Trajectory Controllability of Non-linear Fractional Stochastic System involving statedependent delay and impulsive effects

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Abstract

This paper studies the analysis of Trajectory (T-) controllability for the fractional order neutral stochastic impulsive integrodifferential system involving statedependent delay (SDD) and impulsive effects. Sufficient conditions are designed to illustrate the evaluation of T-controllability via Gronwall's inequality. It is exhibited that the proposed protocol can explicitly drive the results by Mönch fixed point technique and semigroup theory. As a final point, the derived scheme is validated through an example.

Keywords: Fractional impulsive system, Gronwall's inequality, Nonlinear stochastic system, State-dependent delay, T-controllability.

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1 Introduction

The fractional calculus, which deals with integral and derivatives of noninteger order, is a generalization of classical calculus. One of the most effective methods for describing long-range interactions, power laws, long-memory processes, and geometrical scaling rules is fractional calculus. Therefore, fractional differential equations (FDEs) are the corresponding mathematical models. The variety of applications of FDEs can be observed, for example, in the fields of mechanics (viscoelasticity theory), biology (protein modeling), robotics, signal processing, traffic and control systems, finance, and economy. For more details on FDEs, we refer the reader to the books [1, 2, 3, 4, 5], and the references therein.

Moreover, the stochastic differential systems provide a powerful tool in formulation and analysis of the phenomenon which fluctuates due to random influences or noise and therefore this theory can be successfully applied to various problems not only in mathematics but outside also, see [6, 7, 8, 9]. For example, the Wiener process is utilized the noise essential to the stock exchange, where the millions of agents react independently and behave irrationally (see [10]). In recent years, scholars have focused especially on solving the stochastic dynamical models with mixed fBm, Rosenblatt process, and Poisson jumps; see [11, 12, 13, 14] and references therein.

On the other hand, the most important aspect of mathematical control theory is delivered by the notion of controllability. The controllability problem is searching for a suitable control function that steers the proposed dynamical model to a desired final state. For fractional stochastic evolution equations, the theory and applications of the existence of mild solutions and controllability are investigated in [15]. Ahmed et al. [16, 17] established the exact null controllability and boundary controllability of Hilfer fractional stochastic differential equations (SDEs) with fBm. The approximate controllability of a semilinear stochastic integrodifferential system with nonlocal conditions is studied in [18] by utilizing the Sadovskii's fixed point theorem. The new notion of T-controllability was first introduced by Chalishajar et al. [19], to detect for control that steers the system along a prescribed trajectory to the final state instead of navigating a given initial state to the required final destination. The advantages of T-controllability include; minimizing the cost involved in steering the system from an initial state to desired final state and safeguarding the system. For example, while launching a rocket in space sometimes it may be desirable to have a precise path along with the desired destination for cost-effectiveness and collision avoidance. So naturally, T-control is the strongest notion than all other existing control definitions. The first and second order T-controllability in infinite dimension with numerical simulation was initiated by Chalishajar et at. [20, 21]. A few years back, Malik and George [22] looked into the T-controllability of a fractional order system. Recently, Malik and his team [23, 24], studied the T-controllability for nonlinear FDEs by employing Gronwall's inequality. Very recently, Chalishajar and his team [25, 26, 27], investigated the T-controllability of stochastic dynamical system with deviated argument using Rosenblatt process and Poisson jumps.

After the success of theory and applications of fractional calculus for both deterministic and stochastic systems, how to extend them to the case of involving various delays, naturally became a predominant research field. Only a few kinds of results have been studied in previous research regarding the topic of T-controllability for fractional impulsive stochastic systems, particularly with finite and infinite delays. To best of our knowledge, when the semigroups appeared in above fractional stochastic neutral integrodifferential systems are noncompact, it is not easy to obtain the corresponding compact resolvent operators. Also, there is no published paper has considered the impulsive fractional stochastic neutral integrodifferential systems incorporating time and SDD along with nonlocal conditions. Motivated by these statements, it is essential to consider this type of interesting problem. The analysis also takes into account the contributions, highlighted below:

• A significant number of previous research on fractional systems have been published with delay, such as finite, infinite, or without delay. Consequently, it is essential to pay consideration attention to the analysis of fractional stochastic systems with time and SDD.

• Many of previous results on fractional stochastic integrodifferential systems have been published without taking into account nonlocal and impulsive effects. The study of the T-contollability of fractional systems involving impulsive and nonlocal behavior is more essential.

• The semigroups appeared in the stochastic systems are compact [28, 29, 30], assuming the corresponding compact resolvent operator. We have proved the results using noncompact resolvent operator.

• The aim of this work is to study the mild solutions for a class of T-controllability for the fractional order neutral stochastic impulsive integrodifferential systems involving nonlocal condition and time and SDD by using noncompact semigroup in a Hilbert space. Furthermore, under some suitable assumptions, the considered system's T-controllability is established using generalized Gronwall's inequality.

The article is structured as follows: Section 2 describes the essential preliminaries and some notations. Section 3 proves the existence of mild solution for fractional order neutral stochastic impulsive integrodifferential systems involving nonlocal conditions and time and SDD through Hausdorff measure of noncompactness (HMNC) and the Mönch fixed point theorem. Section 4 demonstrates T-controllability results using Gronwall's inequality. Section 5 justifies the proposed theoretical results with the aid of an example.

2 Problem Formulation and Preliminaries

Consider the fractional order neutral stochastic impulsive integrodifferential system involving SDD and impulsive effects,

$$\begin{aligned}
{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[\mathfrak{z}(\nu)-\mathfrak{l}(\nu,\mathfrak{z}_{\nu})\right] &= \Big[\mathscr{A}\mathfrak{z}(\nu)+\mathscr{B}\mathfrak{u}(\nu)+\int_{0}^{\nu}\lambda(\nu-\zeta)\mathfrak{m}(\zeta,\mathfrak{z}_{\zeta})d\zeta\Big]d\nu \\
&+ \mathfrak{n}\left(\nu,\mathfrak{z}_{\rho(\nu,\mathfrak{z}_{\nu})}\right)d\omega(\nu),\ \nu\in\mathscr{J}=[0,\mathfrak{b}], \\
\triangle\mathfrak{z}|_{\nu=\nu_{k}} &= \mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}})), \\
\mathfrak{z}(0)+\mu(\mathfrak{z}) &= \mathfrak{z}_{0}=\tilde{\phi}\in\mathfrak{B}. \qquad (2.1)
\end{aligned}$$

${}^{c}\mathcal{D}_{\nu}^{\alpha}$ is the Caputo derivative with order $\alpha\in(0,1)$, the state variable $\mathfrak{z}(\nu)$ in Hilbert space $\mathscr{H}$. $\mathscr{K}$ represents a separable Hilbert space with $\|\cdot\|_{\mathscr{K}}$. Let $\omega(\nu)_{\nu\geq 0}$ denotes a Wiener process involving covariance operator $\mathscr{Q}\geq 0$ and $\mathscr{K}-$valued function described on the space $(\Omega,\Im,\mathcal{P})$ with the filtration $\Im_{\nu}$, $\nu\in\mathscr{J}$ generated by Wiener process with the probability measure $\mathcal{P}$ on Ω. $\mathscr{A}$ represents an infinitesimal generator of C_0 semigroup $\mathcal{T}(\nu)$ for $\nu \geq 0$ on $\mathscr{H}$, control function $\mathfrak{u} \in \mathcal{L}_2(\mathscr{J}, \mathscr{U})$, $\mathscr{U}$ is a Hilbert space and $\mathscr{B}$ is a bounded linear operator from $\mathscr{U}$ to $\mathscr{H}$. $\mathfrak{z}_\zeta : (-\infty, \mathfrak{b}] \to \mathscr{H}$ on the phase space $\mathfrak{B}$ (defined later) denoted by $\mathfrak{z}_\zeta(\theta) = \mathfrak{z}(\zeta + \theta)$ and $\rho : \mathscr{J} \times \mathfrak{B} \to (-\infty, \mathfrak{b}]$ is a continuous function. Let $\mathfrak{l} : \mathscr{J} \times \mathfrak{B} \to \mathscr{H}$, $\mathfrak{m} : \mathscr{J} \times \mathfrak{B} \to \mathscr{H}$ and $\mathfrak{n} : \mathscr{J} \times \mathfrak{B} \to \mathcal{L}_{\mathscr{Q}}(\mathscr{K}, \mathscr{H})$ be suitable functions and $(\lambda(\nu))_{\nu \geq 0}$ is a bounded linear operator. Let $PC(\mathscr{J}, \mathcal{L}_2(\Omega, \Im, \mathcal{P}; \mathscr{H})) = \{\mathfrak{z}(\nu)$ be continuous everywhere except for some ν_{k}, where $\mathfrak{z}(\nu_{\mathsf{k}}^+)$ & $\mathfrak{z}(\nu_{\mathsf{k}}^-)$ exist with $\mathfrak{z}(\nu_{\mathsf{k}}^-) = \mathfrak{z}(\nu_{\mathsf{k}})$, $\mathsf{k} = 1, 2, \cdots, m$ with $\|\mathfrak{z}\|_{PC} = \sup_{\nu \in \mathscr{J}} |\mathfrak{z}(\nu)| < \infty\}$. Also, $\mathcal{I}_{\mathsf{k}} : \mathfrak{B} \to \mathscr{H}$ and $0 = \nu_0 < \nu_1 < \cdots < \nu_m < \nu_{m+1} = \mathfrak{b}$. Furthermore, $B_{\mathfrak{r}}(\mathfrak{z})$ denotes the closed-ball with center at $\mathfrak{z}$ and radius $\mathfrak{r} > 0$.

