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Narayana Prime Cordial Labeling of Twig Graph

A. Asha Rani[1,a)], K.Thirusangu[2,b)], B.J. Balamurugan[3,c)]

[1] *Department of Mathematics with Computer Applications, PSG College of Arts &Science,*
Coimbatore-641014, Tamil Nadu, India.
[2] *Department of Mathematics, SIVET College Gowrivakkam,*
Chennai - 600 073 Tamil Nadu, India.
[3] *School of Advanced Sciences, VIT University, Chennai Campus,*
Chennai 600127, Tamil Nadu, India.

[a)]*Corresponding author: asharani.a@gmail.com*
[b)] *kthirusangu@gmail.com*
[c)]*balamurugan.bj@vit.ac.in*

Abstract: *Let $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ be a graph. The binary numbers 0 and 1 have been allotted to the edges of the graph G through the evaluating functions defined on V and E by ensuring the cordiality conditions. This has been obtained through the prime and the Narayana numbers. Any graph G which admits this labeling is known as Narayana prime cordial graph. In this research paper , we compute the Narayana prime cordial labeling of Twig graphs.*

1. Introduction

Graph labeling is the process of assigning labels to the vertices in the vertex set V and to the edges in the edge set E of the given graph *G* [1,12]. The applications of graph labeling can be found in [10] . In the year 1987, Cahit [6] discussed the cordial labeling of graphs. The Narayana prime cordial labeling of graphs is a recent development in graph theory which was introduced by B.J Murali et. al [11] . The terminologies and concepts used in this paper have been referred to Harary [8]. The developments in graph labeling have been updated by Gallian [7].The study of Narayana prime cordial labeling for various classes of graphs are found in [2,3,4,13] .

In this research article we discuss the existence of the Narayana prime cordial labeling of grid graphs and triangular grid graphs.

2. Preliminaries

The Narayana numbers , closely related to Catalan numbers [9] are defined as follows.

Definition 2.1

" Let N_0 be the set of non negative integers and let $k, n \in N_0$
N(n,k) is the kth Narayana number for a given n is defined as

$$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k+1}, 0 \leq k \leq n \ \ where \ \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

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Narayana numbers $N(n,k)$ for each n=1,2,…7 ,k=1,2,…7 are tabulated below for quick reference:

k \ n	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	3	1				
4	1	6	6	1			
5	1	10	20	10	1		
6	1	15	50	50	15	1	
7	1	21	105	175	105	21	1

Properties 2.2

The divisibility of Narayana numbers depends on the properties[5]:
(i)Let p be prime and let $n = p^m - 1$ for some $m \in N_0$. Then for all$k$, $1 \leq k \leq n-1$, $p \nmid N(n,k)$ and
(ii)Let p be prime and let $n = p^m$for some$m \in N_0$. Then for all$k$, $1 \leq k \leq n-2$, $p \,/\, N(n,k)$."

Definition 2.3

The graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a Twig Graph.

3. Narayana Prime Cordial Labeling of Twig Graphs

We recall the definitions of the Narayana prime cordial labeling of graph and the Narayana prime cordial graph.

Definition 3.1

"Let $G = (V,E)$ be a graph. An injective function f: $V \to N_0$ is said to be a Narayana prime cordial labeling of the Graph G, if the induced edge function $f^*: E \to \{0,1\}$ satisfies the following conditions:

(i) For every$uv \in E$

$$f^*(uv) = 1 \; if \; p/N\big(f(u), f(v)\big), where \; f(u) > f(v) \; and \; f(u) = p^m$$
$$for \; some \; m \in N_0 \; ; 1 \leq f(v) \leq f(u) - 2 \; where \; p \; is \; a \; prime \; number$$
$$= 1 \; if \; p/N\big(f(v), f(u)\big), where \; f(v) > f(u) \; and \; f(v) = p^m$$
$$for \; some \; m \in N_0 \; ; 1 \leq f(u) \leq f(v) - 2 \; where \; p \; is \; a \; prime \; number$$
$$= 0 \; if \; p \nmid N\big(f(u), f(v)\big), where \; f(u) > f(v) \; and \; f(u) = p^m - 1$$

$$for\ some\ m \in N_0\ ; 0 \le f(v) \le f(u) - 1\ where\ p\ is\ a\ prime\ number$$
$$= 0\ if\ p \nmid N\big(f(v), f(u)\big), where\ f(v) > f(u)\ and\ f(v) = p^m - 1$$
$$for\ some\ m \in N_0\ ; 0 \le f(u) \le f(v) - 1\ where\ p\ is\ a\ prime\ number$$

(ii) $\left|e_{f^*}(0) - e_{f^*}(1)\right| \le 1$ where $e_{f^*}(0)$ and $e_{f^*}(1)$ denote respectively the number of edges with the label 0 and the number of edges with label 1."

