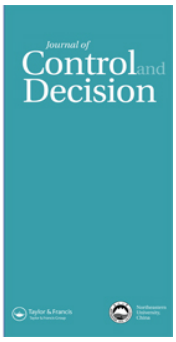
Optimal control of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps

Article *in* Journal of Control and Decision · August 2023

DOI: 10.1080/23307706.2022.2120554



Journal of Control and Decision

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjcd20

Optimal control of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps

K. Ramkumar, K. Ravikumar & E. M. Elsayed

To cite this article: K. Ramkumar, K. Ravikumar & E. M. Elsayed (2022): Optimal control of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps, Journal of Control and Decision, DOI: 10.1080/23307706.2022.2120554

To link to this article: https://doi.org/10.1080/23307706.2022.2120554

Published online: 23 Sep 2022.

JOURNAL OF CONTROL AND DECISION
https://doi.org/10.1080/23307706.2022.2120554

Taylor & Francis
Taylor & Francis Group

Check for updates

Optimal control of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps

K. Ramkumar[a], K. Ravikumar[b] and E. M. Elsayed[c,d]

[a]Department of Mathematics with CA, PSG College of Arts & Science, Coimbatore, India; [b]Department of Mathematics, PSG College of Arts & Science, Coimbatore, India; [c]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia; [d]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, Egypt

ABSTRACT

In this work, the optimal control for a class of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps has been discussed in infinite dimensional space involving the Hilfer fractional derivative. First, we study the existence and uniqueness of mild solution results are proved by the virtue of fractional calculus, successive approximation method and stochastic analysis techniques. Second, the optimal control of the proposed problem is presented by using Balder's theorem. Finally, an example is demonstrated to illustrate the obtained theoretical results.

ARTICLE HISTORY
Received 6 December 2020
Accepted 30 August 2022

KEYWORDS
Hilfer fractional derivative; stochastic integrodifferential systems; Rosenblatt process; Poisson jumps; successive approximation; optimal control

1. Introduction

In recent decades, the fractional differential equations (FDEs) have tremendous applications even though it is originated before seventieth century. FDEs are used in many real-life phenomena. Many mathematicians and scientists are attracted in the field of FDEs, because of their numerous applications in various fields of science and engineering. For more details on FDEs, the reader can refer the articles therein (Kilbas et al., 2006; Podlubny, 1999). FDEs are used as a main tool to model dynamical systems. Stochastic differential equations (SDEs) have attracted great interest from both theoretical and applied disciplines, which have been successfully applied to problems in physics, biology, chemistry and so on, see Mao (1997), Prato and Zabczyk (2014), Ahmed (2015), Cao et al. (2012), Balasubramaniam et al. (2015) and the references therein. The existence of mild solutions for SDEs and SFDEs has been extensively studied in the literature, see Balasubramaniam and Vinayagam (2005), Chadha and Pandey (2015), Anguraj and Ravikumar (2019), Balasubramaniam et al. (2018) and the references therein.

A physicist (Hilfer, 2000), Hilfer introduced a new type of fractional derivative, which includes both Riemann–Liouville and Caputo fractional derivatives later it is named as Hilfer fractional derivative. Hilfer fractional derivative performed in the theoretical simulation of dielectric relaxation in glass forming materials, polymer science, Rouse model, rheological constitute modelling, relaxation and diffusion models for biophysical phenomena. Based on the generalisations, Yang and Wang (2017) extended to study existence results to a class of Hilfer fractional evolution equations with nonlocal conditions. Very recently Gu and Trujillo (2015) obtained the sufficient conditions ensuring the existence of mild solution by using non-compact measure method and Ascolli–Arzela theorem. Ahmed and El-Borai (2018) studied the existence of mild solutions for a class of Hilfer fractional stochastic integrodifferential equations in Hilbert spaces via fixed point theorem.

Nowadays various real-life situations can be modelled by using Poisson jumps. Moreover, many practical applications are used in the field of market crashes, earthquakes, epidemics. For example, if a system jumps from a 'normal state' to 'a other state', the strength of systems is random. To make more realistic model, a jump term is included in any dynamical systems. The study of SDEs driven bt Poisson jumps has considerable attentions (Ahmed & Zhu, 2021; Anguraj & Ravikumar, 2019; Hausenblas, 2006; Luo & Taniguchi, 2009; Tamilalagan & Balasubramaniam, 2014). Recently, Tamilalagan and Balasubramaniam (2014) have investigated, respectively, stochastic fractional evolution inclusions. Very recently Rihan et al. (2017) extended to study the existence of fractional SDEs with Hilfer fractional derivative and Poisson jumps. Balasubramaniam et al. (2018) studied a class of Hilfer fractional fractional stochastic integrodifferential equations with Poisson jumps through the fixed point technique.

The parameter $\mathtt{H}$ characterises all the important properties of the process, when $\mathtt{H} < \frac{1}{2}$ the increments

CONTACT K. Ravikumar ravikumarkpsg@gmail.com Department of Mathematics with CA, PSG College of Arts & Science, Coimbatore 641 014, India

are negatively correlated and the correlation decays more slowly than quadratically; when $\mathrm{H} > \frac{1}{2}$, the increments are positively correlated and the correlation decays so slowly that they are not summable, a situation which is commonly known as the long memory property. Natural candidates are the Hermite processes, and these non-Gaussian stochastic processes appear as limits are called Non-Central Limit theorem. The fBm can be expressed as a Wiener integral with respect to the standard Wiener process, i.e. the integral of a deterministic kernel with respect to a standard Brownian motion, the Hermite process of order 1 is fBm and of order 2 is the Rosenblatt process. The fractional Brownian motion is utilised largely due to its self-similarity, stationary of increments and long-range dependence, for more details , see Biagini et al. (2008), Maslowski and Schmalfuss (2004), Han and Yan (2018), Tamilalagan and Balasubramaniam (2017), Ahmed and Wang (2018), Ahmed et al. (2019), and Ahmed et al. (2021). Tudor (2008) investigated the Rosenblatt process which is a self-similar process with stationary increments and it appears as limit of long-range dependent stationary series in the Non-Central Limit Theorem. Subsequently, Maejima and Tudor (2013) established some new properties of the Rosenblatt distribution. Recently, Shen and Ren (2015) obtained a set of conditions for the existence of solutions of neutral stochastic partial differential equations with Rosenblatt process in Hilbert spaces setting. For more details about the Rosenblatt process, one can refer the articles (Maejima & Tudor, 2013; Shen & Ren, 2015; Tamilalagan & Balasubramaniam, 2017; Tudor, 2008).

Frequently, the optimal control is largely applied to biomedicine, namely, to model the cancer chemotherapy, and recently applied to epidemiological models and medicine , see Urszula and Schattler (2007) and Ivan et al. (2018) and references therein. The main goal of optimal control is to find, in an open-loop control, the optimal values of the control variables for the dynamic system which maximise or minimise a given performance index. If a fractional differential equation describes the performance index and system dynamics, then an optimal control problem is known as a fractional optimal control problem. Using the fractional variational principle and lagrange multiplier technique, Agrawal (2004) discussed the general formulation and solution scheme for Riemann–Liouville fractional optimal control problems. It is remarkable the fixed point technique, which is used to establish the existence results for abstract fractional differential equations, could be extended to address the fractional optimal control problems. Recently, Aicha et al. (2018) studied the optimal controls of impulsive fractional system with Clarke subdifferential. Very recently, using the LeraySchauder fixed point theorem, Balasubramniam and Tamilalagan (2017) studied the solvability and optimal controls for impulsive fractional stochastic integrodifferential equations. Tamilalagan and Balasubramniam (2018) investigated the solvability and optimal controls for fractional stochastic differential equations driven by Poisson jumps in Hilbert space via analytic resolvent operators and Banach contraction mapping principle.

