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An application of multi-objective transportation problem in type-2 Fermatean fuzzy number incorporating the RS-MABAC technique

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ABSTRACT

This study describes two decision making problems in trapezoidal type-2 Fermatean fuzzy numbers. Firstly, to address multi-criteria decision-making problem, we provide a hybrid rank sum and multi-attributive border approximation area comparison approach that ranks alternatives from best to worst contingent upon decision makers' preferences. The second phase is to define a multi-objective transportation problem where the model is reduced to a single objective problem using the data envelopment analysis technique. The simplified single-objective issue is then solved using LINGO-18.0, producing a collection of optimal solutions. Lastly, the suggested method is used to address a medical supply transportation problem, and the outcomes are compared and discussed.

1. Introduction

In contemporary times, uncertainty permeates multiple fields, including technological advancement, science, and everyday life. The growing complexity of ambiguous data, especially in the context of large data, requires strong mathematical frameworks for efficient representation and analysis. To address this challenge, Zadeh [1] introduced fuzzy set theory, which has been widely applied across numerous fields due to its ability to represent imprecise information [2–5]. Subsequently, type-2 fuzzy sets [6], an extension of type-1 fuzzy sets, were introduced to improve the management of uncertainty. T2FS provides greater degrees of freedom than T1FS, therefore enhancing the representation of vagueness and fuzziness in practical applications [7–10]. Despite its benefits, T2FS is comparatively more intricate and presents difficulties in model formulation [11].

In these FS versions, the extent of an element's non-membership is traditionally specified as the complement of its membership degree. This assumption frequently does not sufficiently account for real-world uncertainty. To overcome this limitation, Atanassov [12] introduced the intuitionistic fuzzy set, extending traditional fuzzy set theory. IFS integrate both membership and non-membership degrees, limited to ensure their total equals one, so providing a more thorough depiction of uncertainty. In specific decision-making circumstances, the total of these degrees may surpass one, yet their squared total remains inside one. Inspired by this result, Yager [13] created the Pythagorean fuzzy set, which generalizes the spectrum of membership and non-membership degrees, ensuring that their squared total does not surpass one. This flexibility allows PFS to manage ambiguity more proficiently than IFS. In certain situations, baffling data may occur when the squared total of membership and non-membership degrees surpasses one, while the sum of their cubes remains inside one. For handling that, Senapati and Yager [14] proposed the Fermatean fuzzy set, hence improving decision-making under uncertain conditions. Nonetheless, real-world data may occasionally be missing due to various reasons, such as prolonged durations or diverse sources. In certain instances, conventional two-dimensional fuzzy sets or T1FS may be inadequate for effectively representing uncertainty. Given these advantages, researchers have explored hybrid models combining various fuzzy set variants with T2FS [15–18]. Notable recent extensions include Karmakar et al. [19], who established a type-2 intuitionistic fuzzy for matrix game framework, and Rani and Manivannan [20], who presented a generalized symmetric type-2 intuitionistic fuzzy set with a novel ranking function. Sarkar et al. [21] introduced the type-2 Pythagorean fuzzy set, whereas Mondal and Roy [22] utilized the Choquet integral in the context of interval type-2 Pythagorean fuzzy sets. Umer et al. [23] enhanced the TOPSIS methodology by employing interval type-2 Pythagorean fuzzy numbers, while Mourad et al. [24] introduced an innovative hybrid T2PFS-FWZICbIP strategy for modular self-reconfiguration in robotics.

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Abbreviations employed in this study	
TP	Transportation Problem
MOTP	Multi Objective Transportation Problem
GP	Goal Programming
DEA	Data Envelopment Analysis
DMU	Decision Making Units
MCDM	Multi Criteria Decision Making
MABAC	Multi Attributive Border Approximation area Comparison
RS	Rank Sum
FS	Fuzzy Set
T1FS	Type 1 Fuzzy Set
IFS	Intuitionistic Fuzzy Set
FFS	Fermatean Fuzzy Set
PFS	Pythagorean Fuzzy Set
T2IFS	Type 2 intuitionistic Fuzzy Set
T2PFS	Type 2 Pythagorean Fuzzy Set
TT2NN	Triangular Type 2 Neutrosophic Number
TT2FF	Trapezoidal Type 2 Fermatean Fuzzy

Despite these advancements, existing methods impose constraints on the representation of ambiguous data. Consequently, experts may struggle to articulate their judgments effectively under prevailing uncertainty. To address this limitation, this study introduces an extension of type-2 Pythagorean fuzzy sets, denoted as trapezoidal type-2 Fermatean fuzzy numbers. The TT2FF framework employs trapezoidal membership and non-membership functions, with fundamental operations systematically defined. Additionally, a novel ranking function is proposed, utilizing the α-level cut of membership and non-membership functions to enhance decision-making under uncertainty. To explain the usefulness and efficiency of the recommended TT2FF variables, this study tackles the challenges associated with the MCDM problem and multi objective transportation problem through the numerical example of medical supply transportation problem.

An advanced decision-making technique called multi-criteria group decision making builds upon traditional multi-criteria decision making by taking into account the opinions and viewpoints of several decision-makers [25,26]. Making decisions in many real-world situations requires cooperation from experts, participants, or policymakers who must assess different options according to a number of criteria. MCGDM offers an organized approach to combining different points of view, promoting agreement while resolving competing goals [27,28]. It is especially helpful for issues when evaluating and prioritizing options is necessary.

Finding pertinent criteria, obtaining expert opinions, aggregating preferences, using MCDM approaches, and reaching a conclusion are all steps in the standard MCGDM process. But issues like managing subjective assessments, settling disputes between decision-makers, and guaranteeing equitable weight distribution need to be addressed. We use the multi-attributive border approximation area comparison (MABAC) method, which was created by Pamucar and Cirovic [29] and has a number of benefits over conventional MCDM techniques, to address these problems. This approach is renowned for its precision, ease of computation, and suitability for use with many models of decision-making.

Because of these advantages, the MABAC technique has been widely used in a number of fields, such as risk analysis [30], waste disposal assessment [31], supplier selection [32], the automotive industry [33], and wind farm evaluation [34]. Furthermore, the RS technique for weighing models was established by Stillwell et al. [35], enabling decision-makers to efficiently prioritize criteria. For improved decision analysis, recent research has integrated RS with decision-making algorithms. For instance, Singh and Kushwaha [36] used RS and MoSCoW techniques to agricultural index insurance, while Tripathi et al. [37] proposed a CRITIC-RS-VIKOR framework for assessing renewable energy options. For hospital site selection, Hezam et al. [38] combined RS with the MARCOS approach. Motivated by these developments, we offer a hybrid RS-MABAC framework that employs TT2FF numbers to efficiently rank options, guaranteeing a more sophisticated and reliable decision-making procedure.

The multi-objective transportation problem is an expansion of the fundamental transportation problem in which multiple conflicting objectives are considered at the same time [39,40]. The traditional transportation problem, on the other hand, focuses only on reducing transportation expenses. Real-world logistics and supply chain management require simultaneous optimization of several factors, such as time, expenses, ecological impact, and service quality, in order to reach optimal performance. Given that these goals could conflict, decision-makers must strike a balance to arrive at the best possible answer. The Pareto optimality approach, which involves trade-offs in other objectives when one target is improved, is commonly used to assess MOTP solutions. Numerous sophisticated optimization techniques, including multi-objective evolutionary algorithms, the weight sum approach, and goal programming techniques, are employed to address this degree of complexity [41–43]. Recent research indicates that the data envelopment analysis methodologies offer a more realistic way to arrive at the most effective MOTP solution. DEA, a modern “data-oriented” approach, is used to analyze decision making units, which are groups of two or more entities that convert several inputs into multiple outputs. Because of the DEA-MOTP relation, DEA approaches are preferred for real-world TPs. DEA, or even frontier analysis, was first presented by Charnes et al. [44] in 1978. A lexicographic multi-objective linear programming approach was proposed by Hatami-Marbini et al. [45], which transforms a fuzzy DEA model into a multi-objective linear programming problem. Using DEA cross-efficiency and intuitionistic fuzzy preference relations, Liu et al. [46] investigated a novel group decision-making technique. Bagheri et al. [47] used the DEA technique to solve the MOTP in a fuzzy environment, representing cost coefficients as triangular fuzzy numbers. For MOTP, Akram et al. [48,49] provide an expanded DEA solution. The interpretable DEA for medical picture segmentation was developed by Wu et al. [50]. Since the TT2FF can manage situations where hesitation is a component of the uncertainty and ambiguity, we then incorporate the TT2FF values in the MOTP mathematical model, which is solved using the DEA approach. Table 1 lists more type 2 fuzzy TP and MCDM research.

1.1. Originality and necessity

This study introduces a novel fuzzy concept called the trapezoidal type-2 Fermatean fuzzy number, which is applied to two distinct decision-making problems. The first problem focuses on selecting the most suitable supplier for transporting medical supplies, while the second involves optimizing a multi-objective medical supply transportation problem by minimizing cost and shipping value while maximizing profit. The TT2FF framework employed in this study quickly determines deviations in decision-makers’ evaluations, providing a sophisticated method for managing uncertainty in the prioritization of medical suppliers tasked with the transportation of medicines. A hybrid strategy utilizing the RS-MABAC technique is presented to improve reliability in multi-criteria decision-making. This strategy, owing to its adaptable framework, serves as an effective weapon for addressing intricate decision-making challenges involving numerous conflicting criteria. The TT2FF-RS approach is utilized to ascertain the weights of evaluation criteria derived from expert judgments. In comparison to intuitionistic fuzzy sets and Pythagorean fuzzy sets, TT2FF sets more effectively encapsulate the uncertainty of improper data via trapezoidal membership and non-membership degrees. The data envelopment analysis technique is employed to tackle the multi-objective transportation problem, incorporating two input parameters, cost minimization and shipping value and one output parameter, profit maximization, respectively. Without the need for preset weights, DEA is able to assess an array of inputs and outputs. Three decision-makers supply data on supply, demand, shipping value, cost, and total profit

Table 1
Recent works on type 2 fuzzy environment.

Reference	Uncertainty	Solution method	Application
Sarkar et al. [21]	T2PFS	MCDM	Sustainable transport system selection
Shukla et al. [51]	T2IFS	MCDM: TODIM	Renewable energy resource selection problem
Das et al. [52]	TT2NN	Multi-objective modeling approach	Production industry:location–allocation problem
Roy et al. [53]	LIT2FS	Exigency Vehicle routing	IoT system development for smart city applications
Bind et al. [54]	Normal type 2	Four dimensional TP	Manufacturing Company's TP
Gulen et al. [55]	Interval type-2 fuzzy	AHP and TOPSIS	Digital transformation strategy selection
Present Work	TT2FF	Both MCDM and MCDM	Medical supply transportation problem.

in TT2FF numbers. The issue is subsequently addressed by utilizing the proposed TT2FF-DEA hybrid methodology for MOTP. The efficacy of the hybrid approach for both MCDM and MOTP inside the TT2FF framework is evaluated against existing approaches to illustrate its superiority.

