\{\eta^T(k)\mathcal{J}\eta(k)\}$. Then, it is clear to get that $\mathbb{E}\{\eta^T(k)\mathcal{J}\eta(k)\} \leq \frac{(\upsilon\mathcal{C}_1+\chi_W\varphi)\mu^k}{\chi_1}$. Furthermore, from inequality (12), it is obvious that $\mathbb{E}\{\eta^T(k)\mathcal{J}\eta(k)\} < \mathcal{C}_2$ for every $k \in \{1,2,\ldots,\mathcal{N}\}$. Thus, by the Definition 1 in [25] we conclude that the augmented asynchronous fault detection filtering error system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q},\mathbb{S},\mathbb{R})-\gamma$ dissipative performance subject to $(\mathcal{C}_1,\mathcal{C}_2,\mathcal{N},\gamma,\mathcal{J},\varphi)$. This completes the proof. □

The derived constrains in Theorem 1 shows the stochastic finite-time boundedness with strictly $(\mathbb{Q},\mathbb{S},\mathbb{R})-\gamma$ dissipative performance of system (9). Furthermore, the following theorem ensures that the system (9) with unknown filter gain parameters is stochastically finite-time boundedness with strictly $(\mathbb{Q},\mathbb{S},\mathbb{R})-\gamma$ dissipative performance.

Theorem 2: Let Assumption 1 holds. Let $d, \mu, a, b, \epsilon_1, \mathcal{C}_1 > 0$ be known positive scalars and $\mathbb{Q} \leq 0$, $\mathbb{S}$, $\mathbb{R} = \mathbb{R}^T$, $\mathcal{J} \geq 0$ be known constant matrix, then the augmented asynchronous fault detection filtering error system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q},\mathbb{S},\mathbb{R})-\gamma$ dissipative performance subject to $(\mathcal{C}_1,\mathcal{C}_2,\mathcal{N},\gamma,\mathcal{J},\varphi)$ if there exist positive definite matrices P_{1m}, P_{2m}, P_{3m}, $\bar{P}_{1m}$, $\bar{P}_{2m}$, $\bar{P}_{3m}$, Q_{11}, Q_{12}, Q_{22}, Y_1, Y_2, Y_3 and positive scalars χ_a $(a = 1,2,3)$ such that below mentioned LMIs hold together with (12) for any $m,n \in \mathcal{M}_1$, $p,q \in \mathcal{M}_2$:

$$\begin{bmatrix} [\hat{\Psi}]_{16\times16} & \beta_1\mathcal{N}_{ap}^T & \mathcal{M}_a & \beta_2\mathcal{N}_{bp}^T & \mathcal{M}_b \\ * & -\beta_1 & 0 & 0 & 0 \\ * & * & -\beta_1 & 0 & 0 \\ * & * & * & -\beta_2 & 0 \\ * & * & * & * & -\beta_2 \end{bmatrix} < 0, \quad (30)$$

$$\begin{bmatrix} -\bar{P}_{1m} & -\bar{P}_{2m} & \bar{P}_{1m} & \bar{P}_{2m} \\ * & -\bar{P}_{3m} & \bar{P}_{2m}^T & \bar{P}_{3m} \\ * & * & -\frac{a}{3}I & 0 \\ * & * & * & -\frac{a}{3}I \end{bmatrix} < 0, \quad (31)$$

where

$\hat{\Psi}_{1,1} = -\mu P_{1m} + Q_{11}$, $\hat{\Psi}_{1,2} = -\mu P_{2m} + Q_{12}$,
$\hat{\Psi}_{1,5} = -\bar{\alpha}C_m^T\mathbb{S}$, $\hat{\Psi}_{1,8} = 2A_m^TY_1^T + \bar{\alpha}C_m^T\mathbb{B}_{fp}^T$,
$\hat{\Psi}_{1,9} = 2A_m^TY_3^T + \bar{\alpha}C_m^T\mathbb{B}_{fp}^T$, $\hat{\Psi}_{1,10} = \Theta_\alpha C_m^T\mathbb{B}_{fp}^T$,
$\hat{\Psi}_{1,11} = \Theta_\alpha C_m^T\mathbb{B}_{fp}^T$, $\hat{\Psi}_{1,12} = \sqrt{\epsilon_1^{-1}}A_{dm}^T$,
$\hat{\Psi}_{1,14} = (\sqrt{\epsilon_1}+b)G_m^T$, $\hat{\Psi}_{1,15} = \bar{\alpha}C_m^T\sqrt{-\mathbb{Q}}$,
$\hat{\Psi}_{1,16} = \Theta_\alpha C_m^T\sqrt{-\mathbb{Q}}$, $\hat{\Psi}_{2,2} = -\mu P_{3m} + Q_{22}$,
$\hat{\Psi}_{2,5} = C_{fp}^T\mathbb{S}$, $\hat{\Psi}_{2,8} = 2\mathbb{A}_{fp}^T$, $\hat{\Psi}_{2,9} = 2\mathbb{A}_{fp}^T$,
$\hat{\Psi}_{2,15} = -C_{fp}^T\sqrt{\mathbb{Q}}$, $\hat{\Psi}_{3,3} = -Q_{11}$, $\hat{\Psi}_{3,4} = -Q_{12}$,
$\hat{\Psi}_{3,8} = \sqrt{3}A_{dm}^TY_1^T$, $\hat{\Psi}_{3,9} = \sqrt{3}A_{dm}^TY_3^T$,
$\hat{\Psi}_{3,14} = (\sqrt{\epsilon_1}+b)G_{dm}^T$, $\hat{\Psi}_{4,4} = -Q_{22}$, $\hat{\Psi}_{5,5} = -\mathbb{R} + \gamma I$,
$\hat{\Psi}_{5,8} = 2B_m^Ty_1^T$, $\hat{\Psi}_{5,9} = 2B_m^Ty_3^T$, $\hat{\Psi}_{6,6} = -\mathbb{R} + \gamma I - 2G_{wm}^T\mathbb{S}$,
$\hat{\Psi}_{6,8} = 2E_m^TY_1^T + D_m^T\mathbb{B}_{fp}^T$, $\hat{\Psi}_{6,9} = 2E_m^TY_3^T + D_m^T\mathbb{B}_{fp}^T$,
$\hat{\Psi}_{6,14} = (\sqrt{\epsilon_1}+b)G_{wm}^T$, $\hat{\Psi}_{6,15} = D_m^T\sqrt{-\mathbb{Q}}$,
$\hat{\Psi}_{7,7} = -\mathbb{R} + \gamma I - 2L_m^T\mathbb{S}$, $\hat{\Psi}_{7,8} = 2F_m^TY_1^T + H_m^T\mathbb{B}_{fp}^T$,
$\hat{\Psi}_{7,9} = 2F_m^TY_3^T + H_m^T\mathbb{B}_{fp}^T$, $\hat{\Psi}_{7,14} = (\sqrt{\epsilon_1}+b)L_m^T$,
$\hat{\Psi}_{7,15} = D_m^T\sqrt{-\mathbb{Q}}$, $\hat{\Psi}_{8,8} = \bar{P}_{1m} - Y_1^T - Y_1$,
$\hat{\Psi}_{8,9} = \bar{P}_{2m} - Y_3^T - Y_2$, $\hat{\Psi}_{9,9} = \bar{P}_{3m} - Y_2^T - Y_2$,