To the best of authors knowledge, up to now, no work has been reported to derive the existence results for Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps. The main contributions are summarised as follows:

(1) Stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps are formulated.
(2) Fractional calculus theory is effectively used to derive the existence and uniqueness results on mild solutions via successive approximation method.
(3) The existence of fractional optimal control for stochastic system is also discussed.
(4) An example is provided to illustrate the obtained theoretical results.

2. Preliminaries

In this section, we collect basic concepts, definitions and Lemmas which will be used in the sequel to obtain the main results.

Definition 2.1 (Ahmed & El-Borai, 2018): The Riemann–Liouville fractional integral operator of order $\beta > 0$ for a function f can be defined as

$$I_{0+}^{\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_0^t \frac{f(s)}{(t-s)^{1-\beta}} \mathrm{d}s, \quad t > 0$$

where $\Gamma(\cdot)$ is the Gamma function.

Definition 2.2 (Hilfer, 2000): The Hilfer fractional derivative of type $0 \le p \le 1$ and order $0 < q < 1$ is defined as

$$D_{0+}^{\alpha,\beta} f(t) = I_{0+}^{\alpha(1-\beta)} \frac{\mathrm{d}}{\mathrm{d}t} I_{0+}^{(1-\alpha)(1-\beta)} f(t)$$

In this article, the existence results will be derived for the following equation:

$$\begin{aligned} D_{0+}^{\alpha,\beta} u(t) &= Au(t) + f(t,u(t)) + \int_0^t g(s,u(s))\mathrm{d}Z_{\mathrm{H}}(s) \\ &\quad + \int_Z h(t,u(t),z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z), \quad t \in J' = (0,a], \\ &\quad t \neq t_k, \\ I_{0+}^{(1-\alpha)(1-\beta)} u(0) &= u_0. \end{aligned} \tag{1}$$

Here $D_{0+}^{\alpha,\beta}$ is the Hilfer fractional derivative of $0 \le \alpha \le 1$ and order $0 < \beta < 1$, and $J = [0,a]$. Let A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $\{T(t)\}$, $t \geq 0$

in Hilbert space $\mathcal{X}$ and $f: J \times \mathcal{X} \to \mathcal{X}$, $g: J \times \mathcal{X} \to \mathcal{L}(Q^{1/2}\mathcal{Y}, \mathcal{X})$ and $h: J \times \mathcal{X} \times Z \to \mathcal{X}$ are appropriate functions. In $\widetilde{N}(\mathrm{d}t, \mathrm{d}z) = N(\mathrm{d}t, \mathrm{d}z) - \mathrm{d}t(\lambda \mathrm{d}z)$ the Poisson measure $\widetilde{N}(\mathrm{d}t, \mathrm{d}z)$ denotes the Poisson counting measure.

Let $(\Omega, \Im, \mathbb{P})$ be a complete probability space furnished with complete family of right continuous increasing sub σ-algebras $\{\Im_t, t \in J\}$ satisfying $\Im_t \in \Im$. $\Im_t$ denotes the σ-field generated by $\{Z_{\mathrm{H}(s), s\in[0,t]}\}$ and the $\mathbb{P}$-null sets. $Z_{\mathrm{H}}(t)$ is a Rosenblatt process with parameter $\mathrm{H} \in (\frac{1}{2}, 1)$ on a real separable Hilbert space $\mathcal{Y}$. Let $\mathcal{L}(\mathcal{Y}, \mathcal{X})$ denotes the space of bounded linear operators from $\mathcal{Y}$ into $\mathcal{X}$, whenever $\mathcal{X} = \mathcal{Y}$, we simply denote $\mathcal{L}(\mathcal{Y})$. $Q \in \mathcal{Y}$ represents a non-negative self-adjoint operator. We introduce the subspace $\mathcal{Y}_0 = Q^{\frac{1}{2}}\mathcal{Y}$ of $\mathcal{Y}$ which is endowed with the inner product $< u, v >_{\mathcal{Y}_0} = < Q^{\frac{1}{2}}u, Q^{\frac{1}{2}}v >_{\mathcal{Y}}$ is a Hilbert space. Let $\mathcal{L}_2^0 = \mathcal{L}_2(\mathcal{Y}_0, \mathcal{X})$ be the space of all Hilbert operators from $\mathcal{Y}_0$ into $\mathcal{X}$, $\phi \in \mathcal{L}_2^0$ is called a Q-Hilbert-Schmidt operator, if $\|\phi\|^2_{\mathcal{L}_2^0} = \sum_{n=1}^{\infty} \|Q^{\frac{1}{2}} e_n \phi\|^2 < \infty$, and that the space $\mathcal{L}_2^0$ equipped with inner product $< \phi, \psi >_{\mathcal{L}_2^0} = \sum_{n=1}^{\infty} < \phi e_n, \psi e_n >$ is a separable Hilbert space. Also if $\phi = \psi$, then $\|\phi\|^2_{\mathcal{L}_2^0} = \|\phi Q^{\frac{1}{2}}\|^2 = Tr(\phi Q \phi^*)$. Let $\mathscr{C}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$ be the Banach space of all continuous maps from J into $\mathcal{L}_2(\Omega, \mathcal{X})$ satisfying $\sup_{0\le t\le \tau} \mathbf{E}\|u(t)\|^2 < \infty$. Define $\mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X})) = \{u \in \mathscr{C}((0, a], \mathcal{L}_2(\Omega, \mathcal{X})); \lim_{t\to 0^+} t^{(1-\alpha)(1-\beta)} u(t)$ exists and its finite$\}$, with the norm denoted by $\|u\|^2_{\alpha,\beta} = \sup_{0<t\le T} \|t^{(1-\alpha)(1-\beta)} u(t)\|^2$. Obviously, $\mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$ is a Banach space.

The Wiener–Ito multiple integral of order k with respect to the standard Brownian motion $(w(y))_{y\in\mathbb{R}}$ is given by

$$Z^k_{\mathrm{H}}(t) = \mathbb{C}(\mathrm{H}, k) \int_{\mathbb{R}^+} \left(\int_0^t \prod_{j=1}^{k} (s - y_j)_+^{-(\frac{1}{2} + \frac{1-\mathrm{H}}{k})} \mathrm{d}s \right) \times \mathrm{d}w(y_1) \cdots \mathrm{d}w(y_k) \tag{2}$$

where $x_+ = \max(x, 0)$ and the constant $\mathbb{C}(\mathrm{H}, k)$ is a normalising constant that ensures $\mathbf{E}(Z^k_{\mathrm{H}}(1))^2 = 1$. The process $(Z^k_{\mathrm{H}})_{t\ge 0}$ is called the Hermite process. For $k = 1$, the process given by (2) is the fractional Brownian motion with Hurst parameter $\mathrm{H} \in (\frac{1}{2}, 1)$, further the process is not Gaussian for $k = 2$. Moreover, for $k = 2$ the process given by (3) is called the Rosenblatt process. Consider a time interval $[0, a]$ with arbitrary fixed horizon b and $\{Z_{\mathrm{H}}(t), t \in [0, T]\}$, the one-dimensional Rosenblatt process with parameter $\mathrm{H} \in (\frac{1}{2}, 1)$, Z_{H} has the following integral:

$$Z_{\mathrm{H}}(t) = \mathbb{C}(\mathrm{H}) \int_0^t \int_0^t \left[\int_{y_1 \vee y_2}^t \frac{\partial K^{\mathrm{H}'}}{\partial v}(v, y_1) + \frac{\partial K^{\mathrm{H}'}}{\partial v}(v, y_2) \mathrm{d}v \right] \mathrm{d}w(y_1)\mathrm{d}w(y_2) \tag{3}$$