1.2. Motivation and research gap of the study

- The previous studies utilized Fermatean fuzzy numbers and type 2 fuzzy numbers in optimization problems. Expanding these fuzzy numbers as TT2FF number, provides an improved way to aid in decision-making problems.
- This work is motivated by Sarbari and Jana [56], where they provided a multi-item transportation problem using the MCDM approach in interval type-2 fuzzy environment. With more criteria and alternatives, possible degree used in their paper is quite hard, but there is a need for more effective solutions in this area, so we propose a combined RS-MABAC decision making method.
- In earlier studies, having multi-objectives would have required more computations. However, in this study, we are using the DEA approach, which allows us to turn multi-objectives into a single objective, that requires considerably less computation.
- The applicability of the medical supply transportation problem in the TT2FF context of MCDM and MOTP structure is yet under explored. As a result, this research bridges that gap.

1.3. Contribution and novelty

Research question: How can decision-making models effectively handle trapezoidal type-2 Fermatean fuzzy uncertainty in multi-criteria decision making problem and multi-objective transportation problems?

- An extension of the conventional type 2 fuzzy number, the trapezoidal type 2 Fermatean fuzzy number is introduced and defined in this study to more effectively tackle optimization problems.
- The arithmetic operations and distance measure of the TT2FF are established, and a unique ranking function is developed using the alpha cut of the TT2FF number.
- The two decision-making problems of MCDM and MOTP are handled using the developed TT2FF numbers. The RS-MABAC Method, an expanded MABAC methodology coupled with the rank sum approach, is introduced for MCDM using the TT2FF environment. The fuzzy DEA approach had been implemented to solve the MOTP that incorporates TT2FF number using Lingo 18 software.
- The proposed method is thoroughly described in the flowchart of Fig. 1 and is applied to the medical supply transportation problem. The impacts of this approach are delineated by validating the outcomes with comparison study.

The remainder of the paper is structured as follows: In order to build TT2FF numbers, new ranking functions, distance measures, Section 2 goes over several fundamental definitions. The mathematical model proposed in Section 3 presents RS-MABAC method for solving MCDM in Section 3.1 and an enhanced DEA approach for solving MOTP in Section 3.2. The medical supply transportation problem is numerically shown in Section 4. Comparative studies are conducted in Section 5 to validate our proposed problem. Section 6 presents conclusions and areas for future research.

2. Preliminaries

This section outline basic preliminary definitions for developing the proposed methodologies.

Definition 2.1. Let $\mathbb{U}$ represent the entirety of the values in a universal set. A Fermatean fuzzy set $\mathcal{F}$ is expressed as

$$\mathcal{F} = \left\{ \left(x, \mu_{\mathcal{F}}(x), \vartheta_{\mathcal{F}}(x)\right) | x \in \mathbb{U} \right\}.$$

where $\mu_{\mathcal{F}} : X \to [0,1]$ and $\vartheta_{\mathcal{F}} : X \to [0,1]$ presents the extent of functions related to membership and non-membership functions $x \in \mathbb{U}$ to the set $\mathcal{F}$, encompassing the condition, $0 \le \mu^3_{\mathcal{F}}(x) + \vartheta^3_{\mathcal{F}}(x) \le 1$ for all $x \in \mathbb{U}$. The indeterminacy degree of FFS is computed as

$$\pi_{\mathcal{F}}(x) = \sqrt[3]{1 - (\mu_{\mathcal{F}}(x))^3 - (\vartheta_{\mathcal{F}}(x))^3}; \forall x \in \mathbb{U}.$$

Definition 2.2. Let $\mathbb{U}$ represent the global discourse set. The T2FF set in $\mathbb{U}$ is thus defined as follows:

$$\tilde{\mathcal{F}} = \left\{ \left(x, \mu_{\tilde{\mathcal{F}}}(x), \vartheta_{\tilde{\mathcal{F}}}(x)\right), \left(\mu_{\tilde{\mathcal{F}}}(x), f_{\mu_{\tilde{\mathcal{F}}}}(x), g_{\mu_{\tilde{\mathcal{F}}}}(x)\right), \left(\vartheta_{\tilde{\mathcal{F}}}(x), f_{\vartheta_{\tilde{\mathcal{F}}}}(x), g_{\vartheta_{\tilde{\mathcal{F}}}}(x)\right) | x \in \mathbb{U}, \mu_{\tilde{\mathcal{F}}} \in J^m_x, \vartheta_{\tilde{\mathcal{F}}} \in J^n_x \right\}$$

where, $J^m_x, J^n_x \subseteq [0,1]$ represented as primary membership and non membership degree of $x \in \mathbb{U}$ such that $0 \le \mu^3_{\tilde{\mathcal{F}}}(x) + \vartheta^3_{\tilde{\mathcal{F}}}(x) \le 1$. The function $f_{\mu_{\tilde{\mathcal{F}}}}, g_{\mu_{\tilde{\mathcal{F}}}} : J^m_x, J^n_x \to [0,1]$ and $f_{\vartheta_{\tilde{\mathcal{F}}}}, g_{\vartheta_{\tilde{\mathcal{F}}}} : J^m_x, J^n_x \to [0,1]$ are the secondary membership and non membership function satisfying $0 \le f^3_{\mu_{\tilde{\mathcal{F}}}}(x) + g^3_{\mu_{\tilde{\mathcal{F}}}}(x) \le 1$ and $0 \le f^3_{\vartheta_{\tilde{\mathcal{F}}}}(x) + g^3_{\vartheta_{\tilde{\mathcal{F}}}}(x) \le 1$.

For computational convenience, according to $J^m_x = (x, \mu_{\tilde{\mathcal{F}}}(x)) : \mu_{\tilde{\mathcal{F}}}(x) \in [\bar{\mu}(x), \underline{\mu}(x)]$ and $J^n_x = (x, \vartheta_{\tilde{\mathcal{F}}}(x)) : \vartheta_{\tilde{\mathcal{F}}}(x) \in [\bar{\vartheta}(x), \underline{\vartheta}(x)]$, where $\bar{\mu}(x), \underline{\mu}(x), \bar{\vartheta}(x), \underline{\vartheta}(x)$ represent the lower and upper bounds for the primary membership and non-membership degrees of x, correspondingly.

Definition 2.3. A superior/special type 2 Fermatean fuzzy set on a real number set $\mathbb{R}$ is a trapezoidal type 2 Fermatean fuzzy number, denoted by

$$\tilde{G} = \left(\tilde{\mathfrak{G}}_1, \tilde{\mathfrak{G}}_2, \tilde{\mathfrak{G}}_3; \mu_{\mathfrak{G}}, \vartheta_{\mathfrak{G}}\right). \tag{1}$$

Here $\tilde{\mathfrak{G}}_1, \tilde{\mathfrak{G}}_2, \tilde{\mathfrak{G}}_3$ stands for trapezoidal type 1 Fermatean fuzzy numbers and $\mu_{\mathfrak{G}}, \vartheta_{\mathfrak{G}}$ indicates the membership and non-membership degrees of $\tilde{G}$ accordingly, where

$\tilde{\mathfrak{G}}_1 = \{g_{11}, g_{12}, g_{13}, g_{14}; \mu_{\mathfrak{G}_1}, \vartheta_{\mathfrak{G}_1}\}$, $\tilde{\mathfrak{G}}_2 = \{g_{21}, g_{22}, g_{23}, g_{24}; \mu_{\mathfrak{G}_2}, \vartheta_{\mathfrak{G}_2}\}$, $\tilde{\mathfrak{G}}_3 = \{g_{31}, g_{32}, g_{33}, g_{34}; \mu_{\mathfrak{G}_3}, \vartheta_{\mathfrak{G}_3}\}$ such that $0 \le (\mu_{\mathfrak{G}})^3 + (\vartheta_{\mathfrak{G}})^3 \le 1$, $0 \le (\mu_{\mathfrak{G}_i})^3 + (\vartheta_{\mathfrak{G}_i})^3 \le 1$; i = 1,2,3.

The mathematical representation of membership and non membership function of Eq. (1) is as follows

$$\mu_{\mathfrak{G}_1}(x) = \begin{cases} \frac{\mu_{\mathfrak{G}_1}(x - g_{11})}{(g_{12} - g_{11})} & g_{11} \le x \le g_{12} \\ \mu_{\mathfrak{G}_1} & g_{12} \le x \le g_{13} \\ \frac{\mu_{\mathfrak{G}_1}(g_{14} - x)}{(g_{14} - g_{13})} & g_{13} \le x \le g_{14} \\ 0 & x \le g_{11} \text{ and } x \ge g_{14} \end{cases}$$

$$\vartheta_{\mathfrak{G}_1}(x)=\begin{cases}\frac{(g_{12}-x)+\vartheta_{\mathfrak{G}_1}(x-g_{11})}{(g_{12}-g_{11})} & g_{11}\le x\le g_{12}\\ \vartheta_{\mathfrak{G}_1} & g_{12}\le x\le g_{13}\\ \frac{(x-g_{13})+\vartheta_{\mathfrak{G}_1}(g_{14}-x)}{(g_{14}-g_{13})} & g_{13}\le x\le g_{14}\\ 1 & x\le g_{11} \text{ and } x\ge g_{14}\end{cases}$$

$$\mu_{\mathfrak{G}_2}(x)=\begin{cases}\frac{\mu_{\mathfrak{G}_2}(x-g_{21})}{(g_{22}-g_{21})} & g_{21}\le x\le g_{22}\\ \mu_{\mathfrak{G}_2} & g_{22}\le x\le g_{23}\\ \frac{\mu_{\mathfrak{G}_2}(g_{24}-x)}{(g_{24}-g_{23})} & g_{23}\le x\le g_{24}\\ 0 & x\le g_{21} \text{ and } x\ge g_{24}\end{cases}$$

$$\vartheta_{\mathfrak{G}_2}(x)=\begin{cases}\frac{(g_{22}-x)+\vartheta_{\mathfrak{G}_2}(x-g_{21})}{(g_{22}-g_{21})} & g_{21}\le x\le g_{22}\\ \vartheta_{\mathfrak{G}_2} & g_{22}\le x\le g_{23}\\ \frac{(x-g_{23})+\vartheta_{\mathfrak{G}_2}(g_{24}-x)}{(g_{24}-g_{23})} & g_{23}\le x\le g_{24}\\ 1 & x\le g_{21} \text{ and } x\ge g_{24}\end{cases}$$

$$\mu_{\mathfrak{G}_3}(x)=\begin{cases}\frac{\mu_{\mathfrak{G}_3}(x-g_{31})}{(g_{32}-g_{31})} & g_{31}\le x\le g_{32}\\ \mu_{\mathfrak{G}_3} & g_{32}\le x\le g_{33}\\ \frac{\mu_{\mathfrak{G}_3}(g_{34}-x)}{(g_{34}-g_{33})} & g_{33}\le x\le g_{34}\\ 0 & x\le g_{31} \text{ and } x\ge g_{34}\end{cases}$$

$$\vartheta_{\mathfrak{G}_3}(x)=\begin{cases}\frac{(g_{32}-x)+\vartheta_{\mathfrak{G}_3}(x-g_{31})}{(g_{32}-g_{31})} & g_{31}\le x\le g_{32}\\ \vartheta_{\mathfrak{G}_3} & g_{32}\le x\le g_{33}\\ \frac{(x-g_{33})+\vartheta_{\mathfrak{G}_3}(g_{34}-x)}{(g_{34}-g_{33})} & g_{33}\le x\le g_{34}\\ 1 & x\le g_{31} \text{ and } x\ge g_{34}.\end{cases}$$