Let $\mathfrak{B}$ be a phase space for measurable functions $\Im_0 : \mathscr{J}_0 = (-\infty, 0] \to \mathscr{H}$ with $\|\cdot\|_{\mathfrak{B}}$ fulfills the succeeding conditions:

(a) On $[0, \mathfrak{b})$, if $\gamma : (-\infty, \mathfrak{b}) \to \mathscr{H}$ is continuous and $\gamma_0 \in \mathfrak{B}$, then the following constraints gets satisfied for each $\nu \in [0, \mathfrak{b})$:

(i) $\gamma_\nu \in \mathfrak{B}$;

(ii) $\|\gamma(\nu)\| \leq \mathscr{K}_1 \|\gamma_\nu\|_{\mathfrak{B}}$;

(iii) $\|\gamma_\nu\|_{\mathfrak{B}} \leq \mathscr{K}_2(\nu)\|\gamma_0\|_{\mathfrak{B}} + \mathscr{K}_3(\nu) \sup \|\gamma(\zeta)\|$; $0 \leq \zeta \leq \mathfrak{b}$, where $\mathscr{K}_1 > 0$ is a constant, $\mathscr{K}_2 : [0, \infty) \to [0, \infty)$ symbolizes locally bounded function and $\mathscr{K}_3 : [0, \infty) \to [0, \infty)$ is a continuous function. $\mathscr{K}_1$, $\mathscr{K}_2$ and $\mathscr{K}_3$ are independent of $\mathfrak{z}$.

(b) $\mathfrak{B}$ be a complete space.

Assume the $\Im-$adapted measurable process $\mathfrak{z} : (-\infty, \mathfrak{b}] \to \mathscr{H}$ such that $\Im_0-$adapted process $\mathfrak{z}_0 = \tilde{\phi}(\nu) \in \mathcal{L}_2(\Omega, \mathfrak{B})$ gives

$$\mathbb{E}\|\mathfrak{z}_\nu\|^2_{\mathfrak{B}} \leq \overline{\mathscr{K}_2} \mathbb{E}\|\tilde{\phi}\|^2_{\mathfrak{B}} + \overline{\mathscr{K}_3} \sup_{\nu \in \mathscr{J}} \{\mathbb{E}\|\mathfrak{z}(\zeta)\|^2\},$$

where $\overline{\mathscr{K}_2} = \sup_{\nu \in \mathcal{J}} \mathscr{K}_2(\nu)$, $\overline{\mathscr{K}_3} = \sup_{\nu \in \mathscr{J}} \mathscr{K}_3(\nu)$.

Lemma 2.1. *[28] For each $\nu \in \mathscr{D}$, $\mathscr{D} = (-\infty, 0]$ and $\tilde{\phi} \in \mathfrak{B}$ with $\tilde{\phi}_\nu \in \mathfrak{B}$. Assume that there exists $\mathscr{H}^{\tilde{\phi}} : \mathscr{D} \to [0, \infty)$ for $\nu \in \mathscr{D}$ such that $\mathbb{E}\|\tilde{\phi}_\nu\|^2_{\mathfrak{B}} \leq \mathscr{H}^{\tilde{\phi}}(\nu)\mathbb{E}\|\tilde{\phi}\|^2_{\mathfrak{B}}$. Assume the function $\mathfrak{z} : (-\infty, \mathfrak{b}] \to \mathscr{H}$ such that $\mathfrak{z}_0 = \tilde{\phi}$ and $\mathfrak{z} \in PC(\mathcal{J}, \mathcal{L}_2)$ gives*

$$\mathbb{E}\|\mathfrak{z}_\zeta\|^2_{\mathfrak{B}} \leq \left(\overline{\mathscr{H}_2} + n\right) \mathbb{E}\|\tilde{\phi}\|^2_{\mathfrak{B}} + \overline{\mathscr{H}_3} \sup\{\mathbb{E}\|\mathfrak{z}(\theta)\|^2;\ \theta \in [0, \max\{0, \zeta\}]\},\ \zeta \in (-\infty, \mathfrak{b}).$$

Here $n = \sup_{\nu \in \mathscr{D}} \mathscr{H}^{\tilde{\phi}}(\nu)$, $\overline{\mathscr{H}_2} = \sup_{\nu \in \mathcal{J}} \overline{\mathscr{K}_2}(\nu)$ and $\overline{\mathscr{H}_3} = \sup_{\nu \in \mathcal{J}} \overline{\mathscr{K}_3}(\nu)$.

Definition 2.1. *[28] The fractional integral of order $\kappa > 0$, with the lower limit 0 for a function $\mathfrak{f}$,*

$$\mathcal{I}^\kappa \mathfrak{f}(\nu) = \frac{1}{\Gamma(\kappa)} \int_0^\nu \frac{\mathfrak{f}(\zeta)}{(\nu - \zeta)^{\kappa - 1}} d\zeta, \quad \nu > 0,$$

where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2. *[28] The Caputo derivative of order $\kappa > 0$, with the lower limit 0 for a function $\mathfrak{f}$ given by*

$$^{c}\mathcal{D}_{\nu}^{\alpha}\mathfrak{f}(\nu) = \frac{1}{\Gamma(n-\kappa)}\int_0^{\nu}\frac{\mathfrak{f}^{n}(\zeta)}{(\nu-\zeta)^{\kappa+1-n}}, \quad \nu > 0.$$

Next, we recall some facts of the HMNC $\aleph(\cdot)$ defined on each bounded subset $\mathscr{E}$ of Banach space $\mathbb{X}$ by

$$\aleph(\mathscr{E}) = \inf\{\epsilon > 0;\ \mathscr{E} \text{ has a finite } \epsilon- \text{ net in } \mathbb{X}\}.$$

Lemma 2.2. *[25] Let $\mathbb{X}$ be a real Banach space and $\mathscr{E}, \mathscr{F} \subset \mathbb{X}$ be bounded, the following properties hold:*

(1) $\mathscr{E}$ *is precompact if and only if* $\aleph(\mathscr{E}) = 0$;

(2) $\aleph(\mathscr{E}) = \aleph(\overline{\mathscr{E}}) = \aleph(conv\mathscr{E})$, *where* $\overline{\mathscr{E}}$ *and conv* $\mathscr{E}$ *are the closure and convex hull of* $\mathscr{E}$;

(3) $\aleph(\mathscr{E}) \leq \aleph(\mathscr{F})$ *when* $\mathscr{E} \subset \mathscr{F}$;

(4) $\aleph(\mathscr{E}+\mathscr{F}) \leq \aleph(\mathscr{E}) + \aleph(\mathscr{F})$, *where* $\mathscr{E}+\mathscr{F} = \{\mathsf{x}+\mathsf{y};\ \mathsf{x} \in \aleph(\mathscr{E}),\ \mathsf{y} \in \aleph(\mathscr{F})\}$;

(5) $\aleph(\mathscr{E}) \cup \mathscr{F} \leq \max\{\aleph(\mathscr{E}), \aleph(\mathscr{F})\}$;

(6) $\aleph(\lambda\mathscr{E}) \leq |\lambda|\aleph(\mathscr{F})$ *for any* $\lambda \in \mathbb{R}$;