where $w = \{w(t) : t \in [0, T]\}$ is a Wiener process, $\mathrm{H}' = \frac{\mathrm{H}+1}{2}$ and $\mathbb{C}(\mathrm{H}) = \frac{\mathrm{H}+1}{2}\sqrt{\frac{\mathrm{H}}{2(2\mathrm{H}-1)}}$ is a normalising constant and $K^{\mathrm{H}}(t, s)$ is the kernal given by $K^{\mathrm{H}}(t, s) = \mathbb{C}_{\mathrm{H}} s^{\frac{1}{2}-\mathrm{H}} \int_s^t (v - s)^{\mathrm{H}-\frac{3}{2}} v^{\mathrm{H}-\frac{1}{2}} \mathrm{d}v$ for $t > s$, where $\mathbb{C}_{\mathrm{H}} = \sqrt{\frac{\mathrm{H}(2\mathrm{H}-1)}{\beta(2-2\mathrm{H}, \mathrm{H}-\frac{1}{2})}})$ and $\beta(\cdot, \cdot)$ denotes the Beta function. For basic preliminaries and fundamental results on Rosenblatt process, one can refer the articles therein (Balasubramaniam et al., 2018; Maejima & Tudor, 2013; Podlubny, 1999).

Lemma 2.3 (Tudor, 2008): *If $\phi : [0, a] \to \mathcal{L}_2^0$ satisfies $\int_0^T \|\phi(s)\|^2_{\mathcal{L}_2^0} < \infty$, then the sum* (3) *is well defined as a* X*-valued random variable and*

$$\mathbf{E}\left\| \int_0^t \phi(s)\mathrm{d}Z_Q(s) \right\|^2 \le \mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} \int_0^t \|\phi(s)\|^2_{\mathcal{L}_2^0} \mathrm{d}s.$$

Definition 2.4 (Rihan et al., 2017): An $\mathcal{X}$-valued stochastic process $\{u(t) \in \mathscr{C}(J', \mathcal{L}_2(\Omega, \mathcal{X}))\}$ is said to be a mild solution of the system (1) if the process u satisfies the following integral equation:

$$\begin{aligned} u(t) = {} & S_{\alpha,\beta}(t)u_0 + \int_0^t P_\beta(t - s) \\ & \times \left[f(s, u(s)) + \int_0^s g(r, u(r))\mathrm{d}Z_{\mathrm{H}}(r) \right] \mathrm{d}s \\ & + \int_0^t \int_{\mathcal{U}} P_\beta(t - s)h(s, u(s), z)\widetilde{N}(\mathrm{d}s, \mathrm{d}z), \\ & t \in J' \end{aligned} \tag{4}$$

where

$$S_{\alpha,\beta}(t) = I_{0^+}^{\alpha(1-\beta)} P_\beta(t),$$
$$P_\beta(t) = t^{\beta-1} T_\beta(t),$$
$$T_\beta(t) = \int_0^\infty \beta\theta\psi_\beta(\theta)T(t^\beta\theta)\mathrm{d}\theta,$$

here

$$\psi_\beta(\theta) = \sum_{n=1}^{\infty} \frac{(-\theta^{n-1})}{(n-1)!\Gamma(1 - n\beta)sin(n\pi\alpha)}$$
$$\theta \in (0, \infty)$$

is a function of Wright-type defined on $(0, \infty)$ and

$$\int_0^\infty \theta^\zeta \psi_\beta(\theta)\mathrm{d}\theta = \frac{\Gamma(1 + \zeta)}{\Gamma(1 + \beta\zeta)}, \quad \zeta \in (-1, \infty) \quad \text{and}$$

$\|T(t)\|^2 \le M$.

Lemma 2.5 (Gu & Trujillo, 2015): *The properties of the operators $S_{\alpha,\beta}$ and P_β are given by*

(i) *For any fixed $t > 0$, $S_{\alpha,\beta}$ and P_β are linear and bounded operators, and*

$$\left\|P_\beta(t)x\right\|^2 \le \frac{Mt^{2(\beta-1)}}{\Gamma(\beta)^2} \|x\|^2,$$

$$\left\|S_{\alpha,\beta}(t)x\right\|^2 \le \frac{Mt^{2(\alpha-1)(\beta-1)}}{\Gamma(\alpha(1-\beta)+\beta)^2}\left\|x\right\|^2.$$

(ii) $\{P_\beta(t) : t > 0\}$ *is compact, if* $\{T(t) : t > 0\}$ *is compact.*

3. Existence and uniqueness of mild solution

In this section, we establish the existence and uniqueness of mild solutions for system (1) before starting and proving our main results, we introduce the following hypotheses:

(H1) The maps $f : J \times \mathcal{X} \to \mathcal{X}, g : J \times \mathcal{X} \to \mathcal{L}(Q^{1/2}\mathcal{Y}, \mathcal{X})$ and $h : J \times \mathcal{X} \times Z \to \mathcal{X}$ satisfy, for all $t \in J$ and $u, v \in \mathcal{X}$,

$$\begin{aligned}
&\left\|f(t,u)-f(t,v)\right\|^2 \le K\left(\|u-v\|^2\right),\\
&\quad\times\int_0^s \left\|g(t,u)-g(t,v)\right\|^2 \le K\left(\|u-v\|^2\right),\\
&\quad\times\int_Z \|h(s,u,z)-h(s,v,z)\|^2\,\lambda(\mathrm{d}v)\\
&\vee\left(\int_Z \|h(s,u,z)-h(s,v,z)\|^4\,\lambda(\mathrm{d}v)\right)^{1/2}\\
&\le K\left(\|u-v\|^2\right),\\
&\quad\times\left(\int_Z \|h(s,u,z)-h(s,v,z)\|^4\,\lambda(\mathrm{d}v)\right)^{1/2}\\
&\le K\left(\|x\|^2\right),
\end{aligned}$$

where K is a concave nondecreasing function from $\mathbb{R}_+$ to $\mathbb{R}_+$ such that $K(0) = 0, K(\theta) > 0$ for $\theta > 0$ and $\int_{0^+} \frac{\mathrm{d}\theta}{K(\theta)} = +\infty$.

(H2) For all $t \in J$, there exists a constant k_0 such that

$$\left\|f(t,0)\right\|^2 \vee \int_0^s \left\|g(r,0)\right\|^2 \mathrm{d}r \vee \int_Z \|h(t,0,z)\|^2\,\lambda(\mathrm{d}v) \le k_0.$$

Let the solution $u(t) \in \mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$ of (1) be defined as follows:

$$\begin{aligned}
&\lim_{t\to 0^+} t^{(1-\alpha)(1-\beta)}S_{\alpha,\beta}(t)u_0\\
&= \frac{1}{\Gamma(\alpha(1-\beta))\Gamma(\beta)}\\
&\quad\times\int_0^1 (1-s)^{\alpha(1-\beta)-1}s^{\beta-1}u_0\mathrm{d}s\\
&= \frac{u_0}{\Gamma((\alpha(1-\beta)+\beta))},
\end{aligned}$$

$t^{(1-\alpha)(1-\beta)}u(t)$

$$= \begin{cases}
\dfrac{u_0}{\Gamma((\alpha(1-\beta)+\beta))}, \quad t=0,\\
t^{(1-\alpha)(1-\beta)}S_{\alpha,\beta}(t)u_0 + t^{(1-\alpha)(1-\beta)}\\
\displaystyle\int_0^t P_\beta(t-s)[f(s,u(s))\\
\displaystyle+\int_0^s g(r,u(r))\mathrm{d}Z_{\mathrm{H}}(r)]\mathrm{d}s\\
\displaystyle+t^{(1-\alpha)(1-\beta)}\int_0^t\int_Z\\
P_\beta(t-s)h(s,u(s),z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z),\\
0 < t \le a.
\end{cases} \tag{5}$$