Definition 2.4. Let

$$\tilde{G}=\{(g_{11},g_{12},g_{13},g_{14};\mu_{\mathfrak{G}_1},\vartheta_{\mathfrak{G}_1}),(g_{21},g_{22},g_{23},g_{24};\mu_{\mathfrak{G}_2},\vartheta_{\mathfrak{G}_2}),\\ (g_{31},g_{32},g_{33},g_{34};\mu_{\mathfrak{G}_3},\vartheta_{\mathfrak{G}_3});\mu_{\mathfrak{G}},\vartheta_{\mathfrak{G}}\}$$

$$\tilde{F}=\{(f_{11},f_{12},f_{13},f_{14};\mu_{\mathfrak{F}_1},\vartheta_{\mathfrak{F}_1}),(f_{21},f_{22},f_{23},f_{24};\mu_{\mathfrak{F}_2},\vartheta_{\mathfrak{F}_2}),\\ (f_{31},f_{32},f_{33},f_{34};\mu_{\mathfrak{F}_3},\vartheta_{\mathfrak{F}_3});\mu_{\mathfrak{F}},\vartheta_{\mathfrak{F}}\}$$

be a two TT2FF numbers, Then,

1. Sum of $\tilde{G}\oplus\tilde{F}=$

$$\begin{pmatrix}(g_{11}+f_{11},g_{12}+f_{12},g_{13}+f_{13},g_{14}+f_{14};min\{\mu_{\mathfrak{G}_1},\mu_{\mathfrak{F}_1}\},max\{\vartheta_{\mathfrak{G}_1},\vartheta_{\mathfrak{F}_1}\}),\\ (g_{21}+f_{21},g_{22}+f_{22},g_{23}+f_{23},g_{24}+f_{24};min\{\mu_{\mathfrak{G}_2},\mu_{\mathfrak{F}_2}\},max\{\vartheta_{\mathfrak{G}_2},\vartheta_{\mathfrak{F}_2}\}),\\ (g_{31}+f_{31},g_{32}+f_{32},g_{33}+f_{33},g_{34}+f_{34};min\{\mu_{\mathfrak{G}_3},\mu_{\mathfrak{F}_3}\},\\ \quad max\{\vartheta_{\mathfrak{G}_3},\vartheta_{\mathfrak{F}_3}\});\mu_{\mathfrak{G}+\mathfrak{F}},\vartheta_{\mathfrak{G}+\mathfrak{F}}\end{pmatrix}$$

$$\mu_{\mathfrak{G}+\mathfrak{F}}=min\{min\{\mu_{\mathfrak{G}_1},\mu_{\mathfrak{F}_1}\},min\{\mu_{\mathfrak{G}_2},\mu_{\mathfrak{F}_2}\},min\{\mu_{\mathfrak{G}_3},\mu_{\mathfrak{F}_3}\}\}$$

$$\vartheta_{\mathfrak{G}+\mathfrak{F}}=max\{max\{\vartheta_{\mathfrak{G}_1},\vartheta_{\mathfrak{F}_1}\},max\{\vartheta_{\mathfrak{G}_2},\vartheta_{\mathfrak{F}_2}\},max\{\vartheta_{\mathfrak{G}_3},\vartheta_{\mathfrak{F}_3}\}\}.$$

2. Scalar Multiplication:
   - if $\lambda\ge 0$

$$\lambda\tilde{G}=\begin{pmatrix}\{(\lambda g_{11},\lambda g_{12},\lambda g_{13},\lambda g_{14};\mu_{\mathfrak{G}_1},\vartheta_{\mathfrak{G}_1}),(\lambda g_{21},\lambda g_{22},\lambda g_{23},\lambda g_{24};\mu_{\mathfrak{G}_2},\vartheta_{\mathfrak{G}_2}),\\ (\lambda g_{31},\lambda g_{32},\lambda g_{33},\lambda g_{34};\mu_{\mathfrak{G}_3},\vartheta_{\mathfrak{G}_3});\mu_{\mathfrak{G}},\vartheta_{\mathfrak{G}}\}\end{pmatrix}$$

   - if $\lambda\le 0$

$$\lambda\tilde{G}=\begin{pmatrix}\{(\lambda g_{14},\lambda g_{13},\lambda g_{12},\lambda g_{11};\mu_{\mathfrak{G}_1},\vartheta_{\mathfrak{G}_1}),(\lambda g_{24},\lambda g_{23},\lambda g_{22},\lambda g_{21};\mu_{\mathfrak{G}_2},\vartheta_{\mathfrak{G}_2}),\\ (\lambda g_{34},\lambda g_{33},\lambda g_{32},\lambda g_{31};\mu_{\mathfrak{G}_3},\vartheta_{\mathfrak{G}_3});\mu_{\mathfrak{G}},\vartheta_{\mathfrak{G}}\}\end{pmatrix}.$$

Definition 2.5. The α-cuts of TT2FF membership and non membership function is defined as:

For membership,

$$\tilde{\mathfrak{G}}_1=\begin{cases}\alpha\le\mu_{\mathfrak{G}_1}\\ \alpha\le\frac{\mu_{\mathfrak{G}_1}(x-g_{11})}{(g_{12}-g_{11})}\\ x\ge g_{11}+\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{12}-g_{11})\end{cases}\qquad \tilde{\mathfrak{G}}_1=\begin{cases}\alpha\le\mu_{\mathfrak{G}_1}\\ \alpha\le\frac{\mu_{\mathfrak{G}_1}(g_{14}-x)}{(g_{14}-g_{13})}\\ x\le g_{14}-\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{14}-g_{13}).\end{cases}$$

Hence, $x\in\left[g_{11}+\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{12}-g_{11}),g_{14}-\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{14}-g_{13})\right]$.

Similarly for $\tilde{\mathfrak{G}}_2,\tilde{\mathfrak{G}}_3$, we get,

$$x\in\left[g_{21}+\frac{\alpha}{\mu_{\mathfrak{G}_2}}(g_{22}-g_{21}),g_{24}-\frac{\alpha}{\mu_{\mathfrak{G}_2}}(g_{24}-g_{23})\right]$$
$$x\in\left[g_{31}+\frac{\alpha}{\mu_{\mathfrak{G}_3}}(g_{32}-g_{31}),g_{34}-\frac{\alpha}{\mu_{\mathfrak{G}_3}}(g_{34}-g_{33})\right].$$

For non-membership,

$$\tilde{\mathfrak{G}}_1=\begin{cases}(1-\alpha)\le\vartheta_{\mathfrak{G}_1}\\ \alpha\le\frac{(1-\vartheta_{\mathfrak{G}_1})(x-g_{11})}{(g_{12}-g_{11})}\\ x\ge g_{11}+\frac{\alpha(g_{12}-g_{11})}{(1-\vartheta_{\mathfrak{G}_1})}\end{cases}\qquad \tilde{\mathfrak{G}}_1=\begin{cases}(1-\alpha)\le\vartheta_{\mathfrak{G}_1}\\ \alpha\le\frac{(1-\vartheta_{\mathfrak{G}_1})(g_{14}-x)}{(g_{14}-g_{13})}\\ x\le g_{14}-\frac{\alpha(g_{14}-g_{13})}{(1-\vartheta_{\mathfrak{G}_1})}.\end{cases}$$

Hence, $x\in\left[g_{11}+\frac{\alpha(g_{12}-g_{11})}{(1-\vartheta_{\mathfrak{G}_1})},g_{14}-\frac{\alpha(g_{14}-g_{13})}{(1-\vartheta_{\mathfrak{G}_1})}\right]$.

Similarly for $\tilde{\mathfrak{G}}_2,\tilde{\mathfrak{G}}_3$, we get

$$x\in\left[g_{21}+\frac{\alpha(g_{22}-g_{21})}{(1-\vartheta_{\mathfrak{G}_2})},g_{24}-\frac{\alpha(g_{24}-g_{23})}{(1-\vartheta_{\mathfrak{G}_2})}\right]$$
$$x\in\left[g_{31}+\frac{\alpha(g_{32}-g_{31})}{(1-\vartheta_{\mathfrak{G}_3})},g_{34}-\frac{\alpha(g_{34}-g_{33})}{(1-\vartheta_{\mathfrak{G}_3})}\right].$$

2.1. Alpha level ranking of TT2FF

The alpha level sets of membership and non-membership functions are

$$\begin{aligned}\mathfrak{S}_{\mu_{\mathfrak{G}_1}}&=\frac{1}{2}\int_0^{\mu_{\mathfrak{G}_1}}m(G_1^{\alpha})d\alpha\\ &=\frac{1}{2}\int_0^{\mu_{\mathfrak{G}_1}}\left[g_{11}+\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{12}-g_{11})+g_{14}-\frac{\alpha}{\mu_{\mathfrak{G}_1}}(g_{14}-g_{13})\right]d\alpha\\ &=\frac{(g_{11}+g_{12}+g_{13}+g_{14})\mu_{\mathfrak{G}_1}}{4}.\end{aligned}$$

Similarly for $\tilde{\mathfrak{G}}_2,\tilde{\mathfrak{G}}_3$

$$\mathfrak{S}_{\mu_{\mathfrak{G}_2}}=\frac{(g_{21}+g_{22}+g_{23}+g_{24})\mu_{\mathfrak{G}_2}}{4}$$
$$\mathfrak{S}_{\mu_{\mathfrak{G}_3}}=\frac{(g_{31}+g_{32}+g_{33}+g_{34})\mu_{\mathfrak{G}_3}}{4}.$$

Hence, the sum of membership of $\tilde{\mathfrak{G}}_1,\tilde{\mathfrak{G}}_2,\tilde{\mathfrak{G}}_3$,

$$\begin{aligned}\mathfrak{S}_{\mu_{\mathfrak{G}}}=&\frac{1}{4}[(g_{11}+g_{12}+g_{13}+g_{14})\mu_{\mathfrak{G}_1}+(g_{21}+g_{22}+g_{23}+g_{24})\mu_{\mathfrak{G}_2}\\ &+(g_{31}+g_{32}+g_{33}+g_{34})\mu_{\mathfrak{G}_3}].\end{aligned}\tag{2}$$

Further,

$$\begin{aligned}\mathfrak{S}_{\vartheta_{\mathfrak{G}_1}}&=\frac{1}{2}\int_0^{\vartheta_{\mathfrak{G}_1}}m(G_1^{\alpha})d\alpha\\ &=\frac{1}{2}\int_0^{\vartheta_{\mathfrak{G}_1}}\left[g_{11}+\frac{\alpha(g_{12}-g_{11})}{(1-\vartheta_{\mathfrak{G}_1})},g_{14}-\frac{\alpha(g_{14}-g_{13})}{(1-\vartheta_{\mathfrak{G}_1})}\right]d\alpha\\ &=\frac{(g_{11}+g_{12}+g_{13}+g_{14})(1-\vartheta_{\mathfrak{G}_1})}{4}.\end{aligned}$$