(7) *if* $\mathbb{K} \subset \mathcal{C}(\mathscr{J})$ *is bounded, then*

$$\aleph(\mathbb{K}(\nu)) \leq \aleph(\mathbb{K}) \quad \textit{for all} \quad \nu \in \mathscr{J},$$

where $\mathbb{K}(\nu) = \{\mathfrak{u}(\nu) : \mathfrak{u} \in \mathbb{K} \subset \mathbb{X}\}$. *Further, if* $\mathbb{K}$ *is equicontinuous on* $\mathscr{J}$, *then* $\nu \to \mathbb{K}(\nu)$ *is continuous on* $\mathscr{J}$, *and*

$$\aleph(\mathbb{K}) = \sup\{\mathbb{K}(\nu) : \quad \nu \in \mathscr{J}\};$$

(8) *if* $\mathbb{K} \subset \mathcal{C}(\mathscr{J};\mathbb{X})$ *is bounded and equicontinuous, then* $\nu \to \aleph(\mathbb{K}(\nu))$ *is continuous on* $\mathscr{J}$ *and*

$$\aleph\left(\int_0^{\nu}\mathbb{K}(s)ds\right) \leq \int_0^{\nu}\aleph(\mathbb{K}(s))ds, \forall\nu \in \mathscr{J},$$

where

$$\int_0^{\nu}\mathbb{K}(s)ds = \left\{\int_0^{\nu}\mathfrak{u}(s)ds : \quad \mathfrak{u} \in \mathbb{K}\right\};$$

(9) *let* $\{\mathfrak{u}_n\}_{n=1}^{\infty}$ *be a sequence of Bochner integrable functions from* $\mathscr{J}$ *to* $\mathbb{X}$ *with* $\|\mathfrak{u}_n(\nu)\| \leq \hat{\mathfrak{m}}(\nu)$ *for almost all* $\nu \in \mathscr{J}$ *and every* $n \geq 1$, *where* $\hat{\mathfrak{m}}(\nu) \in \mathcal{L}(\mathscr{J};\mathbb{R}^{+})$, *then the function* $\phi(\nu) = \aleph(\{\mathfrak{u}_n\}_{n=1}) \in \mathcal{L}(\mathscr{J};\mathbb{R}^{+})$ *satisfies*

$$\aleph\left(\left\{\int_0^{\nu}\mathfrak{u}_n(s)ds : \ n \geq 1\right\}\right) \leq 2\int_0^{\nu}\psi(s)ds.$$

Lemma 2.3. *[25] If $\mathbb{K} \subset \mathcal{C}(\mathscr{J};\mathcal{L}_2^0(\mathbb{Y},\mathbb{X}))$, ω is a standard Wiener process, then*

$$\aleph\left(\int_0^\nu \mathbb{K}(s)d\omega(s)\right) \leq \sqrt{T}\aleph(\mathbb{K}(\nu)),$$

where

$$\int_0^\nu \mathbb{K}(s)d\omega(s) = \left\{\int_0^\iota \mathfrak{u}(s)d\omega(s); \forall \mathfrak{u} \in \mathbb{K}, \quad \nu \in \mathscr{J}\right\}.$$

Lemma 2.4. *[25] Suppose that $\mathbb{D}$ is a closed convex subset of $\mathbb{X}$, $0 \in \mathbb{D}$. If the map $\Phi : \mathbb{D} \to \mathbb{X}$ is continuous and of Mönch type, (i.e.) Φ satisfies,*

$$\mathscr{M} \subset \mathbb{D}, \quad \mathscr{M} \text{ is countable}, \quad \mathscr{M} \subset \overline{co}\left(\{0\} \cup \Phi(\mathscr{M})\right),$$

this implies $\overline{\mathscr{M}}$ is compact, then Φ has a fixed point in $\mathbb{D}$.

Definition 2.3. *A stochastic process $\mathfrak{z} : \mathscr{J} \times \mathfrak{B} \to \mathscr{H}$ is known as mild solution for the system (2.1) if the subsequent conditions hold:*

(i) $\mathfrak{z}(\nu)$ *is $\Im_\nu-$adapted and measurable for each $\nu \geq 0$.*

(ii) *For $\mathfrak{z}(\nu) \in \mathscr{H}$,*

$$\begin{aligned}
\mathfrak{z}(\nu) \quad &= \quad \mathbb{U}(\nu)\left[\mathfrak{z}_0 - \mu(\mathfrak{z}) - \mathfrak{l}(0,\overline{\phi})\right] + \mathfrak{l}(\nu,\mathfrak{z}_\nu) + \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\mathfrak{l}(\zeta,\mathfrak{z}_\zeta)d\zeta \\
&+ \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathscr{B}\mathfrak{u}(\zeta)d\zeta \\
&+ \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\lambda(\zeta-\tau)\mathfrak{m}(\tau,\mathfrak{z}_\tau)d\tau\right)d\zeta \\
&+ \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta) \\
&+ \sum_{0<\nu_{\mathsf{k}}<\nu} \mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}})), \nu \in \mathscr{J}. \qquad (2.2)
\end{aligned}$$

3 Main Results

The following hypotheses are taken into consideration

(H1) $\mathscr{A}$ is the infinitesimal generator of a C_0- semigroup of bounded linear operators $\mathcal{T}(\nu)$ in $\mathscr{H}$, there exist constants $\mathscr{M}_\beta$, $\mathscr{M}$ and $\mathscr{M}_{1-\beta} \ni \|\mathscr{A}^{-\beta}\| = \mathscr{M}_\beta$, $\|\mathcal{T}(\nu)\| \leq \mathscr{M}$ and $\|\mathscr{A}^{1-\beta}\mathcal{T}(\nu)\| \leq \mathscr{M}_{1-\beta}$, $\forall$ $\nu \in \mathscr{J}$.

(H2) (i) $\mathfrak{l}$ is continuous and $\exists$ $\mathscr{M}_\mathfrak{l} > 0 \ni$

$$\begin{aligned}
\mathbb{E}\left\|\mathscr{A}^\beta\mathfrak{l}(\nu,\mathfrak{z})\right\|_{\mathscr{H}}^2 &\leq \mathscr{M}_\mathfrak{l}\left(1 + \|\mathfrak{z}\|_\beta^2\right), \\
\mathbb{E}\left\|\mathscr{A}^\beta\mathfrak{l}(\nu,\mathfrak{z}_1) - \mathscr{A}^\beta\mathfrak{l}(\nu,\mathfrak{z}_2)\right\|^2 &\leq \mathscr{M}_\mathfrak{l}\|\mathfrak{z}_1 - \mathfrak{z}_2\|_\beta^2, \quad \mathfrak{z}_1, \mathfrak{z}_2 \text{ and } \mathfrak{z} \in \mathfrak{B}, \quad \nu \in \mathscr{J}.
\end{aligned}$$

(ii) $\exists$ a positive function $\mathbb{K}_{\mathfrak{l}}(\nu) \in \mathcal{L}'(\mathscr{J}, \mathbb{R}^+)$ for arbitrary bounded subset $\mathscr{Q} \subset \mathscr{H}$, the Hausdorff non-compact measure β satisfies

$$\beta(\mathfrak{l}(\nu, \mathscr{Q})) \leq \mathbb{K}_{\mathfrak{l}}(\nu) \sup_{-\mathfrak{r} \leq \theta \leq 0} \beta(\mathscr{Q}(\theta)).$$

(H3) (i) $\mathfrak{m}$ is continuous and $\exists\ \mathscr{M}_{\mathfrak{m}} > 0 \ni$

$$\begin{aligned} \mathbb{E}\left\|\mathscr{A}^{\beta}\mathfrak{m}(\nu, \mathfrak{z})\right\|^2_{\mathscr{H}} &\leq \mathscr{M}_{\mathfrak{m}}\left(1 + \|\mathfrak{z}\|^2_{\beta}\right), \\ \mathbb{E}\left\|\mathscr{A}^{\beta}\mathfrak{m}(\nu, \mathfrak{z}_1) - \mathscr{A}^{\beta}\mathfrak{m}(\nu, \mathfrak{z}_2)\right\|^2 &\leq \mathscr{M}_{\mathfrak{m}}\|\mathfrak{z}_1 - \mathfrak{z}_2\|^2_{\beta}, \quad \mathfrak{z}_1, \mathfrak{z}_2 \ and \ \mathfrak{z} \in \mathfrak{B}, \quad \nu \in \mathscr{J}. \end{aligned}$$

(ii) $\exists$ a positive function $\mathbb{K}_{\mathfrak{m}}(\nu) \in \mathcal{L}'(\mathscr{J}, \mathbb{R}^+)$ for arbitrary bounded subset $\mathscr{Q} \subset \mathscr{H}$, the Hausdorff non-compact measure β satisfies

$$\beta(\mathfrak{m}(\nu, \mathscr{Q})) \leq \mathbb{K}_{\mathfrak{m}}(\nu) \sup_{-\mathfrak{r} \leq \theta \leq 0} \beta(\mathscr{Q}(\theta)).$$