Let us consider the sequence of successive approximations defined as follows:

$$\begin{aligned}
u^0(t) &= t^{(1-\alpha)(1-\beta)}S_{\alpha,\beta}(t)u_0, \quad 0 < t \le a,\\
u^n(t) &= \frac{u_0}{\Gamma((\alpha(1-\beta)+\beta))}, \quad t=0, \quad n=1,2,\ldots,\\
u^n(t) &= t^{(1-\alpha)(1-\beta)}S_{\alpha,\beta}(t)u_0\\
&\quad+t^{(1-\alpha)(1-\beta)}\int_0^t P_\beta(t-s)[f(s,u^{n-1}(s))\\
&\quad+\int_0^s g(r,u^{n-1}(r))\mathrm{d}Z_{\mathrm{H}}(r)]\mathrm{d}s\\
&\quad+t^{(1-\alpha)(1-\beta)}\int_0^t\int_Z P_\beta(t-s)\\
&\quad\times h(s,u^{n-1}(s),z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z),\\
&\quad 0 < t \le a,\ n=1,2,\ldots
\end{aligned} \tag{6}$$

Theorem 3.1: *If the assumptions* (H1)–(H2) *are satisfied, then system* (1) *has a unique mild solution in the space* $\mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$, *provided that*

$$\frac{3M}{(\Gamma(\beta))^2}t^{2(1-\alpha)(1-\beta)}(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C) < 1,$$
$$\frac{1}{2} < \beta < 1\ t \in J.$$

The proof of this theorem is given in Appendix 1.

4. Optimal control

Let $\mathcal{K}$ be reflexive Banach space in which controls $\mathbf{u}$ take values. Let us denote a class of nonempty convex and closed subsets of $\mathcal{K}$ by $2^{\mathcal{K}}/\{\emptyset\}$. The multivalued function $\nu : J \to 2^{\mathcal{K}}/\{\emptyset\}$ is measurable and $\nu(\cdot) \subset \zeta$, where ζ is a bounded set of $\mathcal{K}$. The admissible control set $U_{ad} = \{\mathbf{u}(\cdot) \in \mathcal{L}_2(\zeta)|\mathbf{u}(t) \in \nu(t), a.e.\}$. Then $U_{ad} \neq \emptyset$ and $U_{ad} \subset \mathcal{L}_2(J, \mathcal{K})$ is bounded, closed and convex. Consider the Hilfer fractional stochastic control problem

$$D_{0^+}^{\alpha,\beta}x(t) = Au(t) + B(t)\mathbf{u}(t) + f(t, x(t)) + \int_0^t g(s, x(s))\mathrm{d}Z_{\mathrm{H}}(s)$$

$$+\int_Z h(t,x(t),z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z),$$
$$t\in J'=(0,a],\quad t\neq t_k,$$
$$I_{0+}^{(1-\alpha)(1-\beta)}x(0)=x_0. \tag{7}$$

To prove the existence of optimal control pair of system (7), Let us define the performance index

$$l(x,\mathbf{u})=\int_0^a \mathcal{L}(t,x^n(t),\mathbf{u})\mathrm{d}t,$$

among all the admissible state control pair of system (7), that is, find an admissible state control pair $(x^0,\mathbf{u}^0)\in\mathscr{C}(J,\mathcal{L}_2(\Omega,\mathcal{X}))\times U_{ad}$ such that $l(x^0,\mathbf{u}^0)\leq l(x,\mathbf{u})$, for all $\mathbf{u}\in U_{ad}$, here $x^n(t)$ defines the mild solution of (7) corresponding to $\mathbf{u}\in U_{ad}$. For existence of admissible control of (7), we need the following hypotheses:

(H3) The operator $B\in\mathcal{L}_2(J,\mathcal{L}(\mathcal{K},\mathcal{X}))$; $\|B\|_2$ denotes the norm of operator B in Banach space $\mathcal{L}_2(J,\mathcal{L}(\mathcal{K},\mathcal{X}))$. And also, $B\mathbf{u}\in\mathcal{L}_2(J,\mathcal{X})$ for every $\mathbf{u}\in U_{ad}$.

(H4) The cost functional $l:J\times\mathcal{X}\times\mathcal{K}\to\mathbb{R}\cup\{\infty\}$ such that
- (1) The integrand $l:[0,a]\times\mathcal{X}\times\mathcal{K}\to\mathbb{R}\cup\{\infty\}$ is $\Im_t$-measurable.
- (2) The integrand $l(t,\cdot,\cdot)$ is sequentially lower semicontinuous on $\mathcal{X}\times\mathcal{K}$ for almost all $t\in[0,a]$.
- (3) The integrand $l(t,x,\cdot)$ is convex on $\mathcal{K}$ for each $x\in\mathcal{X}$ and almost all $t\in[0,a]$.
- (4) There exist constants $d\geq 0$, $e>0$, μ_0 is non negative and $\mu_0\in\mathcal{L}^1([0,a];\mathbb{R})$ such that

$$\mu_0(t)+d\mathbf{E}\,\|x\|^2+e\mathbf{E}\,\|\mathbf{u}\|_{\mathrm{U}}^p\leq l(t,x,\mathbf{u}).$$

Lemma 4.1: *Let* (H1)–(H4) *hold. Then the system* (7) *is mildly solvable on J with respect to $\mathbf{u}\in U_{ad}$ and $\frac{1}{2}<\beta<1$, then there exists a constant $\rho>0$ such that $\mathbf{E}\|\mathbf{u}(t)\|^2\leq\rho$ for all $t\in J$.*

Proof: If $\mathbf{u}$ is a mild solution of (7) with respect to $\mathbf{u}\in U_{ad}$, then u satisfies Equation (A3). Applying (H1)–(H4) and Lemma 2.3, we obtain

$$\begin{aligned}
&\mathbf{E}\,\|\mathbf{u}(t)\|^2\\
&\leq\frac{5Ma^{2(1-\alpha)(1-\beta)}}{(\Gamma(\alpha(1-\beta)+\beta))^2}\|x_0\|^2\\
&\quad+\frac{5Ma\,\|B\|^2}{(\Gamma(\beta))^2}\left(\int_0^t(t-s)^{2(\beta-1)(p/(p-1))\mathrm{d}s}\right)^{(p-1)/p}\\
&\quad\times\left(\int_0^t\|u(s)\|^{2p}\,\mathrm{d}s\right)^{1/p}\\
&\quad+\frac{5Mk_0(a+Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1}+C)}{(\Gamma(\beta))^2}\frac{a^{2\beta-1}}{2\beta-1}\\
&\quad+\frac{5Mk_0(a+Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1}+2C)}{(\Gamma(\beta))^2}\\
&\quad\times\int_0^t(t-s)^{2(\beta-1)}K(\mathbf{E}\,\|u(s)\|^2)\mathrm{d}s,
\end{aligned}$$

where $Q_6=(5Ma^{2(\alpha-1)(1-\beta)}/(\Gamma(\alpha(1-\beta)+\beta))^2)\|x_0\|^2+(5Ma\|B\|^2/(\Gamma(\beta))^2)(a^{(2\beta p-p-1)/(p-1)}/((2\beta p-p-1)/(p-1)))^{(p-1)/p}\|\mathbf{u}\|^2_{\mathcal{L}_2(J,\mathcal{K})}+(5Mk_0(a+Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1}+C)/(\Gamma(\beta))^2)(a^{2\beta-1}/(2\beta-1))$ and $Q_7=5M(a+Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1}+2C)/(\Gamma(\beta))^2$. Here, $K(\cdot)$ is concave and $K(0)=0$, we can find a pair of positive constants a_1 and b_1 such that $K(t)\leq a_1+b_1t, t\geq 0$.