Similarly for $\tilde{\mathfrak{G}}_2,\tilde{\mathfrak{G}}_3$,

$$\mathfrak{S}_{\vartheta_{\mathfrak{G}_2}}=\frac{(g_{21}+g_{22}+g_{23}+g_{24})(1-\vartheta_{\mathfrak{G}_2})}{4}$$
$$\mathfrak{S}_{\vartheta_{\mathfrak{G}_3}}=\frac{(g_{31}+g_{32}+g_{33}+g_{34})(1-\vartheta_{\mathfrak{G}_3})}{4}.$$

Hence, the sum of non-membership of $\tilde{\mathfrak{G}}_1, \tilde{\mathfrak{G}}_2, \tilde{\mathfrak{G}}_3$,

$$\mathfrak{S}_{\vartheta_{\mathfrak{G}}} = \frac{1}{4}[(g_{11}+g_{12}+g_{13}+g_{14})(1-\vartheta_{\mathfrak{G}_1}) + (g_{21}+g_{22}+g_{23}+g_{24}) \times (1-\vartheta_{\mathfrak{G}_2}) + (g_{31}+g_{32}+g_{33}+g_{34})(1-\vartheta_{\mathfrak{G}_3})]. \quad (3)$$

The ranking function which is based on (2) and (3) is defined by

$$\begin{aligned}\mathfrak{R}(G) &= \eta(\mathfrak{S}_{\mu_{\mathfrak{G}}}) + (1-\eta)(\mathfrak{S}_{\vartheta_{\mathfrak{G}}}) \\ &= \frac{1}{8}\Big\{ (g_{11}+g_{12}+g_{13}+g_{14})(\mu_{\mathfrak{G}_1} + (1-\vartheta_{\mathfrak{G}_1})) \\ &\quad + (g_{21}+g_{22}+g_{23}+g_{24})(\mu_{\mathfrak{G}_2} + (1-\vartheta_{\mathfrak{G}_2})) \\ &\quad + (g_{31}+g_{32}+g_{33}+g_{34})(\mu_{\mathfrak{G}_3} + (1-\vartheta_{\mathfrak{G}_3})) \Big\}\end{aligned} \quad (4)$$

2.2. Distance measure of TT2FF

The distance measure between two TT2FF is a real function $\phi : TT2FF \to [0,1]$, if it has the characteristics listed below:

- $\phi(\tilde{A}, \tilde{A}) = 0$;
- $\phi(\tilde{A}, \tilde{F}) = \phi(\tilde{F}, \tilde{A})$;
- $\phi(\tilde{A}, \tilde{F}) = \phi(\tilde{A}, \tilde{D}) + \phi(\tilde{D}, \tilde{F})$;

The hamming distance between two TT2FF number

$$\tilde{G} = \{(g_{11}, g_{12}, g_{13}, g_{14}; \mu_{\mathfrak{G}_1}, \vartheta_{\mathfrak{G}_1}), (g_{21}, g_{22}, g_{23}, g_{24}; \mu_{\mathfrak{G}_2}, \vartheta_{\mathfrak{G}_2}), (g_{31}, g_{32}, g_{33}, g_{34}; \mu_{\mathfrak{G}_3}, \vartheta_{\mathfrak{G}_3}); \mu_{\mathfrak{G}}, \vartheta_{\mathfrak{G}}\}$$

$$\tilde{F} = \{(f_{11}, f_{12}, f_{13}, f_{14}; \mu_{\mathfrak{F}_1}, \vartheta_{\mathfrak{F}_1}), (f_{21}, f_{22}, f_{23}, f_{24}; \mu_{\mathfrak{F}_2}, \vartheta_{\mathfrak{F}_2}), (f_{31}, f_{32}, f_{33}, f_{34}; \mu_{\mathfrak{F}_3}, \vartheta_{\mathfrak{F}_3}); \mu_{\mathfrak{F}}, \vartheta_{\mathfrak{F}}\}$$

is defined as

$$H(\tilde{G}, \tilde{F}) = \frac{1}{16}\left\{\begin{aligned}&|(\mu_{\mathfrak{G}_1})^2 g_{11} - (\mu_{\mathfrak{F}_1})^2 f_{11}| + |(\mu_{\mathfrak{G}_1})^2 g_{12} - (\mu_{\mathfrak{F}_1})^2 f_{12}| \\ &+|(\mu_{\mathfrak{G}_1})^2 g_{13} - (\mu_{\mathfrak{F}_1})^2 f_{13}| \\ &+|(\mu_{\mathfrak{G}_1})^2 g_{14} - (\mu_{\mathfrak{F}_1})^2 f_{14}| + |(\vartheta_{\mathfrak{G}_1})^2 g_{11} - (\vartheta_{\mathfrak{F}_1})^2 f_{11}| \\ &+|(\vartheta_{\mathfrak{G}_1})^2 g_{12} - (\vartheta_{\mathfrak{F}_1})^2 f_{12}| \\ &+|(\vartheta_{\mathfrak{G}_1})^2 g_{13} - (\vartheta_{\mathfrak{F}_1})^2 f_{13}| + |(\vartheta_{\mathfrak{G}_1})^2 g_{14} - (\vartheta_{\mathfrak{F}_1})^2 f_{14}| \\ &+|g_{11} - f_{11}| + |g_{12} - f_{12}| \\ &+|g_{13} - f_{13}| + |g_{14} - f_{14}| + |(\mu_{\mathfrak{G}_2})^2 g_{21} - (\mu_{\mathfrak{F}_2})^2 f_{21}| \\ &+|(\mu_{\mathfrak{G}_2})^2 g_{22} - (\mu_{\mathfrak{F}_2})^2 f_{22}| \\ &+|(\mu_{\mathfrak{G}_2})^2 g_{23} - (\mu_{\mathfrak{F}_2})^2 f_{23}| + |(\mu_{\mathfrak{G}_2})^2 g_{24} - (\mu_{\mathfrak{F}_2})^2 f_{24}| \\ &+|(\vartheta_{\mathfrak{G}_2})^2 g_{21} - (\vartheta_{\mathfrak{F}_2})^2 f_{21}| \\ &+|(\vartheta_{\mathfrak{G}_2})^2 g_{22} - (\vartheta_{\mathfrak{F}_2})^2 f_{22}| + |(\vartheta_{\mathfrak{G}_2})^2 g_{23} - (\vartheta_{\mathfrak{F}_2})^2 f_{23}| \\ &+|(\vartheta_{\mathfrak{G}_2})^2 g_{24} - (\vartheta_{\mathfrak{F}_2})^2 f_{24}| \\ &+|g_{21} - f_{21}| + |g_{22} - f_{22}| + |g_{23} - f_{23}| \\ &+|g_{24} - f_{24}| + |(\mu_{\mathfrak{G}_3})^2 g_{31} - (\mu_{\mathfrak{F}_3})^2 f_{31}| \\ &+|(\mu_{\mathfrak{G}_3})^2 g_{32} - (\mu_{\mathfrak{F}_3})^2 f_{32}| + |(\mu_{\mathfrak{G}_3})^2 g_{33} - (\mu_{\mathfrak{F}_3})^2 f_{33}| \\ &+|(\mu_{\mathfrak{G}_3})^2 g_{34} - (\mu_{\mathfrak{F}_3})^2 f_{34}| \\ &+|(\vartheta_{\mathfrak{G}_3})^2 g_{31} - (\vartheta_{\mathfrak{F}_3})^2 f_{31}| + |(\vartheta_{\mathfrak{G}_3})^2 g_{32} - (\vartheta_{\mathfrak{F}_3})^2 f_{32}| \\ &+|(\vartheta_{\mathfrak{G}_3})^2 g_{33} - (\vartheta_{\mathfrak{F}_3})^2 f_{33}| \\ &+|(\vartheta_{\mathfrak{G}_3})^2 g_{34} - (\vartheta_{\mathfrak{F}_3})^2 f_{34}| + |g_{31} - f_{31}| + |g_{32} - f_{32}| \\ &+|g_{33} - f_{33}| + |g_{34} - f_{34}|.\end{aligned}\right. \quad (5)$$

3. Proposed mathematical model

In this section, we will cover two distinct approaches to the decision making problems. Initially, we proposed a hybrid multi-criteria decision-making method for ranking the best alternative among the available possibilities, followed by the proposed DEA strategy for converting a multi-objective transportation issue to a single objective function.

3.1. The TT2FF model for RS-MABAC method

Step 1: Acquiring knowledge about the specified issue from decision experts based on their qualities and alternatives. Develop the TT2FF assessment matrix $\mathfrak{m} = M_{ij}^{\alpha}$, where $M_{ij}^{(\alpha)}$ shows the TT2FF data of alternative A_β on characteristic $\mathfrak{C}_i$ by DMs.

Step 2: According to the relevance of the weight of criterion $\lambda_j = (\lambda_1, \lambda_2, \dots, \lambda_l)$, the overall DM matrix $M_{ij}^{(\alpha)}$ is transformed into an unified aggregated matrix

$$M_{ij} = \prod_{\alpha} M_{ij}^{\alpha(\lambda_j)}. \quad (6)$$

Step 3: Normalize the aggregated matrix using the following formula, which varies depending on the type of each attribute:
For benefit attributes: $N_{ij} = M_{ij}$
For cost attributes: $N_{ij} = M_{ij}^{c}$.

Step 4: Employing the rank sum method to determine the criteria subjective priority.

Step 4.1: Based on Eq. (6), find out the aggregated decision matrix of the given linguistic decision matrix.

Step 4.2: Convert the aggregated TT2FF numbers to score matrix by using Eq. (4).

Step 4.3: Estimate the subjective weight of the criteria using indicator weight $(t - \mathfrak{r}_j + 1)$ and $\mathfrak{r}_j$ is the preference of each indicator.

$$W_j = \frac{(t - \mathfrak{r}_j + 1)}{\sum(t - \mathfrak{r}_j + 1)}. \quad (7)$$

Step 5: The weighted normalized matrix R_{ij} can be calculated as follows by using the normalized matrix N_{ij} and the weights of the attributes W_j.

$$R_{ij} = W_j * N_{ij}.$$

Step 6: Compute the BAA matrix $\mathfrak{G}$. The element can be computed as: $\mathfrak{G} = [G]_{1i} = [R_{ij}]^{\frac{1}{j}}$.

Step 7: The distance matrix $D = [d_{ij}]$ can be computed using Eq. (5) as follows:

$$d_{ij} = \begin{cases} d[R_{ij}, G], & \text{if} \quad R_{ij} > G \\ 0, & \text{if} \quad R_{ij} = G \\ -d[R_{ij}, G], & \text{if} \quad R_{ij} > G. \end{cases} \quad (8)$$

Step 8: Add the values of each alternatives as

$$S_i = \sum d_{ij}.$$

Step 9: Rank the alternatives in relation to S_i.