$\hat{\Psi}_{10,10} = \bar{P}_{1m} - Y_1^T - Y_1,\ \hat{\Psi}_{10,11} = \bar{P}_{2m} - Y_3^T - Y_2,$
$\hat{\Psi}_{11,11} = \bar{P}_{3m} - Y_2^T - Y_2,\ \hat{\Psi}_{12,12} = \hat{\Psi}_{13,13} = \hat{\Psi}_{14,14} = -I,$

$$\mathcal{N}_{ap} = [0 \quad 2N_{ap} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \underbrace{0 \cdots 0}_{5}],$$

$$\mathcal{M}_a = [\underbrace{0 \cdots 0}_{7} \quad M_{ap}^T Y_2^T \quad M_{ap}^T Y_2^T \quad \underbrace{0 \cdots 0}_{7}]^T,$$

$$\mathcal{N}_{bp} = [(\bar{\alpha} + \Theta_\alpha + 1) N_{bp} C_m \ \underbrace{0 \cdots 0}_{4} \ N_{bp} D_m \ N_{bp} H_m \ \underbrace{0 \cdots 0}_{9}],$$

$$\mathcal{M}_b = [\underbrace{0 \cdots 0}_{7} \quad M_{ap}^T Y_2^T \quad M_{ap}^T Y_2^T \ M_{ap}^T Y_2^T \quad M_{ap}^T Y_2^T \quad \underbrace{0 \cdots 0}_{7}]^T.$$

Moreover, the gain matrices of the non-fragile asynchronous fault detection filter are given as $A_{fp} = Y_2^{-1} \mathbb{A}_{fp}$, $B_{fp} = Y_2^{-1} \mathbb{B}_{fp}$ and $\mathbb{C}_{fm} = C_{fp}$.

Proof: To ensure the required results, we consider the matrices in the form as $\bar{P}_m = \begin{bmatrix} \bar{P}_{m1} & \bar{P}_{m2} \\ \bar{P}_{m2}^T & \bar{P}_{m3} \end{bmatrix}$, $P_m = \begin{bmatrix} P_{1m} & P_{2m} \\ P_{2m}^T & P_{3m} \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_2 \end{bmatrix}$ and $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$. Further, by letting $\mathbb{A}_{fm} = A_{fm} y_2^T$, $\mathbb{B}_{fm} = B_{fm} Y_2^T$ and $\mathbb{C}_{fm} = C_{fm}$, using the above given partition matrices $\bar{P}_m, P_m, Q, Y$ and Lemma 2 in [11] to the inequality (10) along with the parameter uncertainties defined in (8), we have

$$\tilde{\Psi} = \hat{\Psi}_{11\times 11} + \beta_1 \mathcal{N}_{ap}^T \Delta(k) \mathcal{M}_a + \beta_2 \mathcal{M}_b^T \Delta(k) \mathcal{N}_{bp}, \quad (32)$$

where the factors of $\hat{\Psi}_{11\times 11}$, $\mathcal{N}_{ap}$, $\mathcal{N}_{bp}$, $\mathcal{M}_a$ and $\mathcal{M}_b$ are defined in Theorem 2. On the other hand, by implementing Lemma 1 , the terms in (32) can be rewritten as

$$\begin{aligned} \tilde{\Psi} =& \hat{\Psi}_{11\times 11} + \beta_1 \mathcal{N}_{ap}^T \mathcal{N}_{ap} + \beta_1 \mathcal{M}_a \mathcal{M}_a^T + \\ & \beta_2 \mathcal{N}_{bp}^T \mathcal{N}_{bp} + \beta_2 \mathcal{M}_2 \mathcal{M}_2^T. \end{aligned} \quad (33)$$

The expression in (33) appears equivalent to the matrix terms in (30). Thus, from (31) we accomplish that Y is non-singular. Therefore, system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance subject to $(\mathcal{C}_1, \mathcal{C}_2, \mathbb{N}, \gamma, \mathcal{J}, \varphi)$, if the LMIs in (30) hold together with (12). Hence, proof of this theorem is completed. □

Remark 2: The main contribution of this paper is to desin a finite-time dissipative based asynchronous fault detection filter for conic-type nonlinear semi-Markovian jump systems with time-delay and random jumping fault signal. As pointed out in Remark 1, the conic-type nonlinearity is more general than the global Lipschitz nonlinearities. Besides, it should be emphasized that in order to overcome the difficulties in dealing with the conic-type nonlinearity with time-delay, random jumping fault signal and semi-Markov process in system (9), a new Lyapunov function (14) is employed to achieve finite-time dissipative based asynchronous fault detection filter design. In addition, there exist some control design for conic-type nonlinearities which is addressed in continuous context [16], [18]. Further, it is noted that the aforementioned results are based on conic-type nonlinearities but asynchronous fault detection filter is not reported for semi-Markovian jump systems.

Here, if we assume the conic-type nonlinear SMJSs (1) without time delay, then the augmented asynchronous fault detection filtering error system is given in the following form

$$\begin{aligned} \eta(k+1) =& \bar{A}_{mp} \eta(k) + \tilde{\alpha}(k) \tilde{A}_m \eta(k) + \bar{B}_{mp} \nu(k) + \bar{\zeta}(k), \\ e(k) =& \bar{C}_{mp} \eta(k) + \tilde{\alpha}(k) \tilde{C}_m \eta(k) + \bar{D}_m \nu(k), \end{aligned} \quad (34)$$

Corollary 1: Let Assumption 1 holds. Let d, μ, a, b, ϵ_1, $\mathcal{C}_1 > 0$ be known positive scalars and $\mathbb{Q} \leq 0$, $\mathbb{S}$, $\mathbb{R} = \mathbb{R}^T$, $\mathcal{J} \geq 0$ be known constant matrix then the augmented asynchronous fault detection filtering error system (34) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance subject to $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{N}, \gamma, \mathcal{J}, \varphi)$ if there exist positive definite matrices P_{1m},P_{2m}, P_{3m}, $\bar{P}_{1m}$, $\bar{P}_{2m}$, $\bar{P}_{3m}$, Y_1, Y_2, Y_3 and positive scalars χ_a $(a = 1, 2)$ such that below mentioned LMIs hold for any $m, n \in \mathcal{M}_1,\ p, q \in \mathcal{M}_2$:

$$\begin{bmatrix} [\widehat{\Psi}]_{12\times 12} & \beta_1 \widehat{\mathcal{N}}_{ap}^T & \widehat{\mathcal{M}}_a & \beta_2 \widehat{\mathcal{N}}_{bp}^T & \widehat{\mathcal{M}}_b \\ * & -\beta_1 & 0 & 0 & 0 \\ * & * & -\beta_1 & 0 & 0 \\ * & * & * & -\beta_2 & 0 \\ * & * & * & * & -\beta_2 \end{bmatrix} < 0, \quad (35)$$

$$\begin{bmatrix} -\bar{P}_{1m} & -\bar{P}_{2m} & \bar{P}_{1m} & \bar{P}_{2m} \\ * & -\bar{P}_{3m} & \bar{P}_{2m}^T & \bar{P}_{3m} \\ * & * & -\frac{a}{3} I & 0 \\ * & * & * & -\frac{a}{3} I \end{bmatrix} < 0, \quad (36)$$

$$\upsilon \mathcal{C}_1 + \chi_W \chi_2 < \mathcal{C}_2 \chi_1 \mu^{-k}, \quad (37)$$