$$\mathbf{E}\,\|x(t)\|^2\leq Q_6+Q_7a_1\frac{a^{2\beta-1}}{2\beta-1}+Q_7b_1\int_0^t(t-s)^{2(\beta-1)}\mathbf{E}\,\|x(s)\|^2\,\mathrm{d}s. \tag{8}$$

By using Gronwall's inequality, we have

$$\mathbf{E}\,\|x(t)\|^2\leq\left[Q_6+Q_7a_1\frac{a^{2\beta-1}}{2\beta-1}\right]\times\exp\left(Q_7b_1\frac{a^{2\beta-1}}{2\beta-1}\right)=\rho<\infty. \tag{9}$$

■

Theorem 4.2: *If the hypotheses* (H1)–(H4) *are satisfied, for every $\mathbf{u}\in U_{ad}$, then there exists a unique mild solution of system* (7) *of the form*

$$\begin{aligned}
x(t)&=S_{\alpha,\beta}(t)x_0+\int_0^t P_\beta(t-s)\left[(s)\mathbf{u}(s)+f(s,x(s))\right.\\
&\quad\left.+\int_0^s g(r,x(r))\mathrm{d}Z_{\mathrm{H}}(r)\right]\mathrm{d}s\\
&\quad+\int_0^t\int_{\mathcal{U}}P_\beta(t-s)h(s,x(s),z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z),\qquad t\in J'
\end{aligned} \tag{10}$$

The proof of this Theorem is similar to that of Theorem 3.1, and one can easily prove that solution of system (7) by using the successive approximation, and hence, it is omitted.

Theorem 4.3: *Assume* (H1)–(H4) *and Theorem* 4.2 *and B as a strongly continuous operator, then the stochastic control problem* (7) *permits at least one optimal control pair.*

The reader can refer the proof of this Theorem in Appendix 2.

5. Example

In this section, we provide an example to verify the theoretical results. Consider the control problem

$$D_{0+}^{\alpha,3/4}[x(t,\xi)]=\frac{\partial^2}{\partial\xi^2}[x(t,\xi)]+B(t)\mathbf{u}(t)\xi$$

$$
\begin{aligned}
&+\frac{e^{-t}x(t,\xi)}{(1+e^{t})(1+x(t,\xi))}\\
&+\int_0^t \frac{sinx(t,\xi)}{t^{1/3}}dZ_{\mathrm{H}}(s)\\
&+\int_Z (1+e^{-t})\cos x(t,\xi,z)\widetilde{N}(ds,dz),\\
&0\le \xi\le\pi,\ u\in U_{ad},\\
&x(t,0)=x(t,\pi)=0,\quad t>0\\
&I^{(1-\alpha)(1/4)}x(0)=x_0,\quad 0<\xi<\pi,\quad 0\le t\le a.
\end{aligned} \tag{11}
$$

where $D_{0+}^{\alpha,3/4}$ is the Hilfer fractional derivative, $0\le\alpha\le 1$, $\beta=3/4$, $a>0$. Let $\{Z_{\mathrm{H}}(t):t\in J\}$ is the Rosenblatt process with parameter $\mathrm{H}\in(\frac{1}{2},1)$. Let $\mathcal{X}=\mathcal{L}_2([0,\pi])$. The operator $A:\mathcal{X}\to\mathcal{X}$ is defined by $Ax=x''$ with the domain $\mathscr{D}(A)=\{x\in\mathcal{X}:x,x'$ absolutely continuous, $x''\in\mathcal{X},x(0)=x(\pi)=0\}$. Then $Ax=\sum_{\infty}^{n=1}-n^2\langle x,x_n\rangle_{x_n}$, $x\in\mathscr{D}(A)$, where $x_n(\xi)=\sqrt{\frac{2}{\pi}}\sin(n\xi)$, $n\in\mathbb{N}$, is the orthogonal set of eigenvectors of A. It is well known that A generates a compact semigroup $(T(t))_{t\ge0}$ in $\mathcal{X}$ and is given by $T(t)x=\sum_{n=1}^{\infty}e^{-n^2t}<x,x_n>x_n$, $x\in\mathcal{X}$. Moreover, for any $x\in\mathcal{X}$, we have $l_{3/4}(t)=\frac{3}{4}\int_0^\infty\theta\Psi_{3/4}(\theta)T(t^{3/4}\theta)d\theta$, $l_{3/4}(t)x=\sum_{n=1}^{\infty}\int_0^\infty\theta\Psi_{3/4}(\theta)\exp(-n^2t^{3/4}\theta)d\theta<x,x_n>x_n$. The nonlinear functions $f:J\times\mathcal{X}\to\mathcal{X}$, $g:J\times\mathcal{X}\to\mathcal{L}(Q^{1/2}\mathcal{Y},\mathcal{X})$, $h:J\times\mathcal{X}\times Z\to\mathcal{X}$ are defined by

$$
\begin{aligned}
f(x)(\xi)&=\frac{e^{-t}x(t,\xi)}{(1+e^{t})(1+x(t,\xi))},\\
g(x)(\xi)&=\frac{sinx(t,\xi)}{t^{1/3}},\\
h(x)(\xi)&=(1+e^{-t})\cos x(t,\xi,z)
\end{aligned}
$$

and assuming that $\int_Z v^2\lambda(dv)<\infty$, $\int_Z v^4\lambda(dv)<\infty$. Clearly the functions f, g and h satisfy the $(H1)-(H2)$. If $B=0$, then problem (11) takes the following abstract form:

$$
\begin{aligned}
D_{0+}^{\alpha,\beta}u(t)&=Au(t)+f(t,u(t))+\int_0^t g(s,u(s))dZ_{\mathrm{H}}(s)\\
&+\int_Z h(t,u(t),z)\widetilde{N}(ds,dz),\\
I_{0+}^{(1-\alpha)(1-\beta)}u(0)&=u_0.
\end{aligned} \tag{12}
$$

All the conditions stated in Theorem 3.1 are satisfied for system (11) can be applied to ensure the existence and uniqueness of the mild solution of (11). The controls are the functions $\mathbf{u}:Tx([0,\pi])\to\mathbb{R}$, such that $\mathbf{u}\in\mathcal{L}_2(Tx([0,\pi]))$. It $U(t)=\{\mathbf{u}\in\mathcal{K}:\|\mathbf{u}\|_{\mathcal{K}}\le\tau_1\}$, where $\tau_1\mathcal{L}_2(J,\mathbb{R}^+)$. We restrict the admissible controls U_{ad} to all $u\in\mathcal{L}_2(Tx([0,\pi]))$ such that $\|u(.,t)\|^2\le\tau_1(t)$ almost everywhere. Let us define $B(t)\mathbf{u}(t)x=\int_{[0,\pi]}k_1(x,\gamma)\mathbf{u}(\gamma,t)d\gamma$ and make the following assumptions:

(i) k_1 is continuous
(ii) $\mathbf{u}\in\mathcal{L}_2(Tx([0,\pi]))$ and $l:J\times\mathcal{X}\times\mathcal{K}\to\mathbb{R}\cup\{\infty\}$ is defined by

$$
\begin{aligned}
l&=(t,x^n(t),\\
\mathbf{u}(t)&=\int_{[0,\pi]}\left(\|x(t)(\xi)\|^2+\|\mathbf{u}(t)(\xi)\|^2\right)d\xi.
\end{aligned}
$$

Then the system (11) can be written as in the form of (7). All the conditions stated in Theorem 4.2 are verified. Therefore, there exists an admissible control pair $(x,\mathbf{u})$ such that the associated cost functional

$$
l(x,\mathbf{u})=\int_0^a\mathcal{L}(t,x^n(t),\mathbf{u}(t))dt,
$$

attains its minimum.