3.2. Extended DEA approach on TT2FF numbers

Data envelopment analysis evaluates the efficiency of DMUs with multiple inputs and outputs. The fuzzy MOTP, in which all of the parameters and variables are represented by TT2FF numbers, has been given DEA approach. Arc features that must be reduced are referred to as fuzzy input, and arc qualities that must be maximized are referred to as fuzzy output. Two different fuzzy efficiency scores are obtained for each arc. The DEA method will be employed to investigate the optimal MOTP solution. In this manner, as a performance criterion for the single objective transportation, two TT2FFESs are computed for each arc. After that, each arc's n objectives are combined into a single attribute. A single objective TP has been determined by Lingo 18 software.

For m different fuzzy inputs x_p, each $\mathrm{DMU_q}$ produces s different fuzzy outputs y_q. The given u_g, v_p are weights of output and input of DMUs. Without loss of generality, all input and output data are assumed to be positive TT2FF number to describe this approach briefly.

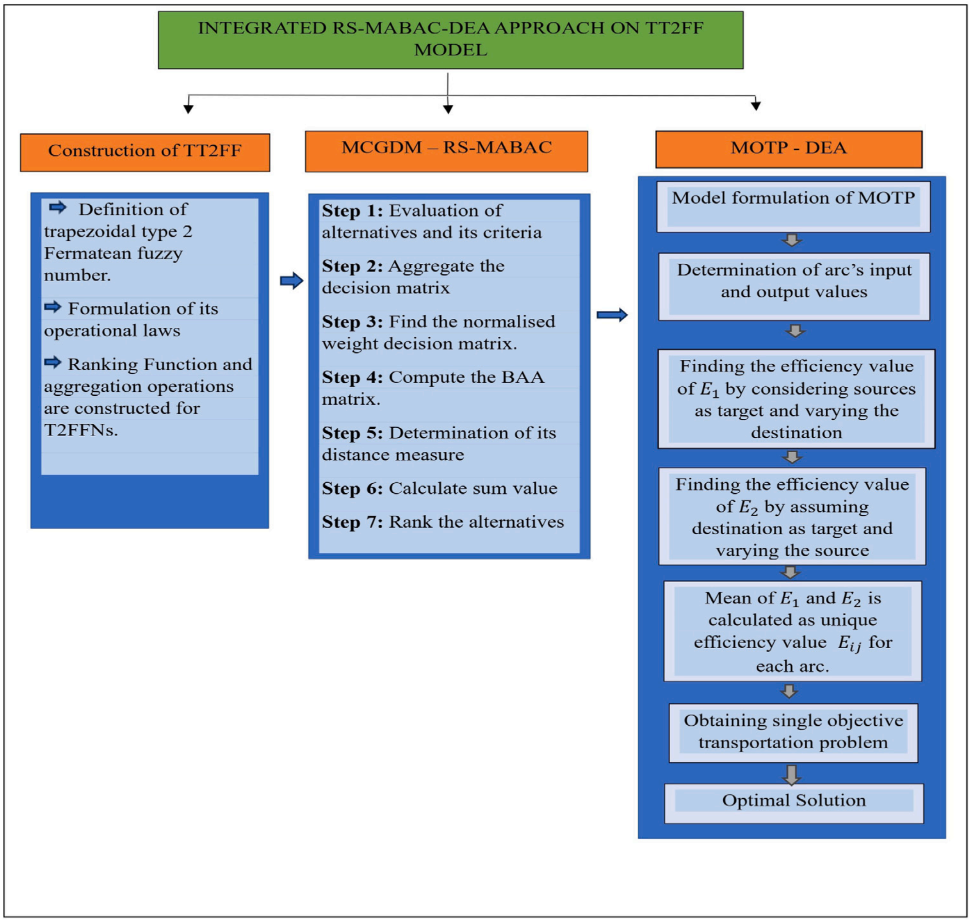

Fig. 1. Overview of the proposed model.

Algorithm 1: Depiction of pseudo code for TT2FFN-RS-MABAC

Input: An array of alternatives and criteria.

Output: Ranking of the alternatives

Step 1: Specify the decision matrix for each of the alternatives based on multiple criteria.

Step 2: Convert the linguistic terms into their corresponding TT2FFNs.

Step 3: Use average aggregation operator to get a collective decision matrix

Step 4: By using the equation (6) transform the decision matrix D into normalised decision matrix D'

Step 5: For j= 1: k

1. Convert the decision matrix into crisp value using the score function.
2. Determine the weight of the criteria w_j as per the equation (7)

End For

Step 6: For i= 1: l

For j= 1: k

Calculate the weighted normalized decision matrix $v_{ij}= w_j * d_{ij}$

Step 7: Compute the BAA matrix.

Step 8: Calculate the distance from the BAA matrix.

Step 9: Identify the total distance.

Step 10: Rank the alternatives.

End For

End For

End

The model to evaluate the relative efficiency is as follows:

$$\begin{cases} \text{Max/Min } \theta_t = \dfrac{\sum_{g=1}^{s} u_g y_{gt}}{\sum_{p=1}^{k} v_p x_{pt}}, \quad p = 1,2,\dots,m \quad g = 1,2,\dots,s \\ \text{subject to} \\ \theta_q = \dfrac{\sum_{g=1}^{s} u_g y_{gq}}{\sum_{p=1}^{k} v_p x_{pq}} \le 1, \quad q = 1,2,\dots,k; \\ u_g, v_p \ge 0. \end{cases}$$

Step 1: For the source p as a target and varying the q destination, the efficiency of $\tilde{E}^1_{pq}$ on the route p to q can be determined using the following linear programming problem:

$$\begin{cases} \tilde{E}^1_{pq} = \text{Max } \dfrac{\sum_{g=1}^{s} u_g y^g_{pq}}{\sum_{h=1}^{k} v_h x^h_{pq}} \\ \text{subject to} \\ \dfrac{\sum_{g=1}^{s} u_g y^g_{pf}}{\sum_{h=1}^{k} v_h x^h_{pf}} \le 1, \quad h = 1,2,\dots,k \quad g = 1,2,\dots,s \\ u_g, v_h \ge 0. \end{cases}$$

Step 2: Similarly for each destination q as a target, the efficiency $\tilde{E}^2_{pq}$ on the route p to q can be determined using the following program:

$$\begin{cases} \tilde{E}^2_{pq} = \text{Max } \dfrac{\sum_{g=1}^{s} u_g y^g_{pq}}{\sum_{h=1}^{k} v_h x^h_{pq}} \\ \text{subject to} \\ \dfrac{\sum_{g=1}^{s} u_g y^g_{fq}}{\sum_{h=1}^{k} v_h x^h_{fq}} \le 1, \quad h = 1,2,\dots,k \quad g = 1,2,\dots,s \\ u_g, v_h \ge 0. \end{cases}$$

The linear form of the above model are

$$\begin{cases} \tilde{E*}^1_{pq} = \text{Max} \sum_{g=1}^{s} u_g y^g_{pq} \\ \text{subject to} \\ \sum_{h=1}^{k} v_h x^h_{pq} = 1, \quad h = 1,2,\dots,k \\ \sum_{g=1}^{s} u_g y^g_{pf} - \sum_{h=1}^{k} v_h x^h_{pf} \le 0, \quad g = 1,2,\dots,s \\ u_g, v_h \ge 0, \end{cases} \tag{9}$$

$$\begin{cases} \tilde{E*}^2_{pq} = \max \sum_{g=1}^{s} u_g y^g_{pq} \\ \text{subject to} \\ \sum_{h=1}^{k} v_h x^h_{pq} = 1, \quad h = 1,2,\dots,k \\ \sum_{g=1}^{s} u_g y^g_{fq} - \sum_{h=1}^{k} v_h x^h_{fq} \le 0, \quad g = 1,2,\dots,s \\ u_g, v_h \ge 0. \end{cases} \tag{10}$$

Step 3: As a result, for every arc (p, q) two fuzzy efficiency scores $\tilde{E}^1_{pq}$ and $\tilde{E}^2_{pq}$ can be determined. The mean of $\tilde{E}^1_{pq}$ and $\tilde{E}^2_{pq}$ are used to derive a new fuzzy efficiency for arc (p, q) as it is shown in relation achieved,

$$E^*_{pq} = \frac{\tilde{E*}^1_{pq} + \tilde{E*}^2_{pq}}{2}. \tag{11}$$

Step 4: The k objectives is converted into a positive one objective $\tilde{E}_{pq}$ in the MOTP. In this way, the FMOTP is converted into the fuzzy single objective TP as follows

$$\begin{cases} \text{Maximum } Z = \sum_{p=1}^{m} \sum_{q=1}^{n} E^*_{pq} y_{pq} \\ \text{subject to} \\ \sum_{q=1}^{n} y_{pq} = a_p, \quad p = 1,2,\dots,m \\ \sum_{i=1}^{m} y_{pq} = b_q, \quad q = 1,2,\dots,n \\ y_{pq} \ge 0, \quad \text{for all } p \text{ and } q. \end{cases} \tag{12}$$

Step 5: Determine the trapezoidal type 2 Fermatean fuzzy optimum solution by solving the above model.

4. Numerical example

To illustrate the practicality of the proposed method, let us consider a medical supply transportation problem in Fig. 2 which delivers the medical products from three sources ($S1, S2, S3$) to three destinations ($D1, D2, D3$) by one of the four medical suppliers ($A1, A2, A3, A4$) is how the decision maker intends to accomplish the target goals. The target functions to optimize include:

- Optimum medical suppliers
- Minimize the transportation costs
- Maximize the transportation profits
- Minimize the Shipment value

This problem deals with two distinct types of issues. Four medical suppliers are offered for the conveyance of medicine from sources to destinations in this problem of medical transportation. Transportation policy considers several criteria that influence the selection of the best medical supplier. The decision makers intend to rank the suppliers based on five criteria, summarized as follows:

- **Supply Capacity (C1):** Supply capacity is a commitment given by suppliers to always have enough capacity to make items as agreed upon with firms.
- **Product Cost (C2):** A major deciding element when choosing a supplier is the cost of medical supplies. It is important to take into account the whole cost of ownership in addition to the supplier's pricing, discounts, and payment arrangements.
- **Logistic Speed (C3):** Identifying a supplier with the necessary speed and capabilities to quickly deliver high-quality medical supplies.

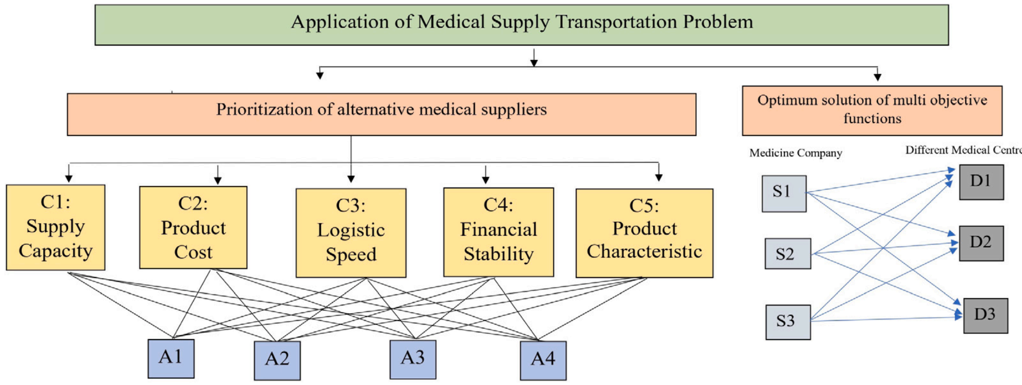

Fig. 2. Medical supply transportation problem.