(H4) (i) ν is continuous and $\exists\ \mathscr{M}_{\mu} > 0 \ni$

$$\begin{aligned} \mathbb{E}\left\|\mathscr{A}^{\beta}\mu(\nu, \mathfrak{z})\right\|^2_{\mathscr{H}} &\leq \mathscr{M}_{\mu}\left(1 + \|\mathfrak{z}\|^2_{\beta}\right), \\ \mathbb{E}\left\|\mathscr{A}^{\beta}\mu(\nu, \mathfrak{z}_1) - \mathscr{A}^{\beta}\mu(\nu, \mathfrak{z}_2)\right\|^2 &\leq \mathscr{M}_{\mu}\|\mathfrak{z}_1 - \mathfrak{z}_2\|^2_{\beta}, \quad \mathfrak{z}_1, \mathfrak{z}_2 \ and \ \mathfrak{z} \in \mathfrak{B}, \quad \nu \in \mathscr{J}. \end{aligned}$$

(ii) $\exists$ a positive function $\mathbb{K}_{\mu}(\nu) \in \mathcal{L}'(\mathscr{J}, \mathbb{R}^+)$ for arbitrary bounded subset $\mathscr{Q} \subset \mathscr{H}$, the Hausdorff non-compact measure β satisfies

$$\beta(\mu(\nu, \mathscr{Q})) \leq \mathbb{K}_{\mu}(\nu) \sup_{-\mathfrak{r} \leq \theta \leq 0} \beta(\mathscr{Q}(\theta)).$$

(H5) $\mathcal{I}_{\mathsf{k}} : \mathfrak{B} \to \mathscr{H}$ is continuous and $\exists\ \mathscr{M}_{\mathsf{k}} > 0$ such that $\mathfrak{z} \in \mathfrak{B}$,

$$\begin{aligned} \mathbb{E}\|\mathcal{I}_{\mathsf{k}}(\mathfrak{z})\|^2 &\leq \mathscr{M}_{\mathsf{k}}\left(\mathbb{E}\|\mathfrak{z}\|^2\right), \\ \lim_{\mathfrak{r} \to \infty} \inf \frac{\mathscr{M}_{\mathsf{k}}(\mathfrak{r})}{\mathfrak{r}} &= \eta_{\mathsf{k}} < \infty, \quad \mathsf{k} = 1, 2, \cdots, n. \end{aligned}$$

(H6) $\mathfrak{n} : \mathscr{J} \times \mathfrak{B} \to \mathscr{H}$ satisfies the following:

(i) Let $\mathfrak{z} : (-\infty, \mathfrak{b}) \to \mathscr{H}$ be such that $\mathfrak{z}_0 = \tilde{\phi}$ and $\mathfrak{z}/\mathscr{J} \in PC$. Also, $\nu \to \mathfrak{n}(\nu, \mathfrak{z}_{\rho(\nu, \mathfrak{z}_{\nu})})$ is measurable on $\mathscr{J}$ and for every $\zeta \in \mathscr{J}$, $\nu \to \mathfrak{n}(\zeta, \mathfrak{z}_{\nu})$ is continuous.

(ii) The continuous non-decreasing function $\mathscr{M}_{\mathfrak{n}} : [0, \infty) \to (0, \infty)$ and $\exists\ m : \mathscr{J} \to [0, \infty) \ni$

$$\mathbb{E}\|\mathfrak{n}(\nu, \mathfrak{z})\|^2 \leq m(\nu)\mathscr{M}_{\mathfrak{n}}\left(\|\mathfrak{z}\|^2_{\mathfrak{B}}\right), \quad (\nu, \mathfrak{z}) \in \mathscr{J} \times \mathfrak{B}.$$

(iii) $\mathfrak{n}$ is continuous and $\exists\ \mathscr{M}_{\mathfrak{n}} \in \mathcal{L}^1(\mathscr{J}, \mathbb{R}^+)$ such that

$$\mathbb{E}\|\mathfrak{n}(\nu, \mathfrak{z}_1) - \mathfrak{n}(\nu, \mathfrak{z}_2)\|^2 \leq \mathscr{M}_{\mathfrak{n}}\|\mathfrak{z}_1 - \mathfrak{z}_2\|^2_{\beta}, \quad \mathfrak{z}_1, \mathfrak{z}_2 \ and \ \mathfrak{z} \in \mathfrak{B}, \quad \nu \in \mathscr{J}.$$

(iv) $\exists$ a positive function $\mathbb{K}_{\mathfrak{n}}(\nu) \in \mathcal{L}'(\mathscr{J}, \mathbb{R}^+)$ for arbitrary bounded subset $\mathscr{Q} \subset \mathscr{H}$, the Hausdorff non-compact measure β satisfies

$$\beta(\mathfrak{n}(\nu, \mathscr{Q})) \leq \mathbb{K}_{\mathfrak{n}}(\nu) \sup_{-\mathfrak{r} \leq \theta \leq 0} \beta(\mathscr{Q}(\theta)), \ \mathbb{K}^*_{\mathfrak{n}} = \sup_{\nu \in \mathscr{J}} \mathbb{K}_{\mathfrak{n}}(\nu).$$

Theorem 3.1. *If the hypotheses (H1)-(H6) hold, then there exists at least one mild solution of the system (2.1).*

Proof. Let $\mathfrak{B}_{\mathfrak{r}} = \{\mathfrak{z} \in PC(\mathscr{J}, \mathcal{L}_2)\}$ be the space furnished with uniform convergence topology. Lemma 2.1 yields that $\mathbb{E}\|\mathfrak{z}_\nu\|^2_{\mathfrak{B}} \leq (\mathscr{H}_2 + \eta)\mathbb{E}\|\tilde{\phi}\|^2_{\mathfrak{B}} + \overline{\mathscr{H}_3}\mathfrak{r} := \mathfrak{r}^*$, we have

$$\begin{aligned}(\Phi\mathfrak{z})(\nu) &= \mathbb{U}(\nu)\left[\mathfrak{z}_0 - \mu(\mathfrak{z}) - \mathfrak{l}(0,\overline{\phi})\right] + \mathfrak{l}(\nu,\mathfrak{z}_\nu) + \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\mathfrak{l}(\zeta,\mathfrak{z}_\zeta)d\zeta \\ &+ \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathscr{B}\mathfrak{u}(\zeta)d\zeta + \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\lambda(\zeta-\tau)\mathfrak{m}(\tau,\mathfrak{z}_\tau)d\tau\right)d\zeta \\ &+ \int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta) + \sum_{0<\nu_{\mathsf{k}}<\nu}\mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}})), \nu \in \mathscr{J}.\end{aligned}$$

Then the problem of finding mild solution for (2.1) is reduced to finding the fixed point of Φ. Let $\mathfrak{B}_{\mathfrak{r}} = \{\mathfrak{z} \in \mathfrak{B} : \|\mathfrak{z}\|^2_{\mathfrak{B}} \leq \mathfrak{r}\}$ stands for the closed ball with center at $\mathfrak{z}$ and radius $\mathfrak{r} > 0$ in $\mathfrak{B}$. We may divide the proof into several steps.

Step 1: We prove that $\exists\ \mathfrak{r} \ni \Phi$ maps $\mathfrak{B}_{\mathfrak{r}}$ into $\mathfrak{B}_{\mathfrak{r}}$.

$$\begin{aligned}\mathbb{E}\|(\Phi\mathfrak{z})(\nu)\|^2 &\leq 9\mathbb{E}\left\|\mathbb{U}(\nu)\mathfrak{z}_0\right\|^2 + 9\mathbb{E}\left\|\mathbb{U}(\nu)\mu(\mathfrak{z})\right\|^2 + 9\mathbb{E}\left\|\mathbb{U}(\nu)\mathfrak{l}(0,\Phi)\right\|^2 + 9\mathbb{E}\left\|\mathfrak{l}(\nu,\mathfrak{z}_\nu)\right\|^2 \\ &+ 9\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\mathfrak{l}(\zeta,\mathfrak{z}_\zeta)d\zeta\right\|^2 \\ &+ 9\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathscr{B}\mathfrak{u}(\zeta)d\zeta\right\|^2 \\ &+ 9\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\lambda(\zeta-\tau)\mathfrak{m}(\tau,\mathfrak{z}_\tau)d\tau\right)d\zeta\right\|^2 \\ &+ 9\mathbb{E}\left\|\sum_{0<\nu_{\mathsf{k}}<\nu}\mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}}))\right\|^2 \\ &+ 9\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta)\right\|^2 \\ &\leq 9\mathscr{M}^2\mathbb{E}\|\mathfrak{z}_0\|^2 + 9\mathscr{M}^2\mathscr{M}_\mu(1+\mathfrak{r}^*) + 9\mathscr{M}^2\mathbb{E}\left\|\mathfrak{l}(0,\Phi)\right\|^2 + 9\mathscr{M}^2_\beta\mathscr{M}_{\mathfrak{l}}(1+\mathfrak{r}^*) \\ &+ \frac{9\mathscr{M}^2_{1-\beta}\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{l}}(1+\mathfrak{r}^*) + \frac{9\mathscr{M}^2_{\mathfrak{B}^2}\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\|\mathfrak{u}\|^2_{\mathcal{L}^2_{\mathcal{F}}} \\ &+ \frac{9\mathscr{M}^2\lambda^*\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{m}}(1+\mathfrak{r}^*) + \frac{9m\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{n}}(1+\mathfrak{r}^*) + 9\mathscr{M}^2 n\sum_{i=1}^n \mathscr{M}_{\mathsf{k}}\mathfrak{r}^*.\end{aligned}$$