6. Conclusion

In this manuscript, the problem for optimal control of Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps has been addressed. Some innovative sufficient conditions are derived by using successive approximation technique. Furthermore, the existence of optimal control of the proposed problem is presented by using Balder's theorem. At last, an example has been established to validate the obtained theoretical results. In future, optimal control problem for Hilfer fractional stochastic integrodifferential systems is with Sobolev type.

Remark 6.1: The technique used here can be extended to investigate the optimal control for a class of nonlocal impulsive Hilfer fractional stochastic integrodifferential systems driven by Rosenblatt process and Poisson jumps of the form

$$
\begin{aligned}
D_{0+}^{\alpha,\beta}x(t)&=Au(t)+B(t)\mathbf{u}(t)+f(t,x(t))\\
&+\int_0^t g(s,x(s))dZ_{\mathrm{H}}(s)\\
&+\int_Z h(t,x(t),z)\widetilde{N}(dt,dz),\quad t\in J'=(0,a],\\
&t\ne t_k,\\
I_{t_i^+}^{(1-\alpha)(1-\beta)}u(t_i^+)&=u(t_i^-)+I_i(u(t_i^-)),\\
&i=1,2,\dots,m.\\
I_{0+}^{(1-\alpha)(1-\beta)}u(0)+p(u)&=u_0.
\end{aligned} \tag{13}
$$

Let the functions f, g, h are defined as in Equation (1) and also define $p:J\to\mathcal{X}$, $0=t_0<t_1<t_2\cdots<t_m<t_{m+1}=b$, $I_i:\mathcal{X}\to\mathcal{X}$ are impulsive functions which characterise the jump of the solutions at impulse point t_i. Hence, all the hypotheses of Theorem 4.3 are satisfied, then the stochastic control problem (13) permits at least one optimal control pair.

Acknowledgments

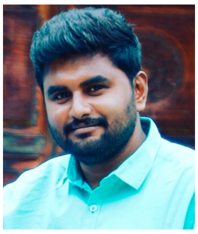
The authors would like to thank the helpful comments and suggestions of the Associate Editor and the anonymous reviewers, which have improved the presentation of this paper.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributors

K. Ramkumar received M.Sc. degree from Ramakrishna Mission, Vivekanandha College, Madras University, and Ph.D. from PSG College of Arts and Science, Coimbatore, Bharathiar University, India. Currently working as an assistant professor at PSG College of Arts and Science, Coimbatore. The research interest includes control theory, stochastic differential systems.



K. Ravikumar received M.Sc. degree from Ramakrishna Mission, Vivekanandha College, Madras University, and Ph.D. from PSG College of Arts and Science, Coimbatore, Bharathiar University, India. Currently working as an assistant professor at PSG College of Arts and Science, Coimbatore. The research interest includes control theory, stochastic differential/integro-differential systems.

E. M. Elsayed received Ph.D. degree in pure Mathematics from Faculty of Science, Mansoura University, Egypt, in 2006. Currently working as a professor of Mathematics, Mathematics Department, Faculty of Science, King Abdulaziz University, Kingdom of Saudi Arabia. The research interest includes Difference Equations including Local stability, Global stability, Semi-cycles, Oscillation, Periodicity; Differential Equations.

References

Agrawal, O. P. (2004). A general formulation and solution scheme for fractional optimal control problems. *Nonlinear Dynamics*, *38*(1–4), 323–337. https://doi.org/10.1007/s11071-004-3764-6

Ahmed, H. M. (2015). Semilinear neutral fractional stochastic integrodifferential equations with nonlocal conditions. *Journal of Theoretical Probability*, *28*(2), 667–680. https://doi.org/10.1007/s10959-013-0520-1

Ahmed, H. M., & El-Borai, M. M. (2018). Hilfer fractional stochastic integrodifferential equations. *Applied Mathematics and Computation*, *331*, 182–189. https://doi.org/10.1016/j.amc.2018.03.009

Ahmed, H. M., El-Borai, M. M., & Ramadan, M. E. (2019). Boundary controllability of nonlocal Hilfer fractional stochastic differential systems with fractional Brownian motion and Poisson jumps. *Advances in Difference Equations*, *2019*(1), 1–23. https://doi.org/10.1186/s13662-019-2028-1

Ahmed, H. M., El-Borai, M. M., & Ramadan, M. E. (2021). Noninstantaneous impulsive and nonlocal Hilfer fractional stochastic integrodifferential equations with fractional Brownian motion and Poisson jumps. *International Journal of Nonlinear Sciences and Numerical Simulation*, *22*(7–8), 927–942. https://doi.org/10.1515/ijnsns-2019-0274

Ahmed, H. M., & Wang, J. (2018). Exact null controllability of Sobolev-type Hilfer fractional stochastic differential equations with fractional Brownian motion and Poisson jumps. *Bulletin of the Iranian Mathematical Society*, *44*(3), 673–690. https://doi.org/10.1007/s41980-018-0043-8

Ahmed, H. M., & Zhu, Q. (2021). The averaging principle of Hilfer fractional stochastic delay differential equations with Poisson jumps. *Applied Mathematics Letters*, *112*, 106755. https://doi.org/10.1016/j.aml.2020.106755

Aicha, H., Nieto, J. J., & Amar, D. (2018). Solvability and optimal controls of impulsive Hilfer fractional delay evolution inclusions with Clarke subdifferential. *Journal of Computational and Applied Mathematics*, *344*, 725–737. https://doi.org/10.1016/j.cam.2018.05.031

Anguraj, A., & Ravikumar, K. (2019). Existence and stability results for impulsive stochastic functional integrodifferential equations with Poisson jumps. *Journal of Applied Nonlinear Dynamics*, *8*(3), 407–417. https://doi.org/10.5890/JAND.2019.09.005

Balasubramaniam, P., Kumaresan, N., Ratnavelu, K., & Tamilalagan, P. (2015). Local and global existence of mild solution for impulsive fractional stochastic differential equations. *Bulletin of the Malaysian Mathematical Sciences Society*, *38*(2), 867–884. https://doi.org/10.1007/s40840-014-0054-4

Balasubramaniam, P., Saravanakumar, S., & Ratnavelu, K. (2018). Study a class of Hilfer fractional stochastic integrodifferential equations with Poisson jumps. *Stochastic Analysis and Applications*, *36*(6), 1–16. https://doi.org/10.1080/07362994.2018.1524303

Balasubramniam, P., & Tamilalagan, P. (2017). The solvability and optimal controls for impulsive fractional stochastic integrodifferential equations via resolvent operators. *Journal of Optimization Theory and Applications*, *174*(1), 139–155. https://doi.org/10.1007/s10957-016-0865-6

Balasubramaniam, P., & Vinayagam, D. (2005). Existence of solutions of nonlinear neutral stochastic differential inclusions in Hilbert space. *Stochastic Analysis and Applications*, *23*(1), 137–151. https://doi.org/10.1081/SAP-200044463

Biagini, F., Hu, Y., Oksendal, B., & Zhang, T. (2008). *Stochastic calculus for fractional Brownian motion and applications*. Springer Science & Business Media.

Cao, J., Yang, Q., & Huang, Z. (2012). On almost periodic mild solutions for stochastic functional differential equations. *Nonlinear Analysis, Theory, Methods & Applications*, *13*(1), 275–286.