Table 2
Linguistic terms and its corresponding T2FFNs.

Linguistic terms	T2FFNs
(VP) Very Poor	[(0.10,0.22,0.24,0.35;0.5, 0.1), (0.15,0.22,0.24,0.30; 0.4, 0.2),(0.20,0.22,0.24,0.25; 0.4, 0.3);0.4, 0.1]
(P) Poor	[(0.30,0.42, 0.44,0.55; 0.6,0.1), (0.35,0.42, 0.44,0.50;0.5,0.3), (0.40, 0.42, 0.44,0.45;0.4,0.3);0.4,0.3]
(M) Medium	[(0.50,0.62, 0.64, 0.75; 0.7,0.2), (0.55, 0.62, 0.64, 0.70;0.6,0.3), (0.60, 0.62, 0.64, 0.65;0.6,0.3);0.6,0.3]
(G) Good	[(0.70, 0.82, 0.84, 0.95; 0.8,0.3), (0.75, 0.82, 0.84,0.90;0.7,0.4), (0.80, 0.82, 0.84,0.85;0.7,0.4); 0.7,0.4]
(VG) Very Good	[(0.90, 1.02,1.04, 1.15; 0.9,0.4), (0.95, 1.02,1.04, 1.10; 0.8,0.5),(1.00,1.02,1.04,1.05;0.8,0.5); 0.8,0.5]

Table 3
TT2FF judgmental matrix by three decision makers.

DM	Alternatives	*C1*	*C2*	*C3*	*C4*	*C5*
JM1	*A1*	P	M	G	VG	G
	A2	VG	G	G	G	M
	A3	M	VP	M	M	M
	A4	VG	VG	G	M	G
JM 2:	*A1*	P	G	G	VG	VG
	A2	VG	G	M	G	G
	A3	M	P	M	M	P
	A4	VG	VG	M	G	VG
JM 3:	*A1*	M	G	G	G	M
	A2	VG	G	G	VG	M
	A3	M	M	G	M	G
	A4	G	G	M	G	G

Table 4
TT2FF judgmental matrix for subjective weight.

DM	*C1*	*C2*	*C3*	*C4*	*C5*
JM1	M	G	G	G	VG
JM2	G	P	M	VG	M
JM3	G	M	M	M	VG

- **Financial Stability (C4):** Deciding on a financially stable supplier for medical supplies is critical to ensuring a consistent supply of essential products, especially during crisis situations.
- **Product Characteristics (C5):** The quality of medical supplies is vital for patient safety and medical results. Taking into account of the supplier's reputation, certification and quality assurance processes, as well as product testing and validation.

4.1. Numerical computation on RS-MABAC

At this point, we have four medical suppliers, five criteria and three decision makers. The decision makers evaluate the alternative with respect to the criteria are represented by five point linguistic scale in Tables 3 and 4 by using Table 2 that shows the linguistic term and its related trapezoidal type 2 Fermatean fuzzy number. Considering the weight of criteria as $(0.33, 0.33, 0.33, 0.33, 0.33)$ for five criteria. The proposed technique is implemented to identify the best medicine supplier, and the specific phases are as follows, The collective TT2FF evaluation matrix is built using the converted TT2FF matrices. A run-through of the combined outcomes are calculated and further determine the attribute weights based on Eq. (7) of TT2FF-RS technique, Table 6 presents the results. The outcomes of the TT2FF weighted normalized matrix are documented in Table 5 in accordance with the normalized matrix and attribute weights. Determine the BAA matrix and normalized Hamming distance using Eq. (8), between the BAA matrix and the weighted normalized matrix. The total of each S_i's value is presented in Table 7 . The greatest S_i value belongs to the ideal option.

As a result the medical suppliers are ranked as $A2 > A4 > A1 > A3$. Hence $A2$ is preferable than other suppliers shown in Fig. 3.

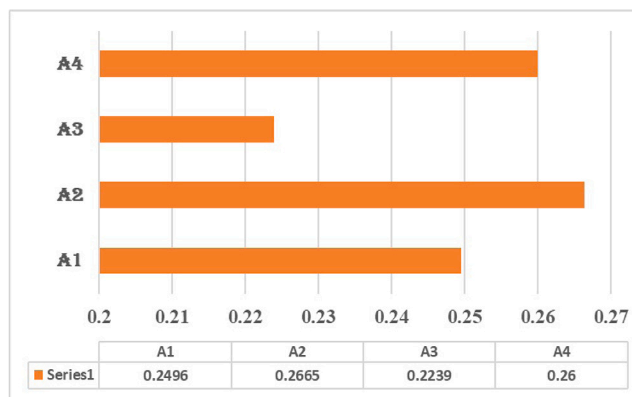

Fig. 3. Ranking of medical suppliers.

4.2. Computation on DEA approach

To solve the medicine transportation problem which delivers the medical products from three sources $(S1, S2, S3)$ to three destinations $(D1, D2, D3)$ and their corresponding cost, shipping value and profit are provided in Table 8. All three of the fuzzy qualities that are now associated with each arc (p, q) need be transformed into a positive fuzzy attribute E_{ij}, in order to choose the best solution. Each arc must thus be viewed as a DMU with two inputs and one output. As a matter of

$$
\begin{cases}
E^{1}_{A1D3} = Max\left[\begin{array}{l}
u_1([(400,425,445,470;0.8,0.2)(405,425,445,460;0.76,0.34)(410,425,445,450;0.64,0.4);0.64,0.4])/\\
v_1([(600,675,700,750;0.7,0.2)(665,675,700,740;0.6,0.3)(670,675,700,730;0.5,0.4);0.5,0.4])\\
+v_2([(2,3.25,3.55,5;0.5,0.15)(2.5,3.25,3.55,4.5;0.55,0.25)(3,3.25,3.55,4;0.55,0.25);0.55,0.25])
\end{array}\right]\\
\text{subject to}\\
\left[\begin{array}{l}
u_1([(700,755,800,900;0.6,0.15)(720,755,800,890;0.5,0.2)(740,755,800,880;0.4,0.3);0.4,0.3])/\\
v_1([(400,425,445,470;0.55,0.11)(405,425,445,460;0.42,0.23)(410,425,445,450;0.32,0.2);0.32,0.2])\\
+v_2([(4,5.25,5.45,6.5;0.6,0.2)(4.5,5.25,5.45,6;0.5,0.3)(5,5.25,5.45,5.5;0.4,0.3);0.4,0.3])
\end{array}\right]\leq 1;\\
\left[\begin{array}{l}
u_1([(600,630,670,700;0.82,0.2)(610,630,670,690;0.73,0.34)(620,630,670,680;0.62,0.37);0.62,0.37])/\\
v_1([(500,545,575,600;0.65,0.11)(510,545,575,590;0.53,0.24)(520,545,575,580;0.45,0.32);0.45,0.32])\\
+v_2([(1.5,2.75,2.95,4;0.72,0.14)(2,2.75,2.95,3.5;0.64,0.23)(2.5,2.75,2.95,3;0.5,0.32);0.5,0.32])
\end{array}\right]\leq 1;\\
\left[\begin{array}{l}
u_1([(400,425,445,470;0.8,0.2)(405,425,445,460;0.76,0.34)(410,425,445,450;0.64,0.4);0.64,0.4])/\\
v_1([(600,675,700,750;0.7,0.2)(665,675,700,740;0.6,0.3)(670,675,700,730;0.5,0.4);0.5,0.4])\\
+v_2([(2,3.25,3.55,5;0.5,0.15)(2.5,3.25,3.55,4.5;0.55,0.25)(3,3.25,3.55,4;0.55,0.25);0.55,0.25])
\end{array}\right]\leq 1;\\
u_1, v_1, v_2 \geq 1
\end{cases}
\tag{13}
$$

Box I.

Table 5
Weight normalized decision matrix.

Criteria	Alternatives	Normalized value
C1	A1	[(0.1, 0.13, 0.14, 0.17; 0.6, 0.3)(0.11, 0.13, 0.14, 0.15; 0.5, 0.3)(0.12, 0.13, 0.14, 0.14; 0.4, 0.3); 0.4, 0.3]
	A2	[(0.24, 0.28, 0.28, 0.31; 0.9, 0.5)(0.26, 0.28, 0.28, 0.3; 0.8, 0.5)(0.27, 0.28, 0.28, 0.28; 0.8, 0.5)0.8, 0.5]
	A3	[(0.14, 0.17, 0.17, 0.2; 07, 0.3)(0.5, 0.17, 0.17, 0.19; 0.6, 0.3)(0.6, 0.17, 0.17, 0.18; 0.6, 0.3)0.6, 0.3]
	A4	[(0.22, 0.26, 0.26, 0.29; 0.8, 0.5)(0.24, 0.26, 0.26, 0.28; 0.7, 0.5)(0.25, 0.26, 0.26, 0.26; 0.7, 0.5)0.7, 0.5]
C2	A1	[(0.04, 0.05, 0.05, 0.06; 0.7, 0.4)(0.05, 0.05, 0.05, 0.06; 0.6, 0.4)(0.05, 0.05, 0.05, 0.05; 0.6, 0.4)0.6, 0.4]
	A2	[(0.05, 0.06, 0.06, 0.07; 0.8, 0.4)(0.05, 0.06, 0.06, 0.06; 0.7, 0.4)(0.06, 0.06, 0.06, 0.06; 0.7, 0.4)0.7, 0.4]
	A3	[(0.02, 0.03, 0.03, 0.04; 0.5, 0.3)(0.02, 0.03, 0.03, 0.03; 0.4, 0.3)(0.03, 0.03, 0.03, 0.03; 0.4, 0.3), 0.4, 0.3]
	A4	[(0.06, 0.07, 0.07, 0.08; 0.8, 0.5)(0.06, 0.07, 0.07, 0.07; 0.7, 0.5)(0.07, 0.07, 0.07, 0.07; 0.7, 0.5)0.7, 0.5]
C3	A1	[(0.09, 0.11, 0.11, 0.12; 0.8, 0.4)(0.1, 0.11, 0.11, 0.12; 0.7, 0.4)(0.1, 0.11, 0.11, 0.11; 0.7, 0.4)0.7, 0.4]
	A2	[(0.08, 0.1, 0.1, 0.11; 0.7, 0.4)(0.08, 0.09, 0.09, 0.1; 0.6, 0.4)(0.09, 0.09, 0.09, 0.09; 0.6, 0.4)0.6, 0.4]
	A3	[(0.07, 0.09, 0.09, 0.11; 0.7, 0.4)(0.08, 0.09, 0.09, 0.1; 0.6, 0.4)(0.09, 0.09, 0.09, 0.09; 0.6, 0.4)0.6, 0.4]
	A4	[(0.07, 0.09, 0.09, 0.11; 0.7, 0.4)(0.08, 0.09, 0.09, 0.1; 0.6, 0.4)(0.09, 0.09, 0.09, 0.09; 0.6, 0.4)0.6, 0.4]
C4	A1	[(0.17, 0.19, 0.19, 0.22; 0.8, 0.5)(0.18, 0.19, 0.19, 0.21; 0.7, 0.5)(0.19, 0.19, 0.19, 0.2; 0.7, 0.5)0.7, 0.5]
	A2	[(0.15, 0.18, 0.18, 0.2; 0.8, 0.5)(0.16, 0.18, 0.18, 0.19; 0.7, 0.5)(0.17, 0.18, 0.18, 0.18; 0.7, 0.5)0.7, 0.5]
	A3	[(0.1, 0.12, 0.13, 0.15; 0.7, 0.3)(0.11, 0.12, 0.13, 0.14; 0.6, 0.3)(0.15, 0.15, 0.15, 0.16; 0.6, 0.4)0.6, 0.4]
	A4	[(0.13, 0.15, 0.15, 0.8; 0.7, 0.4)(0.14, 0.15, 0.15, 0.17; 0.6, 0.4)(0.15, 0.15, 0.15, 0.16; 0.6, 0.4)0.6, 0.4]
C5	A1	[(0.23, 0.27, 0.27, 0.31; 0.7, 0.5)(0.24, 0.27, 0.27, 0.29; 0.6, 0.5)(0.26, 0.27, 0.27, 0.28; 0.6, 0.5)0.6, 0.5]
	A2	[(0.19, 0.23, 0.23, 0.27; 0.7, 0.4)(0.2, 0.23, 0.23, 0.25; 0.6, 0.4)(0.22, 0.23, 0.23, 0.24; 0.6, 0.4)0.6, 0.4]
	A3	[(0.16, 0.2, 0.21, 0.24; 0.6, 0.4)(0.17, 0.2, 0.21, 0.23; 0.5, 0.4)(0.19, 0.2, 0.21, 0.21; 0.4, 0.4)0.4, 0.4]
	A4	[(0.25, 0.29, 0.3, 0.3; 0.8, 0.5)(0.27, 0.29, 0.3, 0.32; 0.7, 0.5)(0.28, 0.29, 0.3, 0.3; 0.7, 0.5)0.7, 0.5]