$$\chi_1 \leq \hat{P}_m \leq \chi_2, \quad (38)$$

where

$\widehat{\Psi}_{1,1} = -\mu P_{1m},\ \widehat{\Psi}_{1,2} = -\mu P_{2m},\ \widehat{\Psi}_{1,3} = -\bar{\alpha} C_m^T \mathbb{S},$
$\widehat{\Psi}_{1,6} = 2A_m^T Y_1^T + \bar{\alpha} C_m^T \mathbb{B}_{fp}^T,\ \widehat{\Psi}_{1,7} = 2A_m^T Y_3^T + \bar{\alpha} C_m^T \mathbb{B}_{fp}^T,$
$\widehat{\Psi}_{1,8} = \Theta_\alpha C_m^T \mathbb{B}_{fp}^T, \widehat{\Psi}_{1,9} = \Theta_\alpha C_m^T \mathbb{B}_{fp}^T, \widehat{\Psi}_{1,10} = (\sqrt{\epsilon_1} + b) G_m^T,$
$\widehat{\Psi}_{1,11} = \bar{\alpha} C_m^T \sqrt{-\mathbb{Q}},\ \widehat{\Psi}_{1,12} = \Theta_\alpha C_m^T \sqrt{-\mathbb{Q}},$
$\widehat{\Psi}_{2,2} = -\mu P_{3m} + Q_{22},\ \widehat{\Psi}_{2,3} = C_{fp}^T \mathbb{S},\ \widehat{\Psi}_{2,6} = 2\mathbb{A}_{fp}^T,$
$\widehat{\Psi}_{2,7} = 2\mathbb{A}_{fp}^T,\ \widehat{\Psi}_{2,11} = -C_{fp}^T \sqrt{\mathbb{Q}},\ \widehat{\Psi}_{3,3} = -\mathbb{R} + \gamma I,$
$\widehat{\Psi}_{3,6} = 2B_m^T y_1^T, \widehat{\Psi}_{3,7} = 2B_m^T y_3^T, \widehat{\Psi}_{4,4} = -\mathbb{R} + \gamma I - 2G_{wm}^T \mathbb{S},$
$\widehat{\Psi}_{4,6} = 2E_m^T Y_1^T + D_m^T \mathbb{B}_{fp}^T,\ \widehat{\Psi}_{4,7} = 2E_m^T Y_3^T + D_m^T \mathbb{B}_{fp}^T,$
$\widehat{\Psi}_{4,10} = (\sqrt{\epsilon_1} + b) G_{wm}^T,\ \widehat{\Psi}_{4,11} = D_m^T \sqrt{-\mathbb{Q}},$
$\widehat{\Psi}_{5,5} = -\mathbb{R} + \gamma I - 2L_m^T \mathbb{S},\ \widehat{\Psi}_{5,6} = 2F_m^T Y_1^T + H_m^T \mathbb{B}_{fp}^T,$
$\widehat{\Psi}_{5,7} = 2F_m^T Y_3^T + H_m^T \mathbb{B}_{fp}^T,\ \widehat{\Psi}_{5,10} = (\sqrt{\epsilon_1} + b) L_m^T,$
$\widehat{\Psi}_{5,11} = D_m^T \sqrt{-\mathbb{Q}},\ \widehat{\Psi}_{6,6} = \bar{P}_{1m} - Y_1^T - Y_1,$
$\widehat{\Psi}_{6,7} = \bar{P}_{2m} - Y_3^T - Y_2,\ \widehat{\Psi}_{7,7} = \bar{P}_{3m} - Y_2^T - Y_2,$
$\widehat{\Psi}_{8,8} = \bar{P}_{1m} - Y_1^T - Y_1,\ \widehat{\Psi}_{8,9} = \bar{P}_{2m} - Y_3^T - Y_2,$
$\widehat{\Psi}_{9,9} = \bar{P}_{3m} - Y_2^T - Y_2,\ \widehat{\Psi}_{10,10} = \widehat{\Psi}_{11,11} = \widehat{\Psi}_{12,12} = -I,$

$$\widehat{\mathcal{N}}_{ap} = [0 \quad 2N_{ap} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \underbrace{0 \cdots 0}_{5}],$$

$$\widehat{\mathcal{M}}_a = [\underbrace{0 \cdots 0}_{5} \quad M_{ap}^T Y_2^T \quad M_{ap}^T Y_2^T \quad \underbrace{0 \cdots 0}_{5}]^T,$$

$$\widehat{\mathcal{N}}_{bp} = [(\bar{\alpha} + \Theta_\alpha + 1)N_{bp}C_m \; 0 \; 0 \; N_{bp}D_m \; N_{bp}H_m \; \underbrace{0 \cdots 0}_{7}],$$

$$\widehat{\mathcal{M}}_b = [\underbrace{0 \cdots 0}_{5} \; M_{ap}^T Y_2^T \; M_{ap}^T Y_2^T \; M_{ap}^T Y_2^T \; M_{ap}^T Y_2^T \; \underbrace{0 \cdots 0}_{3}]^T.$$

Moreover, the gain matrices of the non-fragile asynchronous fault detection filter are given as $A_{fp} = Y_2^{-1}\mathbb{A}_{fp}$, $B_{fp} = Y_2^{-1}\mathbb{B}_{fp}$ and $\mathbb{C}_{fm} = C_{fp}$.

Proof: The proof of this corollary is similar to Theorem 2 and hence it is neglected. □

IV. VALIDATION

In this section, we present two numerical examples such as Pulse-Width-Modulation (PWM)-driven boost converter model and R-L-C circuit model to show the efficiency of proposed non-fragile asynchronous fault detection filter design for conic-type nonlinear SMJSs with and without time delay, respectively. For the numerical purpose, we use MATLAB LMI control toolbox to solve the LMIs obtained in the previous section.

Example 1: Consider a PWM-driven boost converter model in FIGURE 1 from [27]. Here, $s(t)$ specifies a switching

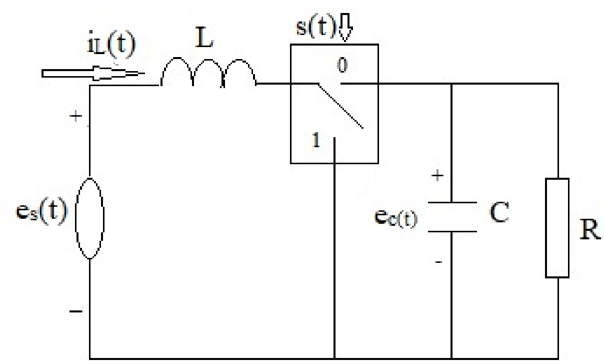

FIGURE 1: Pulse-Width-Modulation-driven boost converter model

mode which is operated by a PWM-driven boost converter, the inductance is denoted by L, C indicates the capacitance, R denotes the load resistance, the current is represented by $i_L(t)$ over the inductance, $e_s(t)$ represents the capacitor's terminal voltage and every cycle interval is denoted by T where its switch will occur only once during this period. Moreover, PWM system manages the switch mode $\varrho(k)$ and it follows a semi-Markov series. Furthermore, we take $\kappa = t/T$, $L_1 = I/T$ and $C_1 = C/T$. Then, PWM-driven booster model equation is expressed by

$$\dot{e}_C(\kappa) = -\frac{1}{RC_1}e_C(\kappa) + (1 - s(\kappa))\frac{1}{C_1}i_L(\kappa) \tag{39}$$

$$\dot{i}_L(\kappa) = -(1 - s(\kappa))\frac{1}{L_1}e_C(\kappa) + s(\kappa)\frac{1}{L_1}e_s(\kappa). \tag{40}$$