If we assume that $\Phi(\mathfrak{B}_{\mathfrak{r}}) \nsubseteq \mathfrak{B}_{\mathfrak{r}}$, then for every positive constant $\mathfrak{r} > 0 \ni \mathbb{E}\|\Phi\mathfrak{z}^{\mathfrak{r}}\|^2 > \mathfrak{r}$,

$$\begin{aligned}\mathfrak{r} &< \mathbb{E}\|(\Phi\mathfrak{z}^{\mathfrak{r}})(\nu)\|^2 \leq 9\mathscr{M}^2\mathbb{E}\|\mathfrak{z}_0\|^2 + 9\mathscr{M}^2\mathscr{M}_\mu(1+\mathfrak{r}^*) + 9\mathscr{M}^2\mathbb{E}\left\|\mathfrak{l}(0,\Phi)\right\|^2 + 9\mathscr{M}^2_\beta\mathscr{M}_{\mathfrak{l}}(1+\mathfrak{r}^*) \\ &+ \frac{9\mathscr{M}^2_{1-\beta}\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{l}}(1+\mathfrak{r}^*) + \frac{9\mathscr{M}^2_{\mathfrak{B}^2}\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\|\mathfrak{u}\|^2_{\mathcal{L}^2_{\mathcal{F}}} + 9\mathscr{M}^2 n\sum_{i=1}^n \mathscr{M}_{\mathsf{k}}\mathfrak{r}^* \\ &+ \frac{9\mathscr{M}^2\lambda^*\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{m}}(1+\mathfrak{r}^*) + \frac{9m\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\mathscr{M}_{\mathfrak{n}}(1+\mathfrak{r}^*)\end{aligned}$$

Dividing by $\mathfrak{r}$ throughout and let $\mathfrak{r} \to \infty$,

$$1 < 9\left[\mathscr{M}^2\mathscr{M}_\mu + \mathscr{M}^2_\beta\mathscr{M}_{\mathfrak{l}} + \frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\left(\mathscr{M}^2_{1-\beta}\mathscr{M}_{\mathfrak{l}} + \lambda^*\mathscr{M}_{\mathfrak{m}} + m\mathscr{M}_{\mathfrak{n}}\right) + \mathscr{M}^2 n\sum_{\mathsf{k}=1}^n \eta_{\mathsf{k}}\right],$$

which contradicts our assumption. Thus, $\Phi(\mathfrak{B}_{\mathfrak{r}}) \subset \mathfrak{B}_{\mathfrak{r}}$, $\forall\ \mathfrak{r} > 0$.

Step 2: We prove that Φ is continuous in $\mathfrak{B}_{\mathfrak{r}}$. Let $\{\mathfrak{z}_n\} \to \mathfrak{z}$ in $\mathfrak{B}_{\mathfrak{r}}$ (as $n \to \infty$), then

$$\begin{aligned}
\mathbb{E}\left\|(\Phi\mathfrak{z}^n)(\nu) - (\Phi\mathfrak{z})(\nu)\right\|^2 \quad &\leq \quad 6\mathbb{E}\left\|\mathbb{U}(\nu)\left[\mu(\mathfrak{z}^n) - \mu(\mathfrak{z})\right]\right\|^2 + 6\mathbb{E}\left\|\mathfrak{l}(\nu, \mathfrak{z}^n_\nu) - \mathfrak{l}(\nu, \mathfrak{z}_\nu)\right\|^2 \\
&+ \quad 6\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\left[\mathfrak{l}(\zeta, \mathfrak{z}^n_\zeta) - \mathfrak{l}(\zeta, \mathfrak{z}_\zeta)\right]d\zeta\right\|^2 \\
&+ \quad 6\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\int_0^\zeta \lambda(\zeta-\tau)\left[\mathfrak{m}(\tau, \mathfrak{z}^n_\tau) - \mathfrak{m}(\tau, \mathfrak{z}_\tau)\right]d\tau\right)d\zeta\right\|^2 \\
&+ \quad 6\mathbb{E}\left\|\int_0^\nu (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left[\mathfrak{n}(\zeta, \mathfrak{z}^n_{\rho(\tau,\mathfrak{z}_\tau)}) - \mathfrak{n}(\zeta, \mathfrak{z}_{\rho(\tau,\mathfrak{z}_\tau)})\right]d\omega(\zeta)\right\|^2 \\
&+ \quad 6\mathbb{E}\left\|\sum_{0<\nu_{\mathsf{k}}<\nu}\mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}\left[\mathfrak{z}^n(\nu_{\mathsf{k}}) - \mathfrak{z}(\nu_{\mathsf{k}})\right]\right\|^2 \\
&\leq \quad \Bigg[6\mathscr{M}^2\mathscr{M}_\mu + 6\mathscr{M}_\beta^2\mathscr{M}_{\mathfrak{l}} + \frac{6\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)\Gamma^2(\alpha)}\left[\mathscr{M}_{1-\beta}^2\mathscr{M}_{\mathfrak{l}} + \lambda^*\mathscr{M}_{\mathfrak{m}} + \mathscr{M}_{\mathfrak{n}}\right] \\
&+ \quad 6\mathscr{M}^2\mathscr{M}_{\mathsf{k}}\Bigg]\mathbb{E}\left\|\mathfrak{z}^n - \mathfrak{z}\right\|^2 \\
&\to \quad 0 \text{ as } n \to \infty.
\end{aligned}$$

Thus $\mathbb{E}\left\|(\Phi\mathfrak{z}^n)(\nu) - (\Phi\mathfrak{z})(\nu)\right\|^2 \to 0$ as $n \to \infty$ which implies Φ is continuous.

Step 3: We show that $\Phi(\mathfrak{B}_{\mathfrak{r}})$ is equicontinuous on $\mathscr{J}$. Let $\xi_{\mathsf{k}} \leq \nu_1 < \nu_2 < \xi_{\mathsf{k}+1}$, $\mathsf{k} = 0, 1, 2, \cdots$ and $\mathfrak{z} \in \mathfrak{B}_{\mathfrak{r}}$ then for $\mathfrak{z}_{\mathfrak{B}_{\mathfrak{r}}}$, we have

