DESIGNING AN ECONOMIC REPETITIVE GROUP SAMPLING PLAN FOR LIFE TESTS BASED ON PERCENTILES OF EXPONENTIATED RAYLEIGH DISTRIBUTION

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Abstract

In this study, a unique Repetitive Group Sampling Plan (RGSP) for life tests is presented, emphasizing the use of percentiles obtained from the Exponentiated Rayleigh Distribution. To enhance the accuracy, effectiveness, and general reliability of life testing techniques, the suggested sample plan provides a strong foundation for assessing the longevity and reliability of goods in a variety of industries. By offering a strong framework for evaluating the robustness and lifespan of products in diverse industries, the suggested sample plan seeks to improve the efficiency and reliability of life testing methods. Through the incorporation of statistical techniques grounded in the Exponentiated Rayleigh Distribution, this research advances the fields of reliability assessment and quality control in industrial processes. Table values are calculated by using Python programming. The implementation of these statistical techniques not only enhances the accuracy of life testing but also enables businesses to make more informed decisions regarding product development and quality assurance. As industries increasingly rely on data-driven approaches, this research provides a valuable resource for optimizing performance and minimizing costs

Keywords: Exponentiated Rayleigh Distribution, Percentiles, Life tests, Repetitive Group Sampling Plan.

I. Introduction

Life tests are essential for guaranteeing the durability and dependability of products in the domains of quality control and reliability evaluation. Conventional techniques frequently depend on preset sampling intervals and fixed sample sizes, which may not be sufficient to capture the dynamic character of product life cycles. Our research presents a Repetitive Group Sampling Plan (RGSP) designed specifically for live tests in order to overcome this issue. The main contribution is the application of the Exponentiated Rayleigh Distribution, a robust statistical model with great flexibility and versatility that can be used to assess the dependability of products under different stress scenarios. To estimate a product's life characteristics, life testing entails putting a sample of products under operational stress until they break. Taking into account the inherent variety in product lifetimes, the suggested RGSP offers an efficient and adaptable solution to life testing. Our methodology seeks to provide a more accurate picture of product durability and reliability by including the Exponentiated Rayleigh Distribution, which has been shown to be successful in estimating failure times. Because of its versatility in simulating a wide range of failure time patterns, the Exponentiated Rayleigh Distribution is a development of the classical Rayleigh Distribution—has become more well-known in the field of reliability engineering. This distribution has been effectively used in earlier research to examine life statistics from a variety of industries, including the electronics, automotive, and aerospace sectors. The distribution is well-suited to representing the complicated failure features seen in real-world circumstances since it can handle both growing and decreasing hazard rates.

Adaptive sampling strategies are necessary to account for dynamic changes in product reliability, despite the fact that traditional life testing methods have depended on set sample sizes and constant sampling intervals. This has been highlighted by recent research. Building on this tendency, the study's suggested Repetitive Group Sampling Plan (RGSP) provides a versatile and effective method for life testing. The RGSP attempts to achieve a compromise between statistical rigor and practical implementation by combining percentiles generated from the Exponentiated Rayleigh Distribution. This makes it a helpful tool for industries looking to improve product reliability and quality assurance. To summarize, the limitations of conventional approaches are addressed by this article's groundbreaking Repetitive Group Sampling Plan for life testing, which makes use of the Exponentiated Rayleigh Distribution. Improving dependability assessment and quality control procedures in a variety of manufacturing industries is greatly promising when these statistical approaches are integrated. The attributes repeating group sampling plan for a normal distribution is suggested by [14]. This repeated sampling method functions similarly to a sequential sampling method. He claims that this sampling plan provides the necessary protection for both the producer and the customer, in addition to providing the minimal sample size. Moreover, sequential sampling is less efficient than repeated sampling, but it is still more efficient than a single sampling strategy. Among other academics, examined the factors in a repeated group acceptance sampling plan and contrasted the outcomes with a single sample plan developed by [4]. On the other hand, no effort has been made to investigate attributes repeating sampling plans for an Exponentiated Rayleigh Distribution based on truncated life tests. Assuming that the lifetime of the products follows the Exponentiated Rayleigh Distribution with a known shape parameter and that the quality level is represented by the ratio of mean life to the stipulated life, we designed an attributes repetitive sampling plan for the truncated life tests in this study. There have been notable developments in the fields of reliability evaluation and quality control, and researchers are always working to improve life testing techniques. This review of literature delves into the corpus of work that has been written about a certain subject: "A Repetitive Group Sampling Plan for Life Tests Based on Percentiles of Exponentiated Rayleigh Distribution." This novel method offers a promising path toward enhanced reliability assessment by fusing the ideas of the Exponentiated Rayleigh Distribution with the Repetitive Group Sampling Plans (RGSP).