Chadha, A., & Pandey, D. N. (2015). Existence results for an impulsive neutral stochastic fractional integrodifferential equation with infinite delay. *Nonlinear Analysis*, *128*, 149–175. https://doi.org/10.1016/j.na.2015.07.018

Gu, H., & Trujillo, J. J. (2015). Existence of mild solution for evolution equation with Hilfer fractional derivative. *Applied Mathematics and Computation*, *257*, 344–354. https://doi.org/10.1016/j.amc.2014.10.083

Han, J., & Yan, L. (2018). Controllability of a stochastic functional differential equation driven by a fractional Brownian motion. *Advances in Difference Equations*, *104*(1). https://doi.org/10.1186/s13662-018-1565-3

Hausenblas, E. (2006). SPDEs driven by Poisson random measure with non Lipschitz coefficients:existence

results. *Probability Theory and Related Fields*, *137*(1–2), 161–200.
Hilfer, R. (Ed.) (2000). *Applications of fractional calculus in physics*. World Scientific.
Ivan, A., Nieto, J. J., Silva, C. J., & Torres, D. F. (2018). Ebola model and optimal control with vaccination constraints. *Journal of Industrial & Management Optimization*, *14*(2), 427–446. https://doi.org/10.3934/jimo.2017054
Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). Theory and applications of fractional differential equations. *Elsevier Science*, *204*,
Luo, J., & Taniguchi, T. (2009). The existence and uniqueness for non-Lipschitz stochastic neutral delay evolution equations driven by Poisson jumps. *Stochastics and Dynamics*, *9*(1), 135–152. https://doi.org/10.1142/S0219493709002592
Maejima, M., & Tudor, C. A. (2013). On the distribution of the Rosenblatt process. *Statistics & Probability Letters*, *83*(6), 1490–1495. https://doi.org/10.1016/j.spl.2013.02.019
Mao, X. (1997). *Stochastic differential equations and applications*. Horwood.
Maslowski, B., & Schmalfuss, B. (2004). Random dynamical systems and stationary solutions of differential equations driven by the fractional Brownian motion. *Stochastic Analysis and Applications*, *22*(6), 1577–1607. https://doi.org/10.1081/SAP-200029498
Podlubny, I. (1999). *Fractional differential equations*. Academic Press.
Prato, G. D., & Zabczyk, J. (2014). *Stochastic equations in infinite dimensions*. Cambridge University Press.
Rihan, F. A., Rajivganthi, C., & Muthukumar, P. (2017). Fractional stochastic differential equations with Hilfer fractional derivative: Poisson jumps and optimal control. *Discrete Dynamics in Nature and Society*, 1–11. https://doi.org/10.1155/2017/5394528
Shen, G. J., & Ren, Y. (2015). Neutral stochastic partial differential equations with delay driven by Rosenblatt process in a Hilbert space. *Journal of the Korean Statistical Society*, *44*(1), 123–133. https://doi.org/10.1016/j.jkss.2014.06.002
Tamilalagan, P., & Balasubramaniam, P. (2014). Existence results for semilinear fractional stochastic evolution inclusions driven by Poisson jumps. In P. N. Agrawal, R, N. Mohapatra, U. Singh & H. M. Srivastava (Eds.), *Mathematical analysis and its applications*, Springer Proceedings in Mathematics and Statistics, *143* (pp. 477–487). Springer India.
Tamilalagan, P., & Balasubramaniam, P. (2017). Approximate controllability of fractional stochastic differential equations driven by mixed fractional Brownian motion via resolvent operator. *International Journal of Control*, *90*(8), 1713–1727. https://doi.org/10.1080/00207179.2016.1219070
Tamilalagan, P., & Balasubramniam, P. (2018). The solvability and optimal controls for fractional stochastic differential equations driven by Poisson jumps via resolvent operators. *Applied Mathematics and Optimization*, *77*(3), 443–462. https://doi.org/10.1007/s00245-016-9380-2
Tudor, C. A. (2008). Analysis of the Rosenblatt process. *ESAIM: Probability and Statistics*, *12*, 230–257. https://doi.org/10.1051/ps:2007037
Urszula, L., & Schattler, H. (2007). Antiangiogenic therapy in cancer treatment as an optimal control problem. *SIAM Journal on Control and Optimization*, *46*(3), 1052–1079. https://doi.org/10.1137/060665294
Yang, M., & Wang, Q. (2017). Existence of mild solutions for a class of Hilfer fractional evolution equations with nonlocal conditions. *Fractional Calculus & Applied Analysis*, *20*(3), 679–705. https://doi.org/10.1515/fca-2017-0036

Appendices

Appendix 1

The proof will be given in the following steps:

Proof: **Step 1**: For all $t \in J$, the sequence $u^n(t)$, $n \geq 1$, is bounded. Let $u^0(t) \in \mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$. Let u^0 be a fixed initial approximation to (6). Using (H1)–(H2), Holder's inequality, DM-inequality, we have

$$\begin{aligned}
&\mathbf{E}\left\|u^n(t)\right\|^2 \\
&\leq \frac{4M\|u_0\|^2}{\Gamma((\alpha(1-\beta)+\beta))^2} + \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2} \\
&\quad \times \left(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + C\right) k_0 \frac{a^{2\beta}-1}{2\beta-1} \\
&\quad + \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2}\left(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C\right) \\
&\quad \times \int_0^t (t-s)^{2(\beta-1)} K\left(\mathbf{E}\left\|u^{n-1}(s)\right\|^2\right) \mathrm{d}s.
\end{aligned}$$

where $Q_1 = \frac{4M\|u_0\|^2}{\Gamma((\alpha(1-\beta)+\beta))^2} + \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2}(a + Tr(Q)\mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} + C)k_0\frac{a^{2\beta}-1}{2\beta-1}$ and $C > 0$ is constant. Here, $K(\cdot)$ is concave and $K(0) = 0$, we can find a pair of positive constants a_1 and b_1 such that $K(t) \leq a_1 + b_1 t, t \geq 0$.

$$\begin{aligned}
\mathbf{E}\left\|u^n(t)\right\|^2 \leq Q_2 &+ \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2} \\
&\times \left(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C\right) b_1 \\
&\times \int_0^t (t-s)^{2(\beta-1)} \left(\mathbf{E}\left\|u^{n-1}(s)\right\|^2\right) \mathrm{d}s,
\end{aligned}$$

where $Q_2 = Q_1 + \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2}(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C)a_1 \frac{a^{2\beta-1}}{2\beta-1}$. For any $k \geq 1$,

$$\begin{aligned}
&\max_{1<n<k} \mathbf{E} \sup_{0\leq s\leq t} \left\|u^{n-1}(s)\right\|^2 \\
&\leq \mathbf{E}\left\|u^0(s)\right\|^2 + \max_{1<n<k} \mathbf{E} \sup_{0\leq s\leq t} \left\|u^n(s)\right\|^2, \\
&\leq Q_3 \exp\left(\frac{Q_4 a^{2\beta-1}}{2\beta-1}\right),
\end{aligned}$$

where $Q_3 = Q_2 + \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2}(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C)b_1 \int_0^t (t-s)^{2(\beta-1)}\mathbf{E}\|u^0(s)\|^2 \mathrm{d}s < \infty$ and $Q_4 = \frac{8Mt^{2(1-\alpha)(1-\beta)}}{(\Gamma(\beta))^2}(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C)b_1$. Thus

$$\mathbf{E}\left\|u^n(t)\right\|^2 < \infty, \quad n \geq 1, \quad t \in J,$$

which shows that the sequence $u^n(t)$, $n \geq 1$, is bounded in $\mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$.