$$\begin{aligned}
\mathbb{E}\left\|(\Phi\mathfrak{z})(\nu_2) - (\Phi\mathfrak{z})(\nu_1)\right\|^2 \quad &\leq \quad 7\mathbb{E}\left\|\mathbb{U}(\nu_2-\nu_1)\left[\mathfrak{z}_0 - \mu(\mathfrak{z}) - \mathfrak{l}(0,\Phi)\right]\right\|^2 + 7\mathbb{E}\left\|\mathfrak{l}(\nu_2, \mathfrak{z}_{\nu_2}) - \mathfrak{l}(\nu_1, \mathfrak{z}_{\nu_1})\right\|^2 \\
&+ \quad 7\mathbb{E}\Bigg\|\int_0^{\nu_1}(\nu_1-\zeta)^{\alpha-1}\left[\mathscr{A}\mathbb{V}(\nu_2-\zeta) - \mathscr{A}\mathbb{V}(\nu_1-\zeta)\right]\mathfrak{l}(\zeta, \mathfrak{z}_\zeta)d\zeta \\
&+ \quad \int_0^{\nu_1}\left[(\nu_2-\zeta)^{\alpha-1} - (\nu_1-\zeta)^{\alpha-1}\right]\mathscr{A}\mathbb{V}(\nu_2-\zeta)\mathfrak{l}(\zeta, \mathfrak{z}_\zeta)d\zeta \\
&+ \quad \int_{\nu_1}^{\nu_2}(\nu_2-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu_2-\zeta)\mathfrak{l}(\zeta, \mathfrak{z}_\zeta)d\zeta\Bigg\|^2 \\
&+ \quad 7\mathbb{E}\Bigg\|\int_0^{\nu_1}(\nu_1-\zeta)^{\alpha-1}\left[\mathbb{V}(\nu_2-\zeta) - \mathbb{V}(\nu_1-\zeta)\right]\mathscr{B}\mathfrak{u}(\zeta)d\zeta \\
&+ \quad \int_0^{\nu_1}\left[(\nu_2-\zeta)^{\alpha-1} - (\nu_1-\zeta)^{\alpha-1}\right]\mathbb{V}(\nu_2-\zeta)\mathscr{B}\mathfrak{u}(\zeta)d\zeta \\
&+ \quad \int_{\nu_1}^{\nu_2}(\nu_2-\zeta)^{\alpha-1}\mathbb{V}(\nu_2-\zeta)\mathscr{B}\mathfrak{u}(\zeta)d\zeta\Bigg\|^2 \\
&+ \quad 7\mathbb{E}\Bigg\|\int_0^{\nu_1}(\nu_1-\zeta)^{\alpha-1}\left[\mathbb{V}(\nu_2-\zeta) - \mathbb{V}(\nu_1-\zeta)\right]\left(\int_0^\zeta \lambda(\zeta-\tau)\mathfrak{m}(\tau, \mathfrak{z}_\tau)d\tau\right)d\zeta \\
&+ \quad \int_0^{\nu_1}\left[(\nu_2-\zeta)^{\alpha-1} - (\nu_1-\zeta)^{\alpha-1}\right]\mathbb{V}(\nu_2-\zeta)\left(\int_0^\zeta \lambda(\zeta-\tau)\mathfrak{m}(\tau, \mathfrak{z}_\tau)d\tau\right)d\zeta \\
&+ \quad \int_{\nu_1}^{\nu_2}(\nu_2-\zeta)^{\alpha-1}\mathbb{V}(\nu_2-\zeta)\left(\int_0^\zeta \lambda(\zeta-\tau)\mathfrak{m}(\tau, \mathfrak{z}_\tau)d\tau\right)d\zeta\Bigg\|^2 \\
&+ \quad 7\mathbb{E}\Bigg\|\int_0^{\nu_1}(\nu_1-\zeta)^{\alpha-1}\left[\mathbb{V}(\nu_2-\zeta) - \mathbb{V}(\nu_1-\zeta)\right]\mathfrak{n}\left(\zeta, \mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta)
\end{aligned}$$

$$
\begin{aligned}
&+ \int_0^{\nu_1} \left[(\nu_2-\zeta)^{\alpha-1}-(\nu_1-\zeta)^{\alpha-1}\right]\mathbb{V}(\nu_2-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta)\\
&+ \int_{\nu_1}^{\nu_2}(\nu_2-\zeta)^{\alpha-1}\mathbb{V}(\nu_2-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta)\Bigg\|^2\\
&+ 7\mathbb{E}\left\|\sum_{0<\nu_{\mathsf{k}}<\nu}\left[\mathbb{U}(\nu_2-\nu_{\mathsf{k}})-\mathbb{U}(\nu_1-\nu_{\mathsf{k}})\right]\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}}))\right\|^2.
\end{aligned}
$$

By the continuity of $\mathbb{U}(\nu,\zeta)$ and $\mathbb{V}(\nu,\zeta)$ of the assumptions (H1), and by assuming the hypotheses (H1)-(H5), and Lebesgue dominated convergence theorem, as $\nu_2\to\nu_1$ on $\mathscr{J}$,

$$
\mathbb{E}\left\|(\Phi\mathfrak{z})(\nu_2)-(\Phi\mathfrak{z})(\nu_1)\right\|^2\to 0.
$$

This proves that $(\Phi\mathfrak{B}_{\mathfrak{r}})$ is equicontinuous on $\mathscr{J}$.

Step 4: We show that Mönch condition holds.

Let $\mathcal{B}=\overline{Co}\left(\{0\}\cup(\mathfrak{B}_{\mathfrak{r}})\right)$. For any $D\subset\mathcal{B}$, without loss of generality, we assume that $D=\{\mathfrak{z}^n\}_{n=1}^{\infty}$. It is obvious that, Φ maps D into itself and $D\subset\overline{Co}\left(\{0\}\cup(\mathfrak{B}_{\mathfrak{r}})\right)$ is equicontinuous in $\mathscr{J}$. Now we show that $\beta(D)=0$, where β is the HMNC.

Let us consider $\Psi=\Psi_1+\Psi_2+\Psi_3$, where

$$
\begin{aligned}
\Psi_1(\nu) &= \mathbb{U}(\nu)\left[\mathfrak{z}_0-\mu(\mathfrak{z})-\mathfrak{l}(0,\Phi)\right]+\mathfrak{l}(\nu,\mathfrak{z}(\nu))+\int_0^{\nu}(\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\mathfrak{l}(\zeta,\mathfrak{z}_\zeta)d\zeta,\\
\Psi_2(\nu) &= \int_0^{\nu}(\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\int_0^{\zeta}\lambda(\zeta-\tau)\mathfrak{m}(\tau,\mathfrak{z}_\tau)\right)d\zeta,\\
\Psi_3(\nu) &= \int_0^{\nu}(\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\omega(\zeta)+\sum_{0<\nu_k<1}\mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}})).
\end{aligned}
$$

By Lemma 2.1 and Lemma 2.7, we have

$$
\begin{aligned}
\beta\left(\{(\Psi_1\mathfrak{z}^n)(\nu)\}\right) &\leq \mathscr{M}_\beta^2\int_{\nu_0}^{\nu}\beta\left(\left\{\mathfrak{l}(\zeta,\mathfrak{z}_\zeta^n)\right\}_{n=1}^{\infty}\right)d\zeta\\
&\leq \mathscr{M}_\beta^2\int_{\nu_0}^{\nu}\mathbb{K}_{\mathfrak{l}}(\zeta)\sup_{-\tau\leq\theta\leq 0}\beta\left(\{\mathfrak{z}_\zeta^n(\theta)\}_{n=1}^{\infty}\right)d\zeta\\
&\leq \mathscr{M}_\beta^2\|\mathbb{K}_{\mathfrak{l}}\|_{\mathcal{L}^1(\mathscr{J},\mathbb{R}^+)}\sup_{\nu\in\mathscr{J}}\beta\left(\{\mathfrak{z}^n(\nu)\}_{n=1}^{\infty}\right)\\
\beta\left(\{(\Psi_2\mathfrak{z}^n)(\nu)\}\right) &\leq \frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\int_{\nu_0}^{\nu}\mathbb{K}_{\mathfrak{m}}(\zeta)\sup_{tau\leq\theta\leq 0}\beta\left(\left\{\mathfrak{z}_\zeta^n(\theta)\right\}_{n=1}^{\infty}\right)d\zeta\\
&\leq \frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\|\mathbb{K}_{\mathfrak{m}}\|_{\mathcal{L}^1(\mathscr{J},\mathbb{R}^+)}\sup_{\nu\in\mathscr{J}}\beta\left(\{\mathfrak{z}^n(\nu)\}_{n=1}^{\infty}\right)\\
\beta\left(\{(\Psi_3\mathfrak{z}^n)(\nu)\}\right) &\leq \frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\beta\left(\int_{\nu_0}^{\nu}\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_\zeta)}\right)d\zeta\right)\\
&\leq \frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\mathbb{K}_{\mathfrak{n}}^*\sup_{\nu\in\mathscr{J}}\beta\left(\{\mathfrak{z}^n(\nu)\}_{n=1}^{\infty}\right).
\end{aligned}
$$

Therefore,

$$\begin{aligned}\beta\left(\{(\Psi\mathfrak{z}^{n})(\nu)\}_{n=1}^{\infty}\right) &\leq \beta\left(\{(\Psi_1\mathfrak{z}^{n})(\nu)\}_{n=1}^{\infty}\right)+\beta\left(\{(\Psi_2\mathfrak{z}^{n})(\nu)\}_{n=1}^{\infty}\right)+\beta\left(\{(\Psi_3\mathfrak{z}^{n})(\nu)\}_{n=1}^{\infty}\right)\\ &\leq \left[\mathscr{M}_{\beta}^{2}\|\mathbb{K}_{\mathfrak{l}}\|_{\mathcal{L}^1(\mathscr{J},\mathbb{R}^+)}+\frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\|\mathbb{K}_{\mathfrak{m}}\|_{\mathcal{L}^1(\mathscr{J},\mathbb{R}^+)}+\frac{\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{2\alpha-1}\mathbb{K}_{\mathfrak{n}}^{*}\right]\sup_{\nu\in\mathscr{J}}\beta\left(\{\mathfrak{z}^{n}(\nu)\}_{n=1}^{\infty}\right)\\ &\leq H\sup_{\nu\in\mathscr{J}}\beta(D(\nu)).\end{aligned}$$

Therefore we have,

$$\beta(D)\leq\beta\left(\overline{Co}\left(\{0\}\cup\Phi(D)\right)\right)=\beta(\Phi(D))\leq H\sup_{\nu\in\mathscr{J}}\beta(D(\nu))=H\beta(D)<\beta(D),$$

which implies $\beta(D)=0$, the set D is a relatively compact set. Thus Φ has at least one fixed point $\mathfrak{z}$ in $\mathfrak{B}_{\mathfrak{r}}$. thus the proposed system (2.1) has at least one mild solution. This completes the proof.