The methodology of time-truncated acceptance sample plans has fascinated a number of eminent authors. The truncated life tests in the exponential case developed [5]. Acceptance sampling plan with life test objectives are framed [15]. Using the sampling plans based on the Weibull distribution is proposed [6]. In [7] acceptance sampling plan based on Gamma Distribution is established. For [8, 9] acceptance sampling based on truncated life tests based on Log-Logistic distribution. Acceptance sampling based on truncated life tests for Pareto Distribution are suggested [2].

Acceptance sampling based on truncated life tests for Generalized Rayleigh Distribution introduced [17]. Acceptance sampling based on truncated life tests for Birnbaun-Sauders Distribution [3], Generalized Exponential Distribution [1], and Marshall-Olkin Extended Lomax Distribution [13]. Designing of acceptance sampling plan for life test based on percentiles of exponentiated Rayleigh distributions are developed [10, 11, 12]. Skip-lot sampling plan of type SkSP-T for life test based on percentiles of Exponentiated Rayleigh Distribution suggested [16].

The main goal of the paper is to develop a life-test plan for repetitive group sampling using an exponential Rayleigh distribution and a percentile-based comparison plan. An important distribution for life testing and reliability analysis is the Rayleigh exponential distribution. It shows excellent mathematical consistency and possesses some of the fundamental structural characteristics. The Exponentiated Rayleigh distribution shares many characteristics with the Weibull, gamma, and exponential distributions. The density and distribution functions of ERD have comparable shapes. This is swiftly expanded to the abbreviated plans as a result. The ERD distribution's cumulative function is provided by,

$$F(t;\tau,\theta) = \left[1 - e^{-1/2(t/\tau)^2}\right]^{\theta},\ t>0,\ 1/\tau>0,\ \theta>0 \quad (1)$$

Where the parameters for scale and form are, respectively, τ and θ. The probability density function of a cumulative distribution function is its first derivative. Thus, the ERD's probability density function can be expressed as,

$$f(t;\tau,\theta) = \theta\left[1 - e^{-1/2(t/\tau)^2}\right]^{\theta-1}\left[\frac{t}{\tau^2} e^{-1/2(t/\tau)^2}\right] \quad (2)$$

Based on the ERD percentiles, [10, 11] suggested SSP and DSP for life testing. Then, using the ERD percentiles as a guide, [12] created skip-lot sampling designs for life testing.

II. Repetitive group sampling plans based on truncated test

The following is a description of the suggested features repeating group sampling plan based on abbreviated life tests:
Step 1: For a fixed time t_0, take a random sample of size n from a lot and put it through a life test.
Step 2: If D, the number of failures, is less than or equal to c_1, the acceptance number, then accept the lot. When D surpasses c_2, where $c_2 \geq c_1$, halt the test and reject the lot.
Step 3: If $c_1 < D < c_2$, then repeat step 1.
Three parameters, n, c_1, and c_2, define the repeating group sampling scheme described above. It is significant to remember that the ordinary single sampling plan is reduced to the attributes repetitive group sampling plan, which is a generalization of it when $c_1 = c_2$. The Operating Characteristic (OC) function is used to calculate the likelihood of lot acceptance.

III. Operating Procedure for Repetitive Group Sampling Plan

Plan Parameters Determination:
i. N: Specify the sample size for each group.
ii. c_1: Determine the acceptance number.
iii. c_2: Establish the rejection number.
iv. Maximum Number of Groups (Optional): Decide if a limit on samples is needed.

Initial Sampling:
i. Randomly select N items from the lot.
ii. Inspect each item for defects.

First Sample Evaluation:
i. Count the defective items (d) in the first sample.

Initial Decision:
i. Accept: If $d \leq c_1$, accept the lot.
ii. Reject: If $d \geq c_2$, reject the lot.
iii. Proceed to Step 5: If $c_1 < d < c_2$.

Repeat Sampling (If Necessary):
i. Take another random sample of N items.
ii. Inspect and count defectives in the second sample.
iii. Combine results from both samples.

iv. Accept: If cumulative defectives ≤ c_1.
v. Reject: If cumulative defectives ≥ c_2.
vi. Repeat: If c_1 < cumulative defectives < c_2, repeat steps 5 and 6.

Maximum Number of Groups (Optional):

i. If a maximum is set and no decision is reached within that limit, typically reject the lot.

IV. Operating procedure of Repetitive Group Sampling Plan based on percentiles of ERD

In [14] devised the repeated group sampling plan, which involves drawing a sample and counting the number of defective samples. In order to sentence a lot, a fixed criterion determines whether the lot should be accepted, rejected, or wholly ignored, in which case one must start over with a fresh sample. This process is repeated until the fixed criterion instructs us to accept or reject the lot. The following is the operating procedure for a repeating group sample plan:

Step 1. Select a random sample of size 'n' from a lot of size 'N'.

Step 2. Inspect all the articles included in the sample. Let 'd' be the number of defectives in the sample.