Now the above equation can be rewritten as

$$\dot{x} = A_\kappa^c x, \quad \kappa \in \{1, 2\}, \tag{41}$$

where $x = [e_C, i^L, 1]^T$, $\kappa = 1$ and $\kappa = 2$ represents the modes 1 and 2, respectively.

$$A_1^C = \begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{C_1} & 0 \\ -\frac{1}{L_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2^C = \begin{bmatrix} -\frac{1}{RC_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_1} \\ 0 & 0 & 0 \end{bmatrix}$$

To vouch for the filter design, we assume $L = 1H$, $R = 1\Omega$, $C = 1F$ and $T = 1s$, by setting a specific sampling time $T_s = \frac{T}{10}$, then the system matrices can be obtained as follows:

$$A_1 = \begin{bmatrix} 0.94 & 0.10 & 0.06 \\ -0.30 & 0.95 & -0.30 \\ -0.25 & -0.06 & 0.63 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.93 & 0.08 & 0.07 \\ -0.14 & 0.66 & -0.20 \\ -0.16 & -0.40 & 0.66 \end{bmatrix}.$$

In addition to that, we choose other system matrices as

$$A_{d1} = \begin{bmatrix} -0.01 & 0.056 & 0.1 \\ 0.012 & -0.03 & 0.3 \\ 0.2 & 0.4 & 0.1 \end{bmatrix}, \; A_{d2} = \begin{bmatrix} -0.01 & -0.02 & 0.3 \\ -0.01 & -0.03 & 0.01 \\ 0.3 & 0.02 & 0.03 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.01508 \\ -0.085 \\ -0.0732 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.01045 \\ -0.040 \\ -0.3787 \end{bmatrix}, \; C_1 = \begin{bmatrix} -0.0472 \\ -0.0277 \\ -0.0157 \end{bmatrix}^T,$$

$$C_2 = \begin{bmatrix} -0.012 \\ -0.0611 \\ -0.0379 \end{bmatrix}^T, \; E_1 = \begin{bmatrix} 0.1 \\ 0.4 \\ 0.3 \end{bmatrix}, \; E_2 = \begin{bmatrix} 0.1 \\ 0.4 \\ 0.2 \end{bmatrix}, \; F_1 = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.2 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, \; G_1 = G_2 = \begin{bmatrix} 0.01 \\ -0.01 \\ 0.01 \end{bmatrix}^T, \; G_{d1} = G_{d2} = \begin{bmatrix} -0.01 \\ -0.01 \\ 0.1 \end{bmatrix},$$

$D_1 = 0.2,\ D_2 = 0.1,\ G_{w1} = G_{w2} = 0.1\ H_1 = 0.5,\ H_2 = 2,$
$L_1 = 0.1,\ L_2 = 0.1.$

Moreover, the uncertain parameters are chosen as

$$M_{a1} = M_{b1} = \begin{bmatrix} -0.01 \\ -0.03 \end{bmatrix}, \; M_{a2} = M_{b2} = \begin{bmatrix} -0.02 \\ -0.01 \end{bmatrix},$$
$$N_{a1} = \begin{bmatrix} -0.01 & -0.01 \end{bmatrix}, \; N_{a2} = \begin{bmatrix} -0.03 & -0.02 \end{bmatrix},$$
$$N_{b1} = 0.01, \; N_{b2} = 0.04.$$

Furthermore, we take constant time delay $d = 2$. Moreover, the transition probability rates of semi-Markov process $\varrho(k)$ with $\lambda = 1, 2$ are chosen as $\pi_{11}(k) \in [0.4 \quad 0.6]$ and $\pi_{22}(k) \in [0.3 \quad 0.7]$. Also, the transition probability rates of semi-Markov process $\theta(k)$ with $\lambda, n = 1, 2$ are assumed as $\phi_{11}^n(k) \in [0.34 \quad 0.66]$, $\phi_{22}^n(k) \in [0.25 \quad 0.75]$. Furthermore, the other parameters are chosen as $\mathcal{Q} = -0.1$, $\mathcal{S} = 0.3$, $\mathcal{R} = 2.2$, $\bar{\alpha} = 0.4$, $a = 4$, $b = 5$, $\mu = 1$, $\epsilon_1 = 0.8$ and $\Delta(k) = 0.03sin(k)$. Now, by using aforementioned parameters and solving the LMIs in Theorem 2, the dissipative performance level $\gamma = 0.4011$ is obtained and the non-fragile asynchronous fault detection filter gain matrices are calculated as

$$A_{f1} = \begin{bmatrix} 0.0234 & -0.0037 & -0.0054 \\ 0.0081 & 0.0178 & -0.0072 \\ 0.0050 & -0.0067 & 0.0177 \end{bmatrix}, \; B_{f1} = \begin{bmatrix} -0.1014 \\ -0.1489 \\ -0.0747 \end{bmatrix},$$

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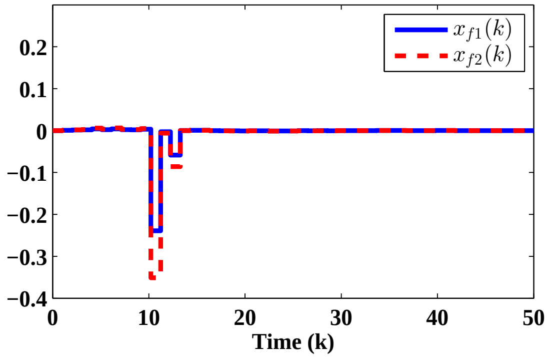
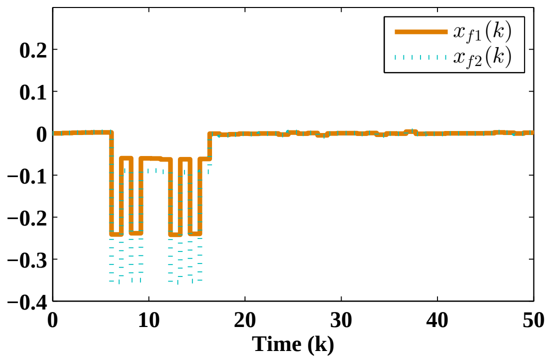
$$A_{f2} = \begin{bmatrix} 0.0216 & -0.0047 & -0.0064 \\ 0.0085 & 0.0192 & -0.0082 \\ 0.0054 & -0.0077 & 0.0193 \end{bmatrix}, \; B_{f2} = \begin{bmatrix} -0.1003 \\ -0.1472 \\ -0.0739 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} 0.0102 \\ -0.0164 \\ -0.0367 \end{bmatrix}^T \text{ and } C_{f2} = \begin{bmatrix} 0.0100 \\ -0.0167 \\ -0.0368 \end{bmatrix}^T.$$



(a)

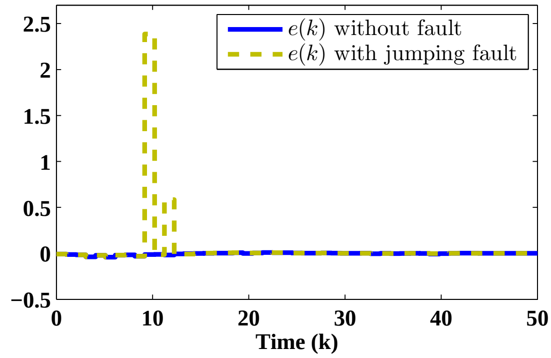


(b)