Step 2: Next show that the sequence u^n, $n \geq 1$, is a Cauchy sequence. From (6), for all $n \geq 1$ and $0 \leq t \leq a$,

$$\begin{aligned}
\mathbf{E}\left\|u^{n+1}(t) - u^n(t)\right\|^2 \leq \frac{3M}{(\Gamma(\beta))^2} t^{2(1-\alpha)(1-\beta)} \\
\times \left(a + Tr(Q)\mathbb{C}_{\mathrm{H}}t^{2\mathrm{H}-1} + 2C\right)
\end{aligned}$$

$$\times \int_0^t (t-s)^{2(\beta-1)} K \times \left(\mathbf{E} \left\| u^n(s) - u^{n-1}(s) \right\|^2 \right) \mathrm{d}s.$$

Let $\Psi_n(t) = \sup_{t\in[0,a]} \mathbf{E}\|u^{n+1}(t) - u^n(t)\|$. Then, we have the above inequality that

$$\Psi_n(t) \le \frac{3M}{(\Gamma(\beta))^2} t^{2(1-\alpha)(1-\beta)} (a + Tr(Q)\mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} + 2C) \times \int_0^t (t-s)^{2(\beta-1)} K\,(\Psi_{n-1}(s))\,\mathrm{d}s, \tag{A1}$$

Choose $a_1 \in [0, a]$ such that

$$Q_5 \int_0^t (t-s)^{2(\beta-1)} K\,(\Psi_{n-1}(s))\,\mathrm{d}s \le Q_5 \int_0^t (t-s)^{2(\beta-1)} \Psi_{n-1}(s)\mathrm{d}s, \quad 0 \le t \le a. \tag{A2}$$

Moreover,

$$\mathbf{E} \left\| u^1(t) - u^0(t) \right\|^2 \le \frac{3M}{(\Gamma(\beta))^2} t^{2(1-\alpha)(1-\beta)} (a + Tr(Q)\mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} + 2C) \times \int_0^t (t-s)^{2(\beta-1)} K \left(\mathbf{E} \left\| u^0(s) \right\|^2 \right) \mathrm{d}s. \tag{A3}$$

If we take the supreme over t, and use Ψ_n

$$\Psi_0(t) = \mathbf{E} \left\| u^1(t) - u^0(t) \right\|^2 + 2C) \times \int_0^t (t-s)^{2(\beta-1)} K \left(\mathbf{E} \left\| u^0(s) \right\|^2 \right) \mathrm{d}s = C_1. \tag{A4}$$

Now, for $n = 1$ in (A1), we have

$$\Psi_1(t) \le Q_5 \int_0^t (t-s)^{2(\beta-1)} C_1 \mathrm{d}s \le Q_5 C_1 \frac{a^2(\beta-1)+1}{2(\beta-1)+1}. \tag{A5}$$

And, for $n = 1$ in (A1), we have

$$\Psi_2(t) \le (Q_5)^2 C_1 \frac{a^2(\beta-1)+1}{2(\beta-1)+1} \frac{a^2}{2!}. \tag{A6}$$

By applying mathematical induction in (A1) and with the above work, we have

$$\Psi_2(t) \le (Q_5)^n C_1 \frac{a^2(\beta-1)+1}{2(\beta-1)+1} \frac{a^n}{n!}, \quad n \ge 1, \quad t \in [0, a_1]. \tag{A7}$$

So, for any $m \ge n \ge 0$,

$$\sup_{t\in[0,a_1]} \mathbf{E} \left\| u^m(t) - u^n(t) \right\|^2 \le \sum_{k=nt\in[0,a_1]}^{\infty} \sup_{t\in[0,a_1]} \mathbf{E} \left\| u^{k+1}(t) - u^k(t) \right\|^2$$

$$\le \sum_{k=n}^{\infty} C_1 \frac{a^2(\beta-1)+1}{2(\beta-1)+1} (Q_5)^n \frac{a^k}{k!} \to 0, \quad \text{as} \quad n \to \infty. \tag{A8}$$

Step 3: The existence and uniqueness of solution for systems (1) are tackled as follows. The Borel–Cantelli Lemma says that $u^n(t) \to u(t)$, as $n \to \infty$ uniformly for $0 \le t \le a$. Thus for all $t \in J$, taking limits on both sides of (6), one can obtain that $u(t)$ is a solution to (1). Next, to show the uniqueness, Let $u_1, u_2 \in \mathscr{C}^{\alpha,\beta}(J, \mathcal{L}_2(\Omega, \mathcal{X}))$ be two solutions on $t \in J$.

$$\mathbf{E}\,\|u_1(t) - u_2(t)\|^2 \le \frac{3M}{(\Gamma(\beta))^2} t^{2(1-\alpha)(1-\beta)} (a + Tr(Q)\mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} + 2C) \times \int_0^t (t-s)^{2(\beta-1)} K\,(\mathbf{E}\,\|u(s) - v(s)\|)\,\mathrm{d}s.$$

Thus, from Bihari inequality, yields that

$$\mathbf{E}\,\|u_1(t) - u_2(t)\|^2 = 0.$$

Therefore, $u_1(t) - u_2(t)$, for all $t \in J$. This complete the proof. ∎

Appendix 2

Proof: If $\inf_{x,\mathbf{u}\in\mathscr{A}_{ad}} l(x, \mathbf{u}) = +\infty$. Then there is nothing to prove. Assume that $\inf_{x,\mathbf{u}\in\mathscr{A}_{ad}} l(x, \mathbf{u}) = \epsilon < \infty$. Using $(H4)$, we have $\epsilon > -\infty$. By definition of infimum, there exists a minimising sequence feasible pair $\{(x^n, \mathbf{u}^n)\}_{n\ge1}$, such that $l(x^n, \mathbf{u}^n) \to \epsilon$ as $n \to +\infty$. Since $\mathbf{u}^n \in U_{ad}$, $\{\mathbf{u}^n\}_{n\ge1} \subset \mathcal{L}_2(J, \mathcal{K})$ is bounded. Thus there exists $\mathbf{u}^0 \in \mathcal{L}_2(J, \mathcal{K})$ and a subsequence extracted from $\mathbf{u}^n$ such that $\mathbf{u}^n \to \mathbf{u}^0$ weakly in $\mathcal{L}_2(J, \mathcal{K})$. Since U_{ad} is closed and convex, the Mazur lemma forces us to conclude that $\mathbf{u}^0 \in U_{ad}$. Suppose that x^m and x^0 are the mild solutions of (7) corresponding to $\mathbf{u}^n$ and $\mathbf{u}^0$, respectively. Form the boundedness of $\mathbf{u}^m$ and $\mathbf{u}^0$, Lemma 4.1, one can verify that there exists a positive number ρ such that $\|x^m\|, \|x^0\| \le \rho$. Then, for $t \in J$, $(p+)/2p < \beta < 1$.

$$\mathbf{E} \left\| x^m(t) - x^0(t) \right\|^2 \le \frac{4M}{(\Gamma(\beta))^2} (a + Tr(Q)\mathbb{C}_{\mathrm{H}} t^{2\mathrm{H}-1} + 2C) \times \int_0^t (t-s)^{2(\beta-1)} K(\mathbf{E} \left\| x^m(s) - x^m(0) \right\|^2)\mathrm{d}s$$

$$+ \frac{4M}{(\Gamma(\beta))^2} \left((p-1) \frac{a^{(2\beta p - p - 1)/(p-1)}}{2\beta p - p - 1} \right)^{(p-1)/p} \times \left(\int_0^a \mathbf{E} \left\| B(s)\mathbf{u}^m(s) - B(s)\mathbf{u}^0(s) \right\|^{2p} \mathrm{d}s \right)^{1/p}$$

Using the continuous operator B and Lebesgue's dominated convergence theorem, we have

$$\mathbf{E} \left\| x^m(t) - x^0(t) \right\|^2 \to 0 \quad \text{as} \quad n \to \infty. \tag{A9}$$

Thus, by $(H4)$ and Balder's theorem, we obtain

$$l(x^0, \mathbf{u}^0) \le \lim_{n\to\infty} l(x^m, \mathbf{u}^n) = m \tag{A10}$$

This shows that l attains its minimum at $\mathbf{u}^0 \in U_{ad}$. ∎