Step 3. If d< c_1, accept the lot If d > c_2, reject the lot If c_1 < d ≤ c_2, repeat the steps 1, 2 and 3.

V. Operating characteristic function for Repetitive Group Sampling Plan

The most often used method for assessing a sample plan's effectiveness and determining the likelihood of adoption is the OC function. It offers the likelihood of the lot being approved. The following is the OC function of RGSP for life tests based on the ERD percentiles:

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{3}$$

[1] Percentile of the lifetime t_q^0 and p depends only on$\delta_0 = t/t_q^0$.

The following is the operation characteristic function for the repetitive group sampling plan

$$L(P) = \frac{P_a}{P_a + P_r} \tag{4}$$

Here P_a and P_r is the binomial model and equation (4) becomes

$$P_a(p) = \frac{\sum_{i=0}^{C_1} \binom{n}{i} P^i (1-P)^{n-i}}{1 - \sum_{i=0}^{C_1} \binom{n}{i} P^i (1-P)^{n-i} + \sum_{i=0}^{C_2} \binom{n}{i} P^i (1-P)^{n-i}} \tag{5}$$

Then, the Average Sample number is

$$ASN\ (p) = ASN\ (R)F \tag{6}$$

The reference plan's average sample number is denoted by R-, while the chance of the reference plan being accepted is indicated by P.

I. Illustration

Suppose that the life time of the electric goods follows ERD. Repetitive group sampling with SSP as reference plan using binomial distribution based on 25th percentile is applied for testing. The parameters for the life testing is as follows: θ=2, t=40 hrs,t0.25=40 hrs, c=2, α=0.05 and β=0.05 then η = 1.56712 from the equation and the ratio is found to be $t/t_{0.25}$=2 by applying the minimum sample size according to the requirements is n=10 and the corresponding OC values L(p) for the Single Sampling plan for the life tests based on percentiles of ERD (n, c, $t/t_{0.25}$=2) = (10,2,1.10) with P^*= 0.95. L(p) is the P value for RGS with SSP for life tests based on the percentiles of ERD as reference plan. For n=10, c_1=2 and c_2=1; the probability of acceptance L(p) values of RGS with

SSP for life tests based on percentiles of ERD are found from eqn. 5. From the illustrations, it is indicated that the actual 25th percentile is almost equal to the required 25th percentile ($t_{0.25}$/ t / $t^0_{0.25}$=1.00) the producer's risk is approximately 0.9800 (1-0.0200). Also, the producer's risk is nearly equal to 0.05 or less and the actual producer risk is large or nearly equal to 1.95 times of the required percentile.

Table 1: *Gives the OC values for sampling plan (n, c_1, c_2, t/$t_{0.25}$) for a given P* under ERD when θ=2*

P*	t/$t_{0.25}$	t/$t_{0.25}$						
		0.7	0.9	1	1.5	2	2.5	3
0.75	0.7	0.3910	0.8421	0.9701	0.9911	0.9980	0.9993	0.9998
0.75	0.9	0.4548	0.8741	0.9660	0.9899	0.9984	0.9994	0.9998
0.75	1	0.4778	0.8989	0.9710	0.9900	0.9987	0.9996	0.9999
0.9	0.7	0.1956	0.7100	0.8989	0.9801	0.9942	0.9990	0.9996
0.9	0.9	0.2421	0.7539	0.9223	0.9847	0.9950	0.9991	0.9998
0.9	1	0.3879	0.8123	0.9527	0.9897	0.9950	0.9993	0.9999
0.95	0.7	0.1333	0.6747	0.9198	0.9697	0.9927	0.9975	0.9990
0.95	0.9	0.1515	0.6981	0.9387	0.9698	0.9943	0.9982	0.9996
0.95	1	0.0925	0.5841	0.8996	0.9596	0.9876	0.9970	0.9985
0.99	0.7	0.0131	0.4555	0.8321	0.9499	0.9732	0.9941	0.9980
0.99	0.9	0.0220	0.4606	0.8387	0.9512	0.9817	0.9955	0.9982
0.99	1	0.0318	0.5001	0.8495	0.9571	0.9860	0.9961	0.9989

VI. Conclusion

This research concludes with a new approach to life testing: a Repetitive Group Sampling Plan (RGSP) based on the percentiles of the Exponentiated Rayleigh Distribution. A strong basis for assessing a product's reliability and durability over time is provided by the sampling strategy's integration of this statistical model. By using the flexibility and adaptability of the Exponentiated Rayleigh Distribution, our proposed RGSP provides a flexible and efficient method to evaluate product lifetimes under different stress conditions. By providing enterprises with a useful tool for guaranteeing the durability and dependability of their products, this research advances quality control and reliability evaluation. The methodology that has been offered shows potential in mitigating the drawbacks of conventional life testing techniques, offering a more precise depiction of product dependability, and supporting the continuous advancement of quality control procedures across various manufacturing industries.

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