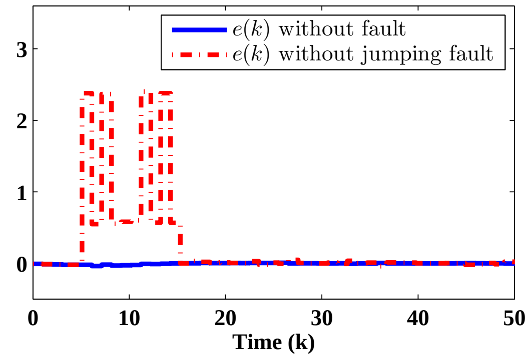
FIGURE 2: Trajectories of filter states. (a) with random jumping fault signal, (b) without random jumping fault signal

We suppose to take the initial condition for the system state and filter state as $x(0) = x_f(0) = [0 \quad 0]^T$. Further we take the disturbance input as $w(k) = \begin{cases} 0.4e^{(-0.1k)} sin(-0.2k) & k \leq 35 \\ 0 & \text{otherwise} \end{cases}$, nonlinear function as $\zeta(k) = \begin{bmatrix} 0.0000075(|(x_1(k)+1)| - |(x_1(k)-1)|) \\ 0 \end{bmatrix}$ and fault signal as $\hat{f}(k) = \begin{cases} 1.2 & 5 \leq k \leq 15 \\ 0 & otherwise \end{cases}$.

Moreover, FIGURE 2 shows the trajectories of the proposed filter states with and without random jumping fault signal. Further, the responses of the error system with random jumping fault signal and without random jumping fault signal are demonstrated in FIGURE 3(a) and FIGURE 3(b), respectively. Also, FIGURE 4(a) exposes the random jumping fault signal $\tilde{f}(k)$ and FIGURE 4(b) shows the fault signal $\hat{f}(k)$. Furthermore, the evolution of the considered system mode is plotted in FIGURE 5 and the evolution of the non-fragile asynchronous fault detection filter mode is expressed in FIGURE 6. Specifically, FIGURE 7 illustrates the residual evaluation function. Whilst, we have selected the threshold value as $\mathrm{J}_{th} = 0.0823$. The residual evaluation function is calculated as $\mathrm{J}_e(k) = \sqrt{\sum_{k=0}^{9} e^T(k)e(k)} = 2.3879$ and it is obvious that $\mathrm{J}_e(k)$ is greater then the threshold J_{th}. On the whole, the fault is detected within the range.



(a)

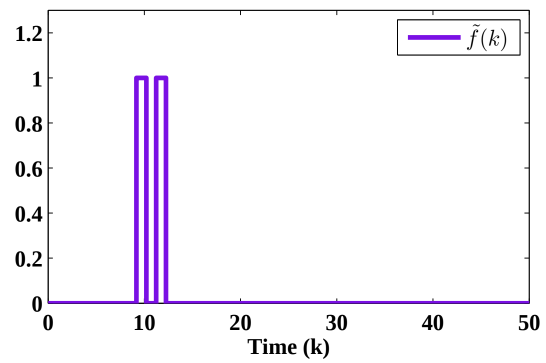


(b)

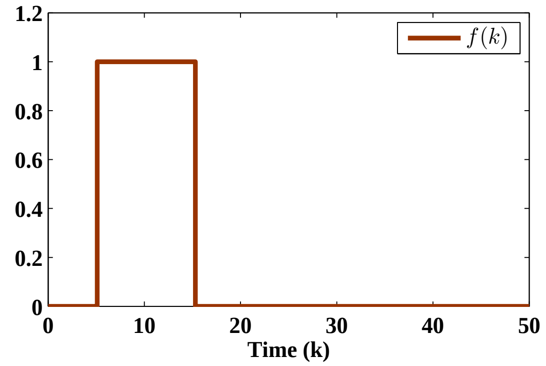
FIGURE 3: The error responses of the system (9). (a) with and without random jumping fault signal, (b) without fault and random jumping fault signal

Finally, the evolution of $x^T(k)\mathcal{J}x(k)$ is depicted in FIGURE 8. Based on FIGURE 8, it is easy to accomplish that the responses of the augmented asynchronous fault detection filtering error system is within the bound $\mathcal{C}_2$. Hence, the augmented asynchronous fault detection filtering error system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance subject to $(0.2, 8.2049, 50, 0.4011, I, 0.01)$ under proposed non-fragile asynchronous fault detection filter design even in the existence of time delay, missing measurements and random jumping fault signal.

Example:2 We consider a R-L-C circuit model to illustrate the efficiency of proposed non-fragile asynchronous fault detection filter problem. Moreover, the modes $\varrho(k)$ and $\theta(k)$ are assumed to obey a semi-Markov process. Specifically, the system parameters are borrowed from [17] and are given as



(a)



(b)

FIGURE 4: Trajectories of the fault signal. (a) the random jumping fault signal, (b) fault signal

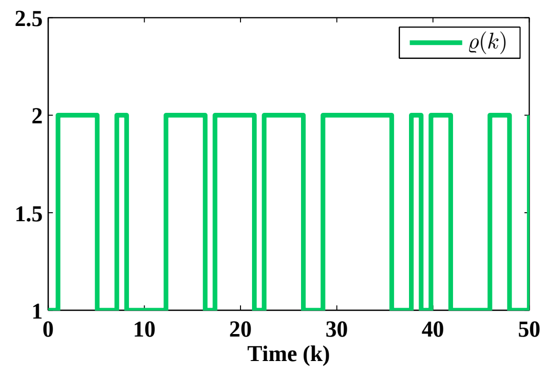


FIGURE 5: State jumping mode

follows:

$$A_1 = \begin{bmatrix} 0.5038 & 0.3171 \\ -0.0051 & -0.0032 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3326 & 0.3363 \\ -0.0034 & -0.0034 \end{bmatrix}.$$

The other parameters are assumed as

$$B_1 = \begin{bmatrix} 0.4509 \\ 0.0056 \end{bmatrix}, B_2 = \begin{bmatrix} 0.6065 \\ 0.0041 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, F_1 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}^T, G_2 = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}^T, G_{d1} = G_{d2} = \begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix}^T,$$

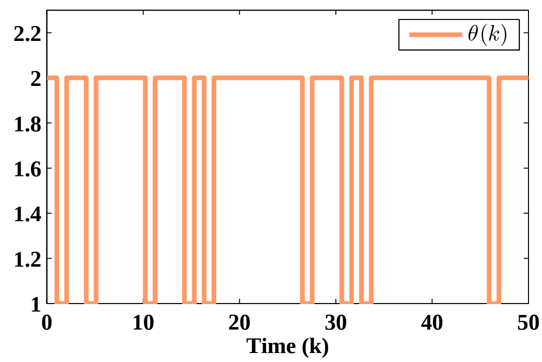


FIGURE 6: Filter jumping mode

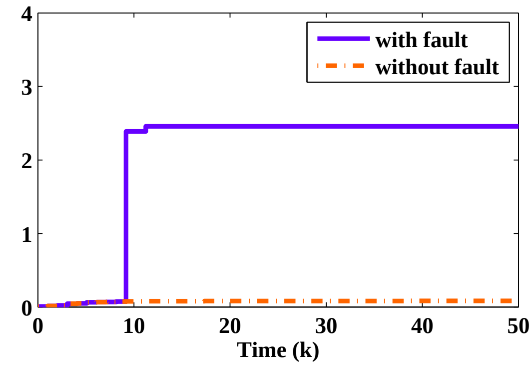


FIGURE 7: Trajectories of the residual evaluation function

$G_{w1} = G_{w2} = 0.1$ $D_1 = -0.1,$ $D_2 = -0.1,$ $H_1 = 2,$ $H_2 = 2,$ $L_1 = 0.1,$ $L_2 = 0.11.$