Definition 3.2

"A graph $G = (V, E)$ which admits a Narayana prime cordial labeling is called a Narayana prime cordial graph."

Theorem:3.3

The twig graph G admits a Narayana prime cordial labeling.

Proof:

Let the vertex set of the Twig graph G be U ∪ V ∪ W where U = $\{u_j : 1 \le j \le n-2\}$,

V = $\{v_i : 1 \le i \le n\}$ and W = $\{w_j : 1 \le j \le n-2\}$ be the vertex set of G.

Let E(G) = $\{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_2 u_1\} \cup \{v_2 w_1\} \cup \{v_i u_{j+1} : 3 \le i \le n-1, 1 \le j \le n-3, i = j+2\} \cup \{v_i w_{j+1} : 3 \le i \le n-1, 1 \le j \le n-3, i = j+2\}$ be the edge set of the twig graph G.

By the definition of an injective function $f: V \to N_0$, the vertices are labelled as follows,

When n is odd

$$f(v_i) = 2^{i+1}; i = 1,3,5, \dots n$$
$$f(v_i) = 2^{i+1} - 1; i = 2,4,6, \dots n-1$$

When n is even

$$f(v_i) = 2^{i+1}; i = 1,3,5, \dots n-1$$
$$f(v_i) = 2^{i+1} - 1; i = 2,4,6, \dots n$$

And the labeling of vertices for the remaining vertices are done as

$$f(u_j) = 2^{(n+2j)}; j = 1,2,3, \dots n-2$$
$$f(w_j) = 2^{(n+1+2j)} - 1; j = 1,2,3, \dots n-2$$

By the definition of induced edge function f* and the properties of Narayana numbers,

$$f^*(v_i v_{i+1}) = \begin{cases} 1\ if\ 2^p > 2^q - 1\ or\ 2^p > 2^q \\ 0\ otherwise \end{cases}; 1 \le i \le n-1$$

$$f^*(u_i v_{i+1}) = \begin{cases} 1\ if\ 2^p > 2^q - 1\ or\ 2^p > 2^q \\ 0\ otherwise \end{cases}; 1 \le i \le n-2$$

$$f^*(w_i v_{i+1}) = \begin{cases} 1\ if\ 2^p > 2^q - 1\ or\ 2^p > 2^q \\ 0\ otherwise \end{cases}; 1 \le i \le n-2$$

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$\left\lfloor\frac{3n-5}{2}\right\rfloor$ edges receive 1 and $\left\lceil\frac{3n-5}{2}\right\rceil$ receive 0 . i.e it satisfies the condition $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ .Therefore the twig graph admits Narayana prime cordial labeling.

For illustration the Narayana prime cordial labeling of twig graph with n=8 and n=7 are shown in figures 1 & 2.

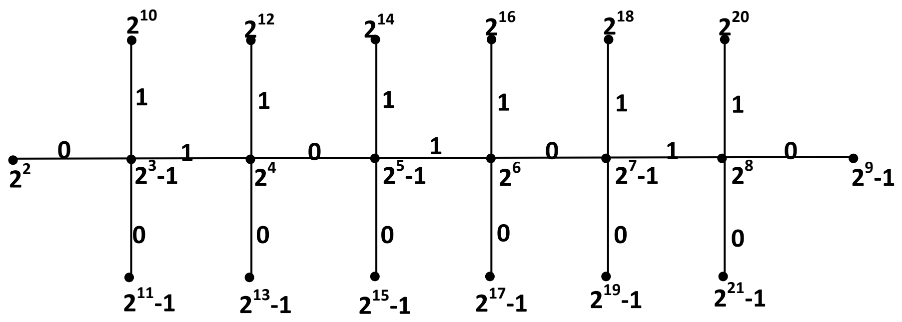

Fig 1. Narayana Prime Cordial Labeling of Even Twig Graph

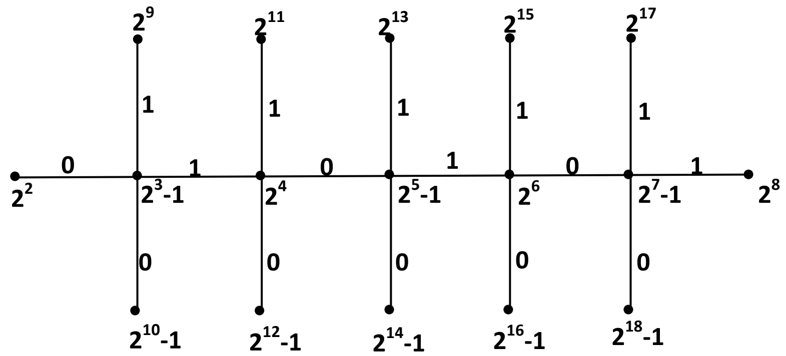

Fig 2. Narayana Prime Cordial Labeling of Odd Twig Graph

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