□

4 Trajectory Controllability

The control system (2.1) is said to be trajectory controllable on $\mathscr{J}$ if for every $\varpi\in\mathfrak{T}$, such that the mild solution $\mathfrak{z}(\cdot)$ of (2.1) satisfies $\varpi(\nu)=\mathfrak{z}(\nu)$ almost everywhere.

Definition 4.1. *Let $\varpi(\nu)$ be the given trajectory on ν. The control system (2.1) is said to be T-controllable on l, if for every $\varpi\in\mathcal{V}$, such that the mild solution $\mathfrak{z}(.)$ of (5.2) satisfies $\varpi(\nu)=\mathfrak{z}(\nu)$ almost everywhere.*

By applying Gronwall's inequality, T-controllability of the system (2.1) gets satisfied.

Theorem 4.1. *If the hypotheses (H1)-(H6) hold, the aforementioned system (2.1) is T-controllable on $\mathscr{J}$.*

Proof. For $\beta\in(0,1)$, we consider the feedback control $\mathfrak{u}(\nu)$ for the prescribed trajectory $\varpi(\nu)$ on $\mathscr{J}$ as

$$\mathfrak{u}(\nu)=\mathscr{B}^{-1}\Big[{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[\varpi(\nu)-\mathfrak{l}(\nu,\varpi_{\nu})\right]-\Big[\mathscr{A}\varpi(\nu)-\int_0^{\nu}\lambda(\nu-\zeta)\mathfrak{m}(\zeta,\varpi_{\zeta})d\zeta\Big]d\nu-\mathfrak{n}\left(\nu,\varpi_{\rho(\nu,\varpi_{\nu})}\right)d\omega(\nu)\Big].\tag{4.1}$$

From (2.1),

$$\begin{aligned}{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[(\mathfrak{z}(\nu)-\varpi(\nu))-\left[\mathfrak{l}(\nu,\mathfrak{z}_{\nu})-\mathfrak{l}(\nu,\varpi_{\nu})\right]\right] &= \mathscr{A}\left[\mathfrak{z}(\nu)-\varpi(\nu)\right]+\int_0^{\nu}\lambda(\nu-\zeta)\left[\mathfrak{m}(\zeta,\nu_{\zeta})-\mathfrak{m}(\zeta,\varpi_{\zeta})\right]d\nu\\ &+ \left[\mathfrak{n}(\nu,\mathfrak{z}_{\rho(\nu,\mathfrak{z}_{\nu})})-\mathfrak{n}(\nu,\varpi_{\rho(\nu,\varpi_{\nu})})\right]d\omega(\nu).\end{aligned}$$

Let $\Psi(\nu) = \mathfrak{z}(\nu) - \varpi(\nu)$,

$$
\begin{aligned}
{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[\Psi(\nu) - [\mathfrak{l}(\nu,\mathfrak{z}_{\nu}) - \mathfrak{l}(\nu,\varpi_{\nu})]\right] &= \mathscr{A}\varpi(\nu) + \int_0^{\nu} \lambda(\nu-\zeta)\left[\mathfrak{m}(\zeta,\mathfrak{z}_{\zeta}) - \mathfrak{m}(\zeta,\varpi_{\zeta})\right] d\nu \\
&+ \left[\mathfrak{g}\left(\nu,\mathfrak{z}_{\rho(\nu,\mathfrak{z}_{\nu})}\right) - \mathfrak{g}\left(\nu,\varpi_{\rho(\nu,\varpi_{\nu})}\right)\right] d\omega(\nu) \\
\triangle\Psi &= \mathcal{I}_{\mathsf{k}}(\Psi(\nu_{\mathsf{k}})) \\
\Psi(0) + [\mu(\mathfrak{z}) - \mu(\varpi)] &= \mathfrak{z}_0 - \varpi_0 = 0.
\end{aligned}
$$

Therefore the mild solution becomes,

$$
\begin{aligned}
\Psi(\nu) \quad = \quad & \mathbb{U}(\nu)\left[\mu(\varpi) - \mu(\mathfrak{z})\right] + \left[\mathfrak{l}(\nu,\mathfrak{z}_{\nu}) - \mathfrak{l}(\nu,\varpi(\nu))\right] + \int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\left[\mathfrak{l}(\zeta,\mathfrak{z}_{\zeta}) - \mathfrak{l}(\zeta,\varpi_{\zeta})\right] d\zeta \\
+ \quad & \int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\int_0^{\zeta} \lambda(\nu-\tau)\left[\mathfrak{m}(\tau,\mathfrak{z}_{\tau}) - \mathfrak{m}(\tau,\varpi_{\tau})\right] d\tau\right) d\zeta + \int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta) \\
\times \quad & \left[\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_{\zeta})}\right) - \mathfrak{n}\left(\zeta,\varpi_{\rho(\zeta,\varpi_{\zeta})}\right)\right] d\omega(\zeta) + \sum_{0<\nu_{\mathsf{k}}<\nu} \mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\Psi(\nu_{\mathsf{k}})), \quad \nu \in \mathscr{J}.
\end{aligned}
$$

Hence the initial data is zero for $\nu \in \mathscr{J}$. Thus we obtain $\varrho(\nu) = 0$.

Hence, $\mathfrak{z}_{\nu} = \varrho_{\nu} + \mathfrak{z}_{\nu}$ and $\varpi_{\nu} = \varpi_{\nu} + \varrho_{\nu}$ on $\mathscr{J}$.

Therefore,

$$
\begin{aligned}
\mathbb{E}\|\Psi(\nu)\|^2 \quad \le \quad & 6\mathbb{E}\left\|\mathbb{U}(\nu)\left[\mu(\varpi) - \mu(\mathfrak{z})\right]\right\|^2 + 6\mathbb{E}\left\|\mathfrak{l}(\nu,\mathfrak{z}_{\nu}) - \mathfrak{l}(\nu,\varpi(\nu))\right\|^2 \\
+ \quad & 6\mathbb{E}\left\|\int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathscr{A}\mathbb{V}(\nu-\zeta)\left[\mathfrak{l}(\zeta,\mathfrak{z}_{\zeta}) - \mathfrak{l}(\zeta,\varpi_{\zeta})\right] d\zeta\right\|^2 \\
+ \quad & 6\mathbb{E}\left\|\int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left(\int_0^{\zeta} \lambda(\nu-\tau)\left[\mathfrak{m}(\tau,\mathfrak{z}_{\tau}) - \mathfrak{m}(\tau,\varpi_{\tau})\right] d\tau\right) d\zeta\right\|^2 \\
+ \quad & 6\mathbb{E}\left\|\int_0^{\nu} (\nu-\zeta)^{\alpha-1}\mathbb{V}(\nu-\zeta)\left[\mathfrak{n}\left(\zeta,\mathfrak{z}_{\rho(\zeta,\mathfrak{z}_{\zeta})}\right) - \mathfrak{n}\left(\zeta,\varpi_{\rho(\zeta,\varpi_{\zeta})}\right)\right] d\omega(\zeta)\right\|^2 \\
+ \quad & 6\mathbb{E}\left\|\sum_{0<\nu_{\mathsf{k}}<\nu} \mathbb{U}(\nu-\nu_{\mathsf{k}})\mathcal{I}_{\mathsf{k}}(\Psi(\nu_{\mathsf{k}}))\right\|^2 \\
\le \quad & \left[6\mathscr{M}^2\mathscr{M}_{\mu} + 6\mathscr{M}_{\beta}^2\mathscr{M}_{\mathfrak{l}} + 6\mathscr{M}^2\mathscr{M}_{\mathsf{k}}\right]\mathbb{E}\|\Psi(\nu)\|^2 + \Big[\frac{6\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)(\Gamma^2(\alpha))}\left[\mathscr{M}_{1-\beta}^2\mathscr{M}_{\mathfrak{l}} + \lambda^*\mathscr{M}_{\mathfrak{m}} + \mathscr{M}_{\mathfrak{n}}\right]\Big] \\
\times \quad & \int_0^{\nu} \mathbb{E}\|\Psi(\zeta)\|^2 d\zeta \\
\le \quad & \mathscr{C}^* \int_0^{\nu} \mathbb{E}\|\Psi(\zeta)\|^2 d\zeta,
\end{aligned}
$$

where

$$
\mathscr{C}^* = \frac{\frac{6\mathscr{M}^2\mathfrak{b}^{2\alpha-1}}{(2\alpha-1)(\Gamma^2(\alpha))}\left[\mathscr{M}_{1-\beta}^2\mathscr{M}_{\mathfrak{l}} + \lambda^*\mathscr{M}_{\mathfrak{m}} + \mathscr{M}_{\mathfrak{n}}\right]}{1 - \left[6\mathscr{M}^2\mathscr{M}_{\mu} + 6\mathscr{M}_{\beta}^2\mathscr{M}_{\mathfrak{l}} + 6\mathscr{M}^2\mathscr{M}_{\mathsf{k}}\right]} \int_0^{\nu} \mathbb{E}\|\Psi(\zeta)\|^2 d\zeta.
$$

By generalized Gronwall's inequality, $\mathbb{E}\|\Psi(\nu)\| = 0$ a.e. As a result, the system (2.1) is T-Controllable on $\mathscr{J}$.