Table 6
Subjective weight of the criteria by RS method.

Criteria	Aggregated criteria matrix	Score matrix	$\mathfrak{r}_j$	$\mathbb{SW}_j$
C_1	[(0.63, 0.75, 0.77, 0.88; 0.7, 0.4)(0.68, 0.75, 0.77, 0.83; 0.6, 0.4)(0.73, 0.75, 0.77, 0.78; 0.6, 0.4), 0.6, 0.4]	1.400	2	0.267
C_2	[(0.48, 0.6, 0.62, 0.73; 0.6, 0.4)(0.53, 0.6, 0.62, 0.68; 0.5, 0.4)(0.58, 0.6, 0.62, 0.63; 0.4, 0.4), 0.4, 0.4]	1.004	5	0.067
C_3	[(0.56, 0.68, 0.7, 0.81; 0.7, 0.4)(0.61, 0.68, 0.7, 0.76; 0.6, 0.4)(0.66, 0.68, 0.7, 0.71; 0.6, 0.4), 0.6, 0.4]	1.278	4	0.133
C_4	[(0.68, 0.81, 0.83, 0.94; 0.7, 0.5)(0.73, 0.81, 0.83, 0.89; 0.6, 0.5)(0.78, 0.81, 0.83, 0.84; 0.6, 0.5), 0.6, 0.5]	1.381	3	0.2
C_5	[(0.74, 0.87, 0.89, 1, 0.7, 0.5; 0.79, 0.87, 0.89, 0.95; 0.6, 0.5)(0.84, 0.87, 0.89, 0.9; 0.6, 0.5), 0.6, 0.5]	1.484	1	0.333

fact, the functions of inputs and outputs for transportation are played by the transportation cost and shipping value serve as inputs and transportation profit as outputs, respectively.

Using the model (9) and (10) the fuzzy efficiency scores $E^{1}_{*\mathsf{pq}}$ should be generated by considering the source p as a target and modifying the destination. Similarly, for $E^{2}_{*\mathsf{pq}}$, consider destination q as the goal and change the source. For example, the model fits the arc (A1,D3) illustrated in the model (13) is given in Box I.

To solve this, the TT2FF numbers are translated to crisp model (14). The problem is solved using LINGO 18 software, yielding E^{1}_{A1D3} as

Table 7
Results on RS-MABAC method.

Alternatives	Overall values	Normalized value	Ranking
A1	2.323	0.2496	3
A2	2.48	0.2665	1
A3	2.084	0.2239	4
A4	2.419	0.2600	2

Table 8
Decision matrix proposed for medical transportation problem.

	D1	D2	D2	Supply
A1				
T.Cost	[(500, 545, 575, 600; 0.65, 0.11) (510, 545, 575, 590; 0.53, 0.24) (520, 545, 575, 580; 0.45, 0.32); 0.45, 0.32]	[(400, 425, 445, 470; 0.55, 0.11) (405, 425, 445, 460; 0.42, 0.23) (410, 425, 445, 450; 0.32, 0.2); 0.32, 0.2]	[(600, 675, 700, 750; 0.7, 0.2) (665, 675, 700, 740; 0.6, 0.3) (670, 675, 700, 730; 0.5, 0.4); 0.5, 0.4]	
T.profit	[(600, 630, 670, 700; 0.82, 0.2) (610, 630, 670, 690; 0.73, 0.34) (620, 630, 670, 680; 0.62, 0.37); 0.62, 0.37]	[(700, 755, 800, 900; 0.6, 0.15) (720, 755, 800, 890; 0.5, 0.2) (740, 755, 800, 880; 0.4, 0.3); 0.4, 0.3]	[(400, 425, 445, 470; 0.8, 0.2) (405, 425, 445, 460; 0.76, 0.34) (410, 425, 445, 450; 0.64, 0.4); 0.64, 0.4]	15
S.value	[(1.5, 2.75, 2.95, 4; 0.72, 0.14) (2, 2.75, 2.95, 3.5; 0.64, 0.23) (2.5, 2.75, 2.95, 3; 0.5, 0.32); 0.5, 0.32]	[(4, 5.25, 5.45, 6.5; 0.6, 0.2) (4.5, 5.25, 5.45, 6; 0.5, 0.3) (5, 5.25, 5.45, 5.5; 0.4, 0.3); 0.4, 0.3]	[(2, 3.25, 3.55, 5; 0.5, 0.15) (2.5, 3.25, 3.55, 4.5; 0.55, 0.25) (3, 3.25, 3.55, 4; 0.55, 0.25); 0.55, 0.25]	
A2				
T.Cost	[(325, 340, 345, 400; 0.8, 0.1) (330, 340, 345, 370; 0.75, 0.3) (335, 340, 345, 350; 0.6, 0.45); 0.6, 0.45]	[(308, 325, 335, 350; 0.76, 0.15) (315, 325, 335, 345; 0.64, 0.25) (320, 325, 335, 340; 0.54, 0.25); 0.54, 0.25]	[(335, 370, 380, 410; 0.63, 0.26) (345, 370, 380, 400; 0.55, 0.35) (355, 370, 380, 390; 0.5, 0.4); 0.5, 0.4]	
T.profit	[(590, 610, 640, 700; 0.9, 0.15) (595, 610, 640, 690; 0.85, 0.24) (600, 610, 640, 680; 0.8, 0.4); 0.8, 0.4]	[(685, 720, 730, 7700; 0.63, 0.26) (695, 7720, 730, 760; 0.55, 0.35) (705, 720, 730, 750; 0.5, 0.4); 0.5, 0.4]	[(900, 925, 935, 960; 0.8, 0.15) (910, 925, 935, 950; 0.75, 0.2) (920, 925, 935, 940; 0.6, 0.34); 0.6, 0.34]	10
S.value	[(3, 4.28, 4.6, 6; 0.8, 0.2) (3.5, 4.28, 4.6, 5.5; 0.7, 0.3) (4, 4.28, 4.6, 5; 0.63, 0.4); 0.63, 0.4]	[(1, 2.15, 2.45, 3.5; 0.8, 0.15) (1.5, 2.15, 2.45, 3; 0.75, 0.2) (2, 2.15, 2.45, 2.5; 0.65, 0.32); 0.65, 0.32]	[(4, 5.32, 5.4, 6; 0.72, 0.14) (4.5, 5.32, 5.4, 5.8; 0.64, 0.25) (5, 5.32, 5.4, 5.5; 0.64, 0.25); 0.64, 0.25]	
A3				
T.Cost	[(420, 437, 447, 470; 0.75, 0.32) (425, 437, 447, 460; 0.63, 0.45) (435, 437, 447, 450; 0.52, 0.4); 0.52, 0.4]	[(425, 440, 475, 500; 0.72, 0.12) (430, 440, 475, 490; 0.65, 0.25) (435, 440, 475, 480; 0.53, 0.32); 0.53, 0.32]	[(555, 580, 590, 620; 0.8, 0.2) (560, 580, 590, 610; 0.7, 0.3) (565, 580, 590, 600; 0.6, 0.4); 0.6, 0.4]	
T.profit	[(500, 550, 570, 700; 0.75, 0.2) (510, 550, 570, 690; 0.6, 0.25) (520, 550, 570, 680; 0.65, 0.25); 0.65, 0.25]	[(365, 388, 398, 420; 0.72, 0.12) (375, 388, 398, 410; 0.65, 0.25) (385, 388, 398, 400; 0.53, 0.32); 0.53, 0.32]	[(1000, 1025, 1045, 1070; 0.65, 0.15) (1010, 1025, 1045, 1060; 0.54, 0.2) (1015, 1025, 1045, 1050; 0.45, 0.32); 0.45, 0.32]	25
S.value	[(2, 3.5, 3.7, 5; 0.6, 0.1) (2.7, 3.5, 3.7, 4.5; 0.55, 0.24) (3.4, 3.5, 3.7, 4; 0.45, 0.34); 0.45, 0.34]	[(3, 4.55, 5.55, 6.8; 0.65, 0.15) (3.5, 4.55, 5.55, 6.5; 0.54, 0.25) (4, 4.55, 5.55, 6; 0.45, 0.25); 0.45, 0.25]	[(5, 6.45, 6.85, 7.5; 0.64, 0.15) (5.5, 6.45, 6.85, 7.2; 0.55, 0.24) (6, 6.45, 6.85, 7; 0.45, 0.34); 0.45, 0.34]	
Demand	15	20	15	

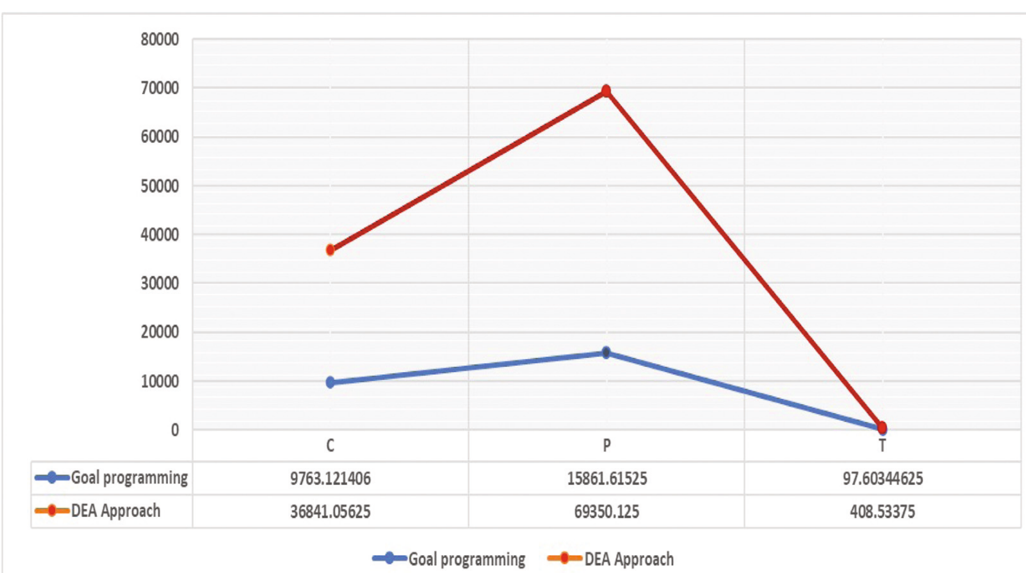

Fig. 4. Comparison graph of MOTP.