Besides, the remaining parameters are taken as in Example IV. Now, by solving the LMIs in Theorem 1 including the aforementioned parameters, the dissipative performance level $\gamma = 0.512$ is obtained and the non-fragile asynchronous fault detection filter gain matrices are calculated as

$$A_{f1} = \begin{bmatrix} 0.0162 & -0.0011 \\ 0.0080 & -0.0033 \end{bmatrix}, A_{f2} = \begin{bmatrix} 0.0164 & -0.0013 \\ 0.0081 & -0.0034 \end{bmatrix},$$

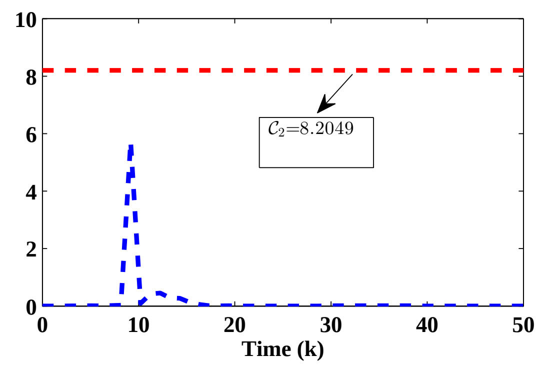


FIGURE 8: Evolution of $x^T(k)\mathcal{J}x(k)$

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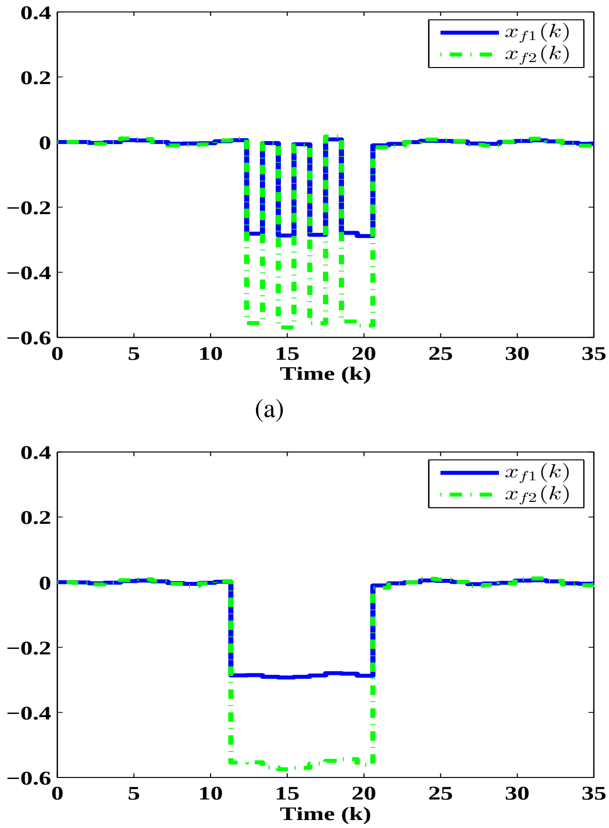
$$B_{f1} = \begin{bmatrix} -0.1053 \\ -0.1802 \end{bmatrix}, \ B_{f2} = \begin{bmatrix} -0.1056 \\ -0.1801 \end{bmatrix}, \ C_{f1} = \begin{bmatrix} -0.0126 \\ 0.0435 \end{bmatrix}^T,$$

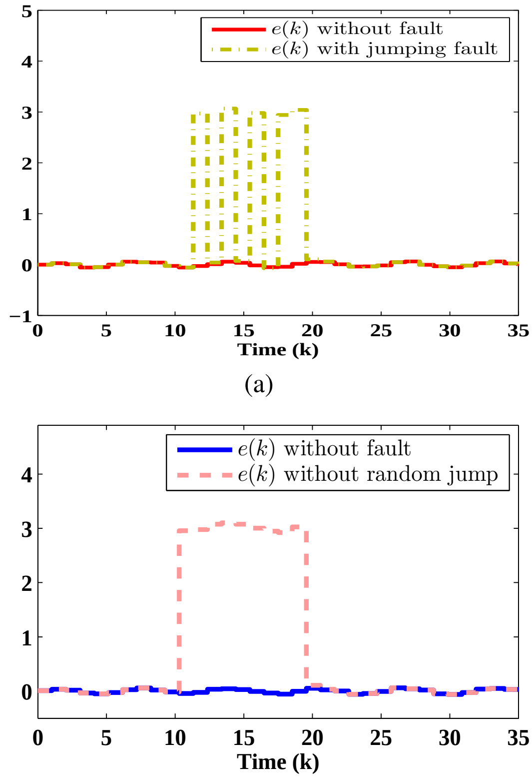
$$C_{f2} = \begin{bmatrix} -0.0139 \\ 0.0433 \end{bmatrix}^T.$$



(b)

FIGURE 9: Trajectories of filter states. (a) with random jumping fault signal, (b) without random jumping fault signal

Let us assume the initial conditions as $x(0) = x_f(0) = [0 \quad 0]^T$ for the system and filter states. Moreover, we choose the disturbance function as in [17], fault signal $\hat{f}(k) = \begin{cases} 1.5 & 10 \leq k \leq 19 \\ 0 & otherwise \end{cases}$ and control input $u(k) = 1$. The corresponding simulation results are exposed in Figs. (9)-(13). Specifically, FIGURE 9 displays the trajectories of system (7) wherein FIGURE 9(a) illustrate the trajectories of $x_{f1}(k)$ and $x_{f2}(k)$ with and without random jumping fault signal and FIGURE 9(b) displays the trajectories of $x_{f1}(k)$ and $x_{f2}(k)$ without fault and random jumping fault signal. The responses of the error system can be viewed in FIGURE 10. In particular, by using a designed non-fragile asynchronous fault detection filter, the error response of the considered system with and without the random jumping fault signal are displayed in FIGURE 10(a). Furthermore, FIGURE 10(b) reveals the trajectories of error system without fault and random jumping fault signal. On the other hand, the fault



(b)

FIGURE 10: Error responses of the system (9). (a) with and without random jumping fault signal, (b) without fault and random jumping fault signal

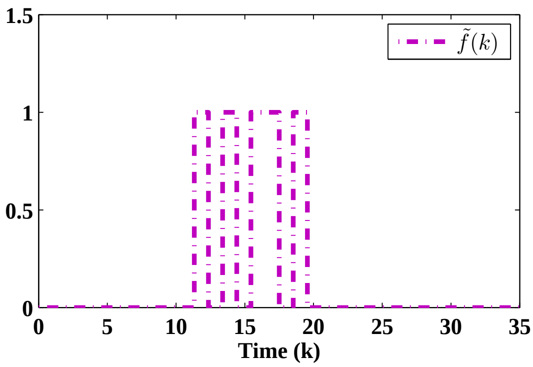
signal with and without random jump is exposed in FIGURE 11(a) and FIGURE 11(b), respectively. Additionally, the residual evaluation function is plotted in FIGURE 12. It is clear from FIGURE 12 that the residual evaluation function $\mathsf{J}_e(k) = \sqrt{\sum_{k=0}^{11} e^T(k)e(k)} = 3.0947$ exceeds the selected threshold $\mathsf{J}_{th} = 0.2463$ in one time step, wherein the ability of the recommended non-fragile asynchronous fault detection filter is precisely displayed. Eventually, the evolution of $x^T(k)\mathcal{J}x(k)$ is shown in FIGURE 13. From FIGURE 13, it is recognizable that the responses of the augmented asynchronous fault detection filtering error system do not exceed the bound value $\mathcal{C}_2$.