□

5 Example

Consider the stochastic fractional integrodifferential equations with impulses and SDD of the from:

$$
\begin{aligned}
&{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[\mathfrak{z}(\nu,\upsilon)-\int_{-\infty}^{\nu}\int_{0}^{\pi} b(\nu-\zeta,\eta,\upsilon)\mathfrak{z}(\zeta,\eta)d\eta d\zeta\right]\\
&=\left[\frac{\partial^2}{\partial\upsilon^2}\mathfrak{z}(\nu,\upsilon)+\mu(\nu,\upsilon)+\int_0^{\nu} b(\nu-\zeta)\frac{\partial^2}{\partial\upsilon^2}\mathfrak{z}(\zeta,\upsilon)d\zeta\right]d\nu\\
&\quad+\left[\int_{-\infty}^{\nu} a(\zeta-\nu)\mathfrak{z}(\zeta-\rho_1(\nu)\rho(\|\mathfrak{z}(\nu)\|),\upsilon)d\zeta\right]d\beta(\nu),\ \nu\in l=[0,b],\\
\Delta\mathfrak{z}(\nu_k,\upsilon)&=\int_{-\infty}^{\nu_k}\mathcal{I}_k(\nu_k-\zeta)\mathfrak{z}(\zeta,\upsilon)d\upsilon,\ k=1,2,...,n,\\
\mathfrak{z}(\nu,0)&=\mathfrak{z}(\nu,\pi)=0,\ \mathfrak{z}(\nu,\upsilon)=\widetilde{\phi}(\nu,\upsilon),\ -a\le\nu\le 0,
\end{aligned}
\tag{5.1}
$$

where $\rho_1:[0,\infty)\to[0,\infty)$, $i=1,2,$. Here $a,b:\mathbb{R}\to\mathbb{R}$ are continuous, $0<\nu_1<\nu_2<\cdots<\nu_m<b$ are prefixed numbers. Let $\beta(\nu)\in\mathscr{H}=\mathcal{L}_2[0,\pi]$ described on $(\Omega,\Im,\mathbb{P})$ and $\widetilde{\phi}\in\mathfrak{B}$. Define $\mathscr{A}\zeta=\zeta$ involving $\mathfrak{D}(\mathscr{A})=\left\{\zeta\in\mathscr{H}:\zeta \text{ and } \frac{\partial}{\partial\mathfrak{z}}\zeta \text{ are absolutely continuous, } \frac{\partial^2}{\partial\mathfrak{z}^2}\zeta,\zeta(0)=\zeta(\pi)=0\right\}$, then $\mathscr{A}$ generators a strongly continuous semigroup $\mathcal{T}(\nu)$, $\nu\ge 0$ given by

$$
\mathcal{T}(\nu)\zeta=\sum_{n=1}^{\infty}e^{-n^2t}\langle\zeta,e_n\rangle e_n,\ \zeta\in\mathscr{H},
$$

and $e_n(\upsilon)=(2/\pi)^{\frac{1}{2}}\sin(n\nu)$, $n=1,2,...$, is orthogonal set of eigenvectors of $\mathscr{A}$. Also, $\mathscr{B}:\mathscr{U}\to\mathscr{H}$ denotes by $\mathscr{B}\mathfrak{u}(\nu)(\upsilon)=\mu(\nu,\upsilon)$, $0\le\upsilon\le\pi$, $\mathfrak{u}\in\mathscr{U}$, where $\mu:[0,1]\times[0,\pi]\to[0,\pi]$ is continuous. Define the operator $\mathfrak{l},\rho:\mathscr{J}\times\mathfrak{B}\to\mathscr{H}$, $\mathfrak{n}:\mathscr{J}\times\mathfrak{B}\to\mathcal{L}_{\mathscr{Q}}(\mathscr{K},\mathscr{H})$ and $\mathcal{I}_{\mathsf{k}}:\mathfrak{B}\to\mathscr{H}$ by

$$
\begin{aligned}
\mathfrak{l}(\nu,\phi)(\upsilon)&=\int_{-\infty}^{\nu}\int_0^{\pi} b(\nu-\zeta,\eta,\upsilon)\mathfrak{z}(\zeta,\eta)d\eta d\zeta,\\
\mathfrak{n}(\nu,\phi)(\upsilon)&=\int_{-\infty}^{\nu} a(\nu)\phi(\nu,\upsilon)d\zeta,\\
\rho(\nu,\phi)(\upsilon)&=\rho_1(\nu)\rho(\|\mathfrak{z}(\nu)\|),\\
\mathcal{I}_k(\phi)(\upsilon)&=\int_{-\infty}^{0}\mathcal{I}_k(-\zeta)\phi(\nu,\upsilon)d\upsilon,\ k=1,2,...,n.
\end{aligned}
$$

Based on above considerations, we can symbolize (5.1) in the abstract from

$$
\begin{aligned}
{}^{c}\mathcal{D}_{\nu}^{\alpha}\left[\mathfrak{z}(\nu)-\mathfrak{l}(\nu,\mathfrak{z}_{\nu})\right]&=\left[\mathscr{A}\mathfrak{z}(\nu)+\mathscr{B}\mathfrak{u}(\nu)+\int_0^{\nu}\lambda(\nu-\zeta)\mathfrak{m}(\zeta,\mathfrak{z}_{\zeta})d\zeta\right]d\nu+\mathfrak{n}\left(\nu,\mathfrak{z}_{\rho(\nu,\mathfrak{z}_{\nu})}\right)d\omega(\nu)\\
\triangle\mathfrak{z}|_{\nu=\nu_{\mathsf{k}}}&=\mathcal{I}_{\mathsf{k}}(\mathfrak{z}(\nu_{\mathsf{k}}))\\
\mathfrak{z}(0)+\mu(\mathfrak{z})&=\mathfrak{z}_0=\tilde{\phi}\in\mathfrak{B}.
\end{aligned}
\tag{5.2}
$$

Besides, $\mathfrak{l},\mathfrak{n},\mathcal{I}_{\mathsf{k}}$ are bounded linear operator, $\|\mathfrak{l}\|^2\le\mathscr{M}_{\mathfrak{l}}$, $\|\mathfrak{n}\|^2\le\mathscr{M}_{\mathfrak{n}}$ and $\|\mathcal{I}_{\mathsf{k}}\|^2\le\mathscr{M}_{\mathfrak{k}}$, for every $k=1,2,...,$. Then, all the assumptions given in Theorem 3.1 are true and we conclude that equation (5.1) has at least one mild solution on $\mathscr{J}$. In addition, $6\mathscr{M}^2\mathscr{M}_{\mu}+6\mathscr{M}_{\beta}^2\mathscr{M}_{\mathfrak{l}}+6\mathscr{M}^2\mathscr{M}_{\mathsf{k}}<1$. In the view of generalized Gronwall's inequality (Lemma 2.6 in [27]), we get $\mathbb{E}\|\Psi(\nu)\|=0$ a.e. and the hypotheses of Theorem 4.1 are fulfilled, so (5.1) is T-controllable on $\mathscr{J}$.

6 Conclusion

This paper is concerned with the T-controllability of fractional order neutral impulsive stochastic integrodifferential systems involving SDD and nonlocal conditions. The results are attained and the T-controllability is constructed and established by semigroup theory, fractional derivatives, fixed point approach and stochastic analysis techniques. To illustrate the significance of developed result, an example is included. Furthermore, the contribution of this paper can be extended to damped dynamical systems with different delay effects.

Declaration of competing interest:

The authors declare that they have no conflicts of interest.

Data availability:

No data was used for the research described in the article.

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