Table 9
E^1_{pq}.

	D1	D2	D3
A1	2.40345	2.54578	1.80751
A2	2.13629	2.54048	1.02708
A3	1.19348	2.81899	1.88086

Table 10
E^2_{pq}.

	D1	D2	D3
A1	2.40345	2.54578	1.80751
A2	2.13629	2.54048	1.02708
A3	1.19348	2.81899	1.88086

1.80751. Similarly, the values of E^1_{pq} and E^2_{pq} may be calculated for all other arcs. Tables 9 and 10 provide the comparable efficiency values for additional arcs.

$$\begin{aligned}
&\text{Maximum } Z = 1247u_1;\\
&\text{subject to}\\
&825.69v_1 + 7.046v_2 = 1;\\
&1384.5u_1 - 1098.9v_1 - 5.838v_2 \le 1;\\
&1508.5u_1 - 747.594v_1 - 9.45v_2 \le 1;
\end{aligned} \tag{14}$$

The new efficiency E^*_{pq} is determined by taking the average of E^1_{pq} and E^2_{pq} as in Eq. (11). Using that in Model (12) to express the single objective function as follows:

$$\begin{cases}
\text{Maximum } = 2.4199x_{11} + 2.0764x_{12} + 1.755x_{13} + 2.3609x_{21}\\
\qquad +2.3042x_{22} + 0.9710x_{23}\\
\qquad +1.2166x_{31} + 2.8462x_{32} + 2.3173x_{33};\\
\text{subject to}\\
x_{11} + x_{12} + x_{13} \le 15;\\
x_{21} + x_{22} + x_{23} \le 10;\\
x_{31} + x_{32} + x_{33} \le 25;\\
x_{11} + x_{21} + x_{31} \le 15;\\
x_{12} + x_{22} + x_{32} \le 20;\\
x_{13} + x_{32} + x_{33} \le 15;
\end{cases} \tag{15}$$

Finally, at the end, by solving the model (15), a trapezoidal type 2 Fermatean fuzzy transportation plan with the maximum efficiency is determined as follows:

$$x_{11} = 5, \quad x_{12} = 10, \quad x_{22} = 10, \quad x_{31} = 10, \quad x_{33} = 15,$$

The fuzzy objective functions of transportation cost, profit and shipment value are presented in Table 11 respectively.

5. Comparison and discussion

Validating the accessible RS-MABAC techniques, we contrast with the CODAS method in computational complexity effectiveness and ability to assess every alternative's relative performance. Then the result of the holding approach is $A1 = -0.0728, A2 = 0.7812, A3 = -0.4369, A4 = 0.7049$. The CODAS technique is ranked as follows: $A2 > A4 > A1 > A3$. Based on the results from the CODAS techniques mentioned above, we conclude that the suggested strategy is reliable and effectively enhances the outcomes.

On the other hand, an extended goal programming technique is now used to compare the proposed DEA technique for the medical supply transportation problem. A popular method for simplifying a TP via multiple objective functions into a single objective function is GP. Reducing the distance between goal functions and an aspiration level vector or that is calculated by the decision maker is the idea behind GP. Assume that the under and over deviations of the objectives f from their aspiration levels are represented by the numbers n^+ and n^-.

$$\begin{cases}
Max/Min\ Z_r = \text{Optimize} \sum_{i=1}^{m}\sum_{j=1}^{n} c^r_{ij} y_{ij},\\
\text{subject to}\\
\sum_{j=1}^{n} y_{ij} \le a_i, \quad i = 1,2,\dots,m\\
\sum_{i=1}^{m} y_{ij} \le b_j, \quad j = 1,2,\dots,n\\
y_{ij} \ge 0, \quad \text{for all } i \text{ and } j.
\end{cases} \tag{16}$$

Here, Z_r represent the corresponding goal functions of model (16) that ought to be minimized and maximized. Thus, GP converts the model (16) into a deviational parameter minimization problem that minimizes the sum of the deviation parameters in the manner described below:

$$\begin{cases}
\text{Minimum} \sum_{r=1}^{m} n^+ + \sum_{r=m+1}^{m+s} n^-\\
\text{subject to}\\
\sum_{i=1}^{m}\sum_{j=1}^{n} c^r_{ij} y_{ij} \le n^+ + Z_r\\
\sum_{i=1}^{m}\sum_{j=1}^{n} c^r_{ij} y_{ij} + n^- \ge Z_r\\
\sum_{j=1}^{n} y_{ij} \le a_i, \quad i = 1,2,\dots,m\\
\sum_{i=1}^{m} y_{ij} \le b_j, \quad j = 1,2,\dots,n\\
n^+ \ge 0, \quad r = 1,2,\dots,g\\
n^- \ge 0, \quad r = g+1, g+2\dots, s\\
y_{ij} \ge 0, \quad \forall\ i \text{ and } j.
\end{cases} \tag{17}$$

The model (17) is a linear program that can be solved using the simplex approach. Hence, by using the GP technique the problem is solved and the values for transportation cost, profit and shipment value are presented in Table 11 and Fig. 4. Therefore, the proposed method is more suitable for finding solutions for MOTP under TT2FF environment.

5.1. Advantage of the proposed model

- **Enhancing version of Uncertainty:** The implementation of trapezoidal type-2 Fermatean fuzzy numbers offers a superior method for addressing errors and uncertainties in decision-making relative to conventional fuzzy methodologies such as intuitionistic fuzzy sets and Pythagorean fuzzy sets .
- **Integrated Decision-Making Model:** The suggested RS-MABAC hybrid method improves reliability in multi-criteria decision-making by efficiently selecting health care suppliers while accounting for various competing factors.
- **Optimization of MOTP:** Through the incorporation of data envelopment analysis, the research enhances medical supply transportation using a multi-objective strategy that reconciles cost reduction, shipping value diminishing, and profit enhancement.
- **Multi-Criteria and Multi-Objective Flexibility:** The framework is engineered to address both multi-criteria decision-making and multi-objective transportation problems, rendering it a versatile tool for logistics and supply chain management.
- **Applicability to Complex Problems:** The suggested hybrid strategy based on TT2FF is flexible and applicable to a range of real-world decision-making situations beyond the transportation of medical supplies.

6. Conclusion

This paper proposes a hybrid technique to solve MCDM with MOTP, using the medical supply transportation problem as a real-world example. Human judgments are typically less accurate than numerical values. In order to describe the uncertain information, we develop a

Table 11
Comparison of optimum results.

	T.Cost	T.Profit	S.Value
Existing GP	[(36919, 2250, 2352, 2483; 0.55, 0.2)	[(34815, 7770, 8225, 9230; 0.6, 0.15)	[(201, 54, 56, 67; 0.6, 0.2)
	(2145, 2250, 2352, 2431; 0.42, 0.3)	(3817, 3995, 4225, 4678; 0.5, 0.2)	(24, 28, 29, 32; 0.5, 0.3)
	(2172, 2250, 2352, 2379; 0.32, 0.4); 0.32, 0.4]	(3918, 3995, 4225; 0.4, 0.32); 0.4, 0.32]	(26, 28, 29, 29; 0.4, 0.34); 0.4, 0.34]
Proposed DEA	[(22105, 23295, 23995, 25200; 0.55, 0.32)	[(36850, 38775, 40025, 43250; 0.6, 0.26)	[(153, 220, 234, 283; 0.6, 0.2)
	(22400, 23295, 23995, 24750; 0.42, 0.45)	(37450, 38775, 40025, 42750; 0.5, 0.35)	(180, 220, 234, 261; 0.5, 0.3)
	(22725, 23295, 23995, 24300; 0.32, 0.4); 0.32, 0.4]	(37975, 38775, 40025, 42250; 0.4, 0.4); 0.4, 0.4]	(207, 220, 234, 240; 0.4, 0.34); 0.4, 0.34]

trapezoidal type 2 Fermatean fuzzy number. As a result, all of the parameters are regarded as TT2FF numbers in both the MCDM and MOTP. Our method has two distinct characteristics. To begin, we first introduce the RS-MABAC technique, an innovative multi-criteria decision-making strategy, which uses a few evaluation criteria for a transportation problem to determine which medical transportation supplier is the best among those that are accessible. Furthermore, the DEA approach was created to sort out the MOTP. Using this approach, each arc in the FFMOTP has been handled as a DMU. Additionally, the objective functions that ought to be minimized and maximized have been used to define the DMU's inputs and outputs values respectively. For each arc, two different efficiency scores have been obtained by solving the DEA models. These efficiency scores have then been averaged to create a unique efficiency score for each arc. In this way, the MOTP has been transformed into a single objective TP. Further, the LINGO-18.0 program was used to create and solve an explicit framework. For that reason, we conclude that our model is very important in real-world scenarios; it provides the decision maker with a unique perspective.

Limitation:

This work has some shortcomings that need to be addressed with future research.

- This study contains limited criteria and choices for correlative MCDM problems, and the assessment index system should include more sustainable criteria.
- The proposed method cannot be utilized to compute the fuzzy optimum solution for unbalanced MOTPs.

Future Study:

Future research will attempt to solve the constraints mentioned above. In addition, we can list the following unresolved issues that will require further investigation and discussion

- The proposed model can be adapted to various transportation models and multi-criteria decision-making methodologies. The transportation model can be expanded to encompass fractional transportation problems, quadratic transportation problems, and four-dimensional transportation issues involving pricing discounts, transit time limits, and breakable or decaying objects.
- Using machine learning approaches to shorten the time needed to compute lengthy decision-making processes and to create additional hybrid MCDM strategies in the expansive and dynamic TT2FF environment.

CRediT authorship contribution statement

P. Anukokila: Writing – review & editing, Visualization, Validation, Supervision, Project administration, Investigation, Conceptualization. **R. Nisanthini:** Writing – original draft, Software, Methodology, Data curation. **B. Radhakrishnan:** Writing – review & editing, Validation, Supervision, Formal analysis, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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