TABLE 1: Maximum value of $\mathcal{C}_2$ for different values of $\mathcal{C}_1$

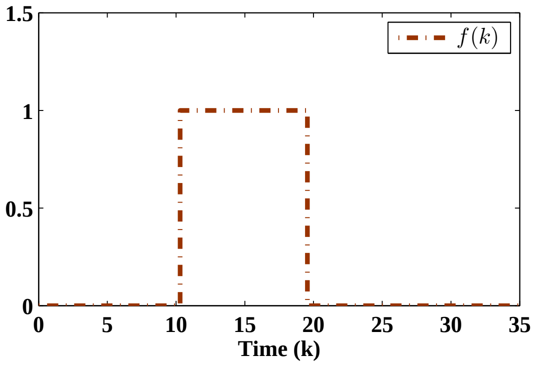
$\mathcal{C}_1$	0.1	0.15	0.2	0.25	0.3
$\mathcal{C}_2$	12.6761	13.1250	13.4512	13.8635	14.1671

TABLE 2: Comparison results

	Minimum of γ
Ref. [17]	2.1
Our paper	0.512



(a)



(b)

FIGURE 11: Trajectories of fault signal.(a) with random jumping fault, (b) without random jumping fault

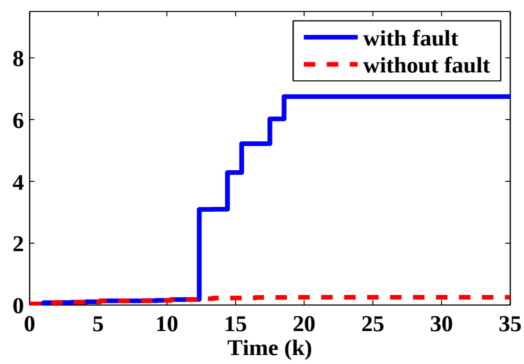


FIGURE 12: Trajectories of residual evaluation function

Moreover, TABLE 1 shows the maximum allowable bound values of $\mathcal{C}_2$ for various values of $\mathcal{C}_1$ with the aid of designed filter. Eventually, from TABLE 2, it is clear to determine that the proposed filter approach in this work yields less conservative results than in [17]. Thus from these simulation results, it can be concluded that the augmented asynchronous fault detection filtering error system (9) is stochastically finite-time bounded with strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance subject to $(0.1, 12.6761, 35, 0.512, I, 0.02)$ under proposed non-fragile asynchronous fault detection filter design.

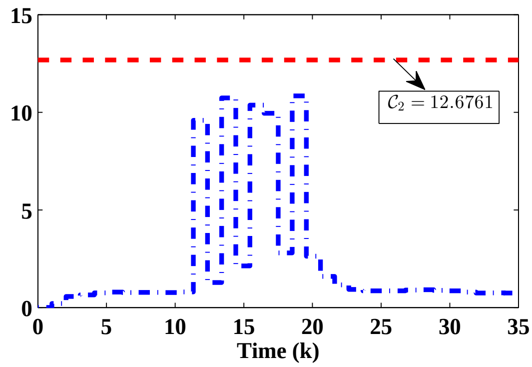


FIGURE 13: Evolution of $x^T(k)\mathcal{J}x(k)$

V. CONCLUSION

In this paper, the finite-time dissipative based non-fragile asynchronous filter design problem is examined for conic-type nonlinear SMJSs with time delay, missing measurements and random jumping fault signal. Specifically, time delay and random jumping fault phenomena are included in conic-type nonlinear term of the considered system. Preciously, a non-fragile asynchronous fault detection filter design has been proposed to lead the asynchronous situation between the state mode and filter mode. Moreover, a group of sufficient conditions is derived with the aid of mode-dependent Lyapunov-Krasovskii functional in terms of LMIs to ensure the stochastic finite-time boundedness with prescribed strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R}) - \gamma$ dissipative performance of the augmented asynchronous fault detection filtering error system. Finally, two numerical examples based on PWM-driven boost converter model and R-L-C circuit model are provided to prove the efficiency of the proposed filter scheme. In our future work, we will extend the proposed approach with some modifications to discuss the robust fault detection filter design problem for conic-type nonlinear positive SMJSs with mismatched quantization and random time varying delay as a potential research direction.

REFERENCES

[1] X. Yu, Y. Fu, P. Li and Y. Zhang, Fault-tolerant aircraft control based on self-constructing fuzzy neural networks and multivariable SMC under actuator faults, *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2324-2335, Aug. 2018.
[2] W. Chen, D. Ding, H. Dong and G. Wei, Distributed resilient filtering for power systems subject to denial-of-service attacks, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 8, pp. 1688-1697, Aug. 2019.
[3] H. Shen, F. Li, Z.-G. Wu and Ju H. Park, Finite-time asynchronous filtering for discrete-time Markov jump systems over a lossy network, *Int. J. Robust Nonlinear Control*, vol. 26, no. 17, pp. 3831-3848, Mar. 2016.
[4] S.H. Kim, Asynchronous dissipative filter design of nonhomogeneous Markovian jump fuzzy systems via relaxation of triple-parameterized matrix inequalities, *Inf. Sci.*, vol. 478, pp. 564-579, Apr. 2019.
[5] F. Li, C. Du, C. Yang, L. Wu L and W. Gui, Finite-time asynchronous sliding mode control for Markovian jump systems. *Automatica*, vol. 109, pp. 108503, Nov 2019.
[6] D.Yao, R. Lu, Y. Xu and L. Wang, Robust H_∞ filtering for Markov jump systems with mode-dependent quantized output and partly unknown transition probabilities, *Signal Proces.*, vol. 137, pp. 328-338, Feb. 2017.

[7] Z. Fei, C. Guan and P. Shi, Further results on H_∞ control for discrete-time Markovian jump time-delay systems, *Int. J. Control*, vol. 90, no. 7, pp. 1505-1517, Aug. 2016.
[8] J. Wang, J. Xia, H. Shen, M. Xing and J.H. Park, H_∞ Synchronization for Fuzzy Markov Jump Chaotic Systems with Piecewise-Constant Transition Probabilities Subject to PDT Switching Rule,*IEEE Trans. Fuzzy Systs.*, DOI:10.1109/TFUZZ.2020.3012761. Jul 2020.
[9] Y. Wang, D. Guo and X. Ji, Filtering design of switched systems with Markovian jump parameters and Lipschitz nonlinearity via fuzzy approach, *IEEE Access*, vol. 7, pp. 142115-142128, Sep. 2019.
[10] W.I. Lee and B.Y. Park, Stabilization of Markovian jump systems with quantized input and generally uncertain transition rates, *IEEE Access*, vol. 9, pp. 83499-83506, June 2021.
[11] M. Shen and S. Fei, A constructive method to static output stabilisation of Markov jump systems, *Int. J. Control*, vol. 88, no.5, pp. 990-1000, May 2015.
[12] G. Zhuang, J. Xia, J.E. Feng, W. Sun and B. Zhang, Admissibilization for implicit jump systems with mixed retarded delays based on reciprocally convex integral inequality and Barbalat's lemma, *IEEE Trans. Systs, Man, Cybern. Systs*, DOI: 10.1109/TSMC.2020.2964057, Jan 2020.
[13] S.H. Kim, Stochastic stability and stabilization conditions of semi-Markovian jump systems with mode transition-dependent sojourn-time distributions, *Inf. Sci.*, vol. 385-386, pp. 314-324, Apr. 2017.
[14] H. Shen, Z.-G. Wu and Ju H. Park, Reliable mixed passive and H_∞ filtering for semi-Markov jump systems with randomly occurring uncertainties and sensor failures, *Int. J. Robust Nonlinear Control*, vol. 25, no. 17, pp. 3231-3251, Oct. 2014.
[15] B. Jiang and C.C. Gao, Decentralized adaptive sliding mode control of large-scale semi-Markovian jump interconnected systems with dead-zone input, *IEEE Trans. Autom. Control*, DOI: 10.1109/TAC.2021.3065658, Mar 2021.
[16] S. He, J. Song and F. Liu, Robust finite-time bounded controller design of time-delay conic nonlinear systems using sliding mode control strategy, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1863-1873, Nov. 2018.
[17] X. Dong, S. He and V. Stojanovic, Robust fault detection filter design for a class of discrete-time conic-type non-linear Markov jump systems with jump fault signals, *IET Control Theory Appl.*, vol. 14, no. 14, pp. 1912-1919, Sep. 2020.
[18] R. Nie, S. He and X. Luan, Finite-time stabilisation for a class of time-delayed Markovian jumping systems with conic non-linearities, *IET Control Theory Appl.*, vol. 13, no. 9, pp. 1279-1283, Jun, 2019.
[19] Q. Fu, S. Zhong, W. Jiang and W. Xie, Projective synchronization of fuzzy memristive neural networks with pinning impulsive control, *J. Frank. Inst.*, vol. 357, no.15, pp.10387-10409, Oct 2020.
[20] L. Wu, W. Luo, Y. Zeng, F. Li and Z. Zheng, Fault detection for underactuated manipulators modeled by Markovian jump systems, *IEEE Trans. Ind. Electron.*, vol. 63, no. 7, pp. 4387-4399, Jul. 2016.
[21] Y. Shi and X. Peng, Fault detection filter design of polytopic uncertain discrete-time singular Markovian jump systems with time-varying delays, *J. Frank. Inst.*, vol. 357, no. 11, pp. 7343-7367, Jul. 2020.
[22] B. Qiao, X. Su, R. Jia, Y. Shi and M.S. Mahmoud, Event-triggered fault detection filtering for discrete-time Markovian jump systems, *Signal Process.*, vol. 152, pp. 384-391, Nov. 2018.
[23] R. Mohajerpoor, H. Abdi and S. Nahavandi, On unknown-input functional observability of linear systems, *In2015 American Control Conference (ACC) IEE*, pp. 3534-3539, Jul 2015.
[24] M. Dai, J. Xia, Ju H. Park, X. Huang and H. Shen, Asynchronous dissipative filtering for Markov jump discrete-time systems subject to randomly occurring distributed delays, *J. Frank. Inst.*, vol. 356, no. 4, pp. 2395-2420, Mar. 2019.
[25] Y. Zhu, X. Song, M. Wang and J. Lu, Finite-time asynchronous H_∞ filtering design of Markovian jump systems with randomly occurred quantization, *Int. J. Control, Autom. Syst.*, vol. 18, no. 9, pp. 450-461, Nov. 2019.
[26] Q. Zheng, S. Xu and Z. Zhang, Asynchronous nonfragile H_∞ filtering for discrete-time nonlinear switched systems with quantization, *Nonlinear Anal. Hybrid Syst.*, vol. 37, pp. 100911, May 2020.
[27] L. Zhang, N. Cui, M. Liu and Y. Zhao, Asynchronous filtering of discrete-time switched linear systems with average dwell time, *IEEE Trans. Circuits Syst. I: Regular Papers*, vol. 58, no. 5, pp. 1109-1118, May 2011.
[28] G. Zhuang, W. Sun, S.F. Su and J. Xia, Asynchronous feedback control for delayed fuzzy degenerate jump systems under observer-based event-driven characteristic, *IEEE Trans. Fuzzy Systs.*, DOI: 10.1109/TFUZ-Z.2020.3027336, Sep 2020.
[29] X. Jiang, G. Xia and Z. Feng, Resilient H_∞ filtering for stochastic systems with randomly occurring gain variations, nonlinearities and channel fadings, *Circuits, Syst., Signal Process.*, vol. 38, pp. 4548-4571, Mar. 2019.
[30] Q. Liu, Z. Wang, X. He, G. Ghinea and F.E. Alsaadi, A resilient approach to distributed filter design for time-varying systems under stochastic nonlinearities and sensor degradation, *IEEE Trans. Signal Process.*, vol. 65, no. 5, pp. 1300-1309, Mar. 2017.
[31] R. Mohajerpoor, L. Shanmugam, H. Abdi, S. Nahavandi and J.H. Park, Delay-dependent functional observer design for linear systems with unknown time-varying state delays, *IEEE Trans. cybern.*, vol. 48, no.7, pp.2036-48, Jul 2017.
[32] Q. Fu, J. Cai and S. Zhong, Robust stabilization of memristor-based coupled neural networks with time-varying delays, *Int. J. Control Autom. Systs.*, vol. 17, no.10, pp. 2666-2676, Oct 2019.
[33] S. Al-Wais, R. Mohajerpoor, L. Shanmugam, H. Abdi and S. Nahavandi, Improved delay-dependent stability criteria for telerobotic systems with time-varying delays, *IEEE Trans. Systs., Man, Cybern. Systs.*, vol. 48 no. 12 pp. 2470-2484, Jun 2017.
[34] M. Zhang, C. Shen, Z.-G. Wu and D. Zhang, Dissipative filtering for switched fuzzy systems with missing measurements, *IEEE Trans. Cyber.*, vol. 50, no. 5, pp. 1931-1940, May 2020.
[35] D. Zhang, S.K. Nguang, D. Srinivasan and L. Yu, Distributed filtering for discrete-time T-S fuzzy systems with incomplete measurements, *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1459-1471, Jun. 2018.
[36] J. Song, Y. Niu and S. Wang, Robust finite-time dissipative control subject to randomly occurring uncertainties and stochastic fading measurements, *J. Frank. Inst.*, vol. 354, no. 9, pp. 3706-3723, Jun 2017.
[37] V. Nithya, R. Sakthivel and F. Alzahrani, Dissipative-based non-fragile filtering for fuzzy networked control systems with switching communication channels, *Appl. Math. Comp.*, vol. 373, pp. 125011, May 2020.
[38] J. Wang, Z. Huang, Z. Wu, J. Cao and H. Shen, Extended dissipative control for singularly perturbed PDT switched systems and its application, *IEEE Trans. Circuits Systs. I: Regular Papers*, vol.67, no.12, pp.5281-5289, Sep 2020.
[39] Y. Zhao, T. Zhang, Y. Fu and L. Ma, Finite-time stochastic H_∞ control for singular Markovian jump systems with (x, v) - dependent noise and generally uncertain transition rates, *IEEE Access*, vol. 7, pp. 64812-64826, May 